TRIGONOMETRY WITH TABLES

PHILLIPS AND STRONG

Florian Cajori
Dec. 19, 1899.

PHILLIPS-LOOMIS MATHEMATICAL SERIES

ELEMENTS OF TRIGONOMETRY

PLANE AND SPHERICAL

BY

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PREFACE

In this work the trigonometric functions are defined as ratios, but their representation by lines is also introduced at the beginning, because certain parts of the subject can be treated more simply by the line method, or by a combination of the two methods, than by the ratio method alone.

Attention is called to the following features of the book:

The simplicity and directness of the treatment of both the Plane and Spherical Trigonometry.

The emphasis given to the formulas essential to the solution of triangles.

The large number of exercises.

The graphical representation of the trigonometric, inverse trigonometric, and hyperbolic functions.

The use of photo-engravings of models in the Spherical Trigonometry.

The recognition of the rigorous ideas of modern mathematics in dealing with the fundamental series of trigonometry.

The natural treatment of the complex number and the hyperbolic functions.

The graphical solution of spherical triangles.

Our grateful acknowledgments are due to our colleague, Professor James Pierpont, for valuable suggestions regarding the construction of Chapter VI.

We are also indebted to Dr. George T. Sellew for making the collection of miscellaneous exercises.

Andrew W. Phillips,
Wendell M. Strong.

Yale University, *December, 1898.*

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PLANE TRIGONOMETRY

CHAPTER I

THE TRIGONOMETRIC FUNCTIONS

ANGLES

1. In Trigonometry the size of an angle is measured by the amount one side of the angle has revolved from the position of the other side to reach its final position.

Thus, if the hand of a clock makes one-fourth of a revolution, the angle through which it turns is one right angle; if it makes one-half a revolution, the angle is two right angles; if one revolution, the angle is four right angles; if one and one-half revolutions, the angle is six right angles, etc.

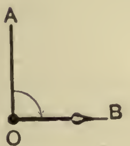


FIG. 1

FIG. 2

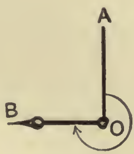


FIG. 3

FIG. 4

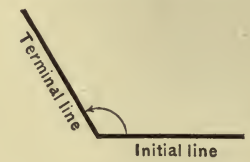
The amount the side *OB* has rotated from *OA* to reach its final position may or may not be equal to the inclination of the lines. In Fig. 1 it is equal to this inclination; in Fig. 4 it is not.

Two angles may have the same sides and yet be different. In Fig. 2

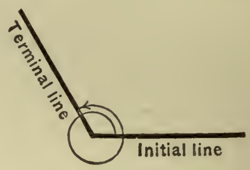
and Fig. 4 the positions of the sides of the angles are the same; yet in Fig. 2 the angle is two right angles, in Fig. 4 it is six right angles. The addition of any number of *complete* revolutions to an angle does not change the position of its sides.

Question.—Through how many right angles does the hour-hand of a clock revolve in $6\frac{1}{2}$ hours? the minute-hand?

Question.—If the fly-wheel of an engine makes 100 revolutions per minute, through how many right angles does it revolve in 1 second?



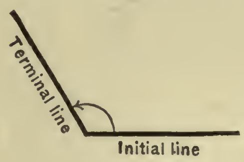
$1\frac{1}{2}$ RIGHT ANGLES



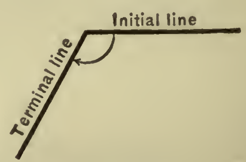
$5\frac{1}{2}$ RIGHT ANGLES

Def.—The first side of the angle—that is, the side from which the revolution is measured—is the **initial line**; the second side is the **terminal line.**

Def.—If the direction of the revolution is *opposite* to that of the hands of a clock, the angle is **positive**; if the *same* as that of the hands of a clock, the angle is **negative.**



POSITIVE ANGLE



NEGATIVE ANGLE

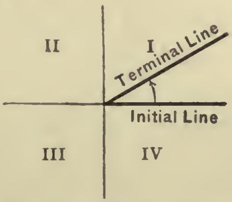
The angles we have employed as illustrations—those described by the hands of a clock—are all negative angles.

2. Angles are usually measured in degrees, minutes, and seconds. A degree is one-ninetieth of a right angle, a minute is one-sixtieth of a degree, a second is one-sixtieth of a minute.

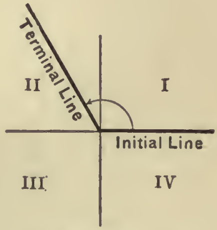
The symbols indicating degrees, minutes, and seconds are ° ′ ″; thus, twenty-six degrees, forty-three minutes, and ten seconds is written 26° 43′ 10″.

3. The plane about the vertex of an angle is divided into four quadrants, as shown in the figure; the first quadrant begins at the initial line.

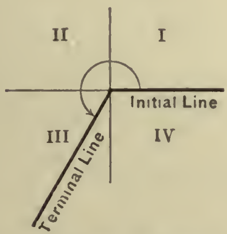
THE FOUR QUADRANTS



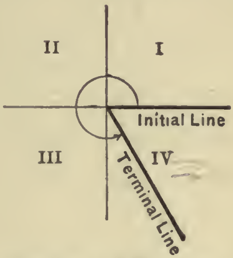
ANGLE IN 1ST QUADRANT



ANGLE IN 2D QUADRANT



ANGLE IN 3D QUADRANT



ANGLE IN 4TH QUADRANT

An angle is said to be in a certain quadrant if its terminal line is in that quadrant.

EXERCISES

4. (1.) Express $2\frac{1}{2}$ right angles in degrees, minutes, and seconds. In what quadrant is the angle?

(2.) What angle less than 360° has the same initial and terminal lines as an angle of 745°?

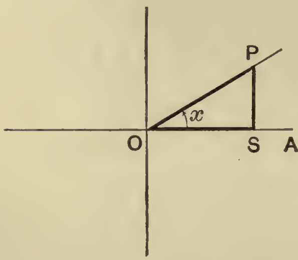
(3.) What positive angles less than 720° have the same sides as an angle of —73°?

(4.) In what quadrant is an angle of —890°?

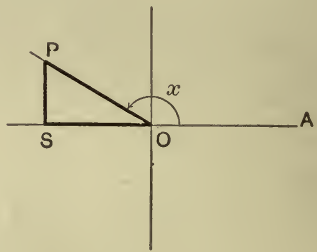
DEFINITIONS OF THE TRIGONOMETRIC FUNCTIONS

5. The trigonometric functions are *numbers*, and are defined as the ratios of lines.

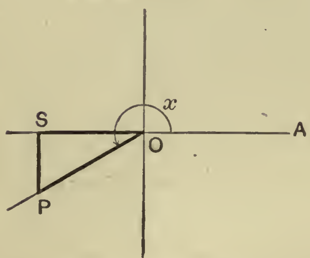
Let the angle *AOP* be so placed that the initial line is horizontal, and from *P*, any point of the terminal line, draw *PS* perpendicular to the initial line.



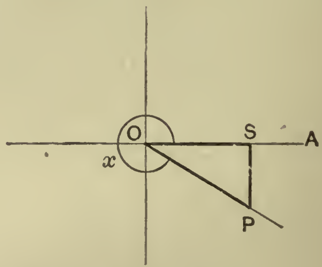
ANGLE IN THE 1ST QUADRANT



ANGLE IN THE 2D QUADRANT



ANGLE IN THE 3D QUADRANT



ANGLE IN THE 4TH QUADRANT

Denote the angle *AOP* by x.

$$\frac{SP}{OP} = \textbf{sine} \text{ of } x \text{ (written } \sin x\text{).}$$

$$\frac{OS}{OP} = \textbf{cosine} \text{ of } x \text{ (written } \cos x\text{).}$$

$$\frac{SP}{OS} = \textbf{tangent of } x \text{ (written } \tan x).$$

$$\frac{OS}{SP} = \textbf{cotangent of } x \text{ (written } \cot x).$$

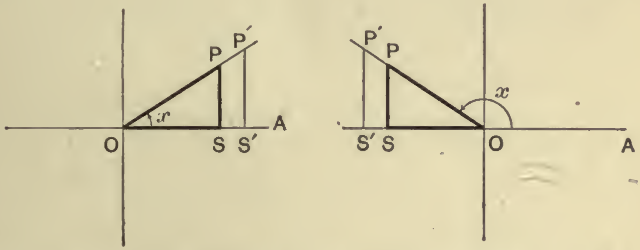
$$\frac{OP}{OS} = \textbf{secant of } x \text{ (written } \sec x).$$

$$\frac{OP}{SP} = \textbf{cosecant of } x \text{ (written } \csc x).$$

To the above may be added the versed sine (written versin) and coversed sine (written coversin), which are defined as follows:

$$\textbf{versin } x = 1 - \cos x; \quad \textbf{coversin } x = 1 - \sin x.$$

The values of the sine, cosine, etc., do not depend upon what point of the terminal line is taken as P, but upon the angle.



For the triangles OSP and $OS'P'$ being similar, the ratio of any two sides of $OS'P'$ is equal to the ratio of the corresponding sides of OSP.

Def.—The sine, cosine, tangent, cotangent, secant, and cosecant of an angle are the **trigonometric functions** of the angle, and depend for their value on the angle alone.

6. A line may by its length and direction represent a number; the *magnitude* of the number is expressed by the *length* of the line; the number is *positive* or *negative* according to the *direction* of the line.

7. In § 5, if the denominators of the several ratios be taken equal to unity, *the trigonometric functions will be represented by lines.*

Thus, $\sin x = \dfrac{SP}{OP} = \dfrac{SP}{1} = SP$ = the number represented by the line, that is, the ratio of the line to its unit of length.

Hence SP may represent the sine of x.

In a similar manner the other trigonometric functions may be represented by lines.

In the following figures a circle of unit radius is described about the vertex O of the angle AOP, this angle being denoted by x. Then from § 5 it follows that

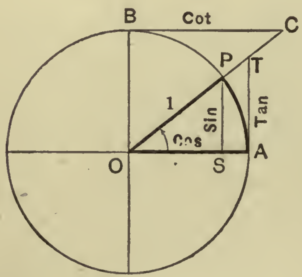


FIG. 1

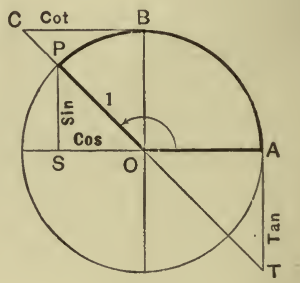


FIG. 2

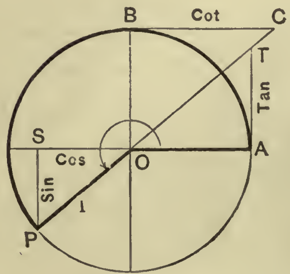


FIG. 3

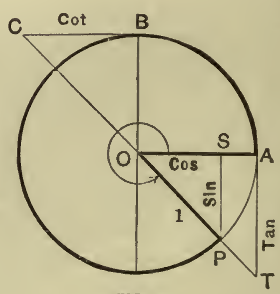


FIG 4

SP represents the **sine** of x.
OS represents the **cosine** of x.
AT represents the **tangent** of x.
BC represents the **cotangent** of x.
OT represents the **secant** of x.
OC represents the **cosecant** of x.

For the sake of brevity, the lines SP, OS, etc., of the preceding figures are often spoken of as the sine, cosine, etc.

Hence, we may also define the trigonometric functions in general terms as follows:

If a circle of unit radius is described about the vertex of an angle,

(1.) The **sine** of the angle is represented by the perpendicular upon the initial line from the intersection of the terminal line with the circumference.

(2.) The **cosine** of the angle is represented by the segment of the initial line extending from the vertex to the sine.

(3.) The **tangent** of the angle is represented by a line tangent to the circle at the beginning of the first quadrant, and extending from the point of tangency to the terminal line.

(4.) The **cotangent** of the angle is represented by a line tangent to the circle at the beginning of the second quadrant, and extending from the point of tangency to the terminal line.

(5.) The **secant** of the angle is represented by the segment of the terminal line extending from the vertex to the tangent.

(6.) The **cosecant** of the angle is represented by the segment of the terminal line extending from the vertex to the cotangent.

The definitions in § 5 are called the *ratio definitions* of the trigonometric functions, and those in § 7 the *line definitions*. The introduction of two definitions for the same thing should not embarrass the student. We have shown that they are equivalent. In some cases it is convenient to use the first definition, and in other cases the second, as the student will observe in the course of this study. It is therefore important that he should become familiar with the use of both.

SIGNS OF THE TRIGONOMETRIC FUNCTIONS

8. Lines are regarded as positive or negative according to their directions. Thus, in the figures of § 5, OS is *positive* if it extends to the *right* of O along the initial line, *negative* if it extends to the *left*; SP is *positive* if it extends *upward* from OA, *negative* if it extends *downward*. OP, the terminal line, is always *positive*.

The above determines, from § 5, the *signs* of the trigonometric functions, since it shows the signs of the two terms of each ratio.

By the line definitions the *signs* may be determined directly. The *sine* and *tangent* are *positive* if measured *upward* from OA, and *negative* if measured *downward*.

The *cosine* and *cotangent* are *positive* if measured to the *right* from OB, and *negative* if measured to the *left*.

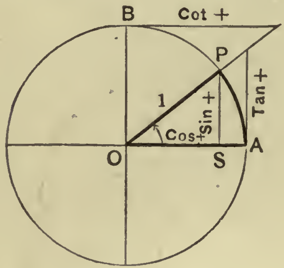


FIG. 1

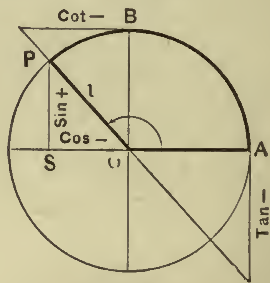


FIG. 2

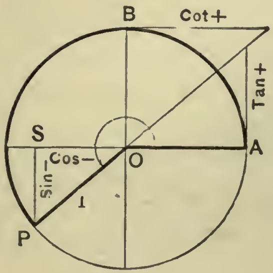


FIG. 3

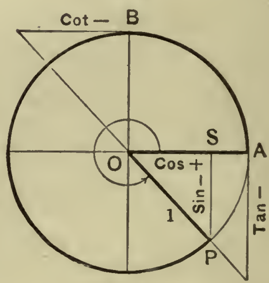


FIG. 4

The *secant* and *cosecant* are *positive* if measured in the *same direction as the terminal line, OP*; negative if measured in the opposite direction.

The signs of the functions of angles in the different quadrants are as follows:

Quadrant	I	II	III	IV
Sine and cosecant	+	+	–	–
Cosine and secant	+	–	–	+
Tangent and cotangent	+	–	+	–

9. It is evident that the values of the functions of an angle depend only upon the *position* of the sides of the angle. If two angles differ by 360°, or any multiple of 360°, the position of the sides is the same, hence the values of the functions are the same.

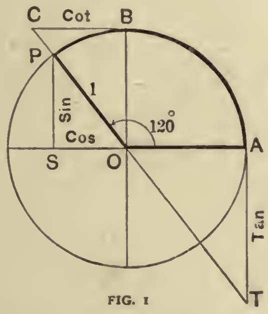


FIG. 1

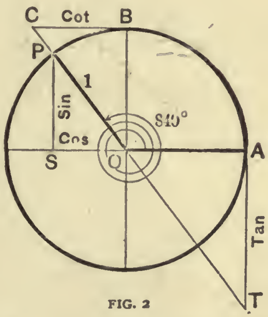


FIG. 2

Thus in Fig. 1 the angle is 120°, in Fig. 2 the angle is 840°, yet the lines which represent the functions are the same for both angles.

EXERCISE

Determine, by drawing the necessary figures, the *sign* of tan 1000°; cos 810°; sin 760°; cot —70°; cos —550°; tan —560°; sec 300°; cot 1560°; sin 130°; cos 260°; tan 310°.

RELATIONS OF THE FUNCTIONS

10. By § 5, whatever may be the length of OP, we have

$$\frac{SP}{OP}=\sin x\,;\ \frac{OS}{OP}=\cos x\,;\ \frac{SP}{OS}=\tan x\,;\ \frac{OS}{SP}=\cot x\,;\ \frac{OP}{OS}=\sec x\,;\ \frac{OP}{SP}=\csc x.$$

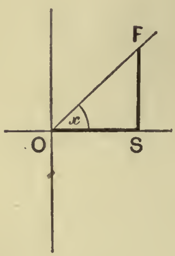


FIG. 1

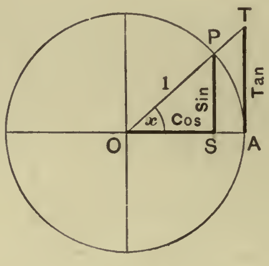


FIG. 2

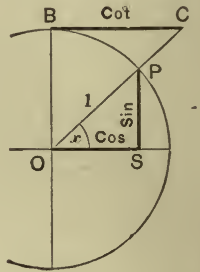


FIG. 3

We have, then, from Figs. 2 and 3,

$$\frac{SP}{OS}=\tan x=\frac{\sin x}{\cos x}; \tag{1}$$

$$\frac{OS}{SP}=\cot x=\frac{\cos x}{\sin x}. \tag{2}$$

Multiplying (1) by (2),

$$\tan x \cot x=1, \tag{3}$$

or $$\tan x=\frac{1}{\cot x};\quad \cot x=\frac{1}{\tan x}.$$

Again, from Figs. 2 and 3,

$$\frac{OP}{OS}=\sec x=\frac{1}{\cos x}; \tag{4}$$

$$\frac{OP}{SP}=\csc x=\frac{1}{\sin x}. \tag{5}$$

From Figs. 2 and 3, $OS^2+SP^2=OP^2$,

or $$\sin^2 x+\cos^2 x=1, \tag{6}$$

and $$\sin^2 x=1-\cos^2 x;\quad \cos^2 x=1-\sin^2 x.$$

Also, $OA^2+AT^2=OT^2$, and $OB^2+BC^2=OC^2$,

or $$1+\tan^2 x=\sec^2 x; \tag{7}$$

$$1+\cot^2 x=\csc^2 x. \tag{8}$$

The angle x has been taken in the first quadrant; the results are, however, true for any angle. The proof is the same for angles in other quadrants, except that SP becomes negative in the third and fourth quadrants, and OS in the second and third.

EXERCISES

11. (1.) Prove $\cos x \sec x = 1$.

(2.) Prove $\sin x \csc x = 1$.

(3.) Prove $\tan x \cos x = \sin x$.

(4.) Prove $\sin x \sqrt{1 - \cos^2 x} = 1 - \cos^2 x$.

(5.) Prove $\tan x + \cot x = \dfrac{1}{\sin x \cos x}$.

(6.) Prove $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$.

(7.) Prove $\dfrac{1}{\cot x \sec x} = \sin x$.

(8.) Prove $\tan x \sin x + \cos x = \sec x$.

12. The formulas (1)–(8) of § 10 are algebraic equations connecting the different functions of the same angle. If the value of one of the functions of an angle is given, we can substitute this value in one of the equations and solve to find another of the functions. Repeating the process, we find a third function, etc.

In solving equation (6), (7), or (8) a square root is extracted; unless something is given which determines whether to choose the positive or negative square root, we get two values for some of the functions. The reason for this is that there are two angles less than 360° for which a function has a given value.

EXERCISES

13. (1.) Given x less than 90° and $\sin x = \frac{1}{2}$; find all the other functions of x.

Solution.—

$$\cos x = \pm\sqrt{1 - \tfrac{1}{4}} = \pm \tfrac{1}{2}\sqrt{3}.$$

Since x is less than 90°, we know that $\cos x$ is positive.

Hence

$$\cos x = +\tfrac{1}{2}\sqrt{3};$$

$$\tan x = \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{3}} = \tfrac{1}{3}\sqrt{3};$$

$$\cot x = \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}} = \sqrt{3};$$

$$\sec x = \frac{1}{\frac{1}{2}\sqrt{3}} = \tfrac{2}{3}\sqrt{3};$$

$$\csc x = \frac{1}{\frac{1}{2}} = 2.$$

(2.) Given $\tan x = -\frac{1}{3}$ and x in quadrant IV; find $\sin x$ and $\cos x$.

Solution.—

$$\frac{\sin x}{\cos x} = -\tfrac{1}{3};$$

hence $$3 \sin x = -\cos x,$$

$$\sin^2 x + \cos^2 x = 1;$$

hence $$10 \sin^2 x = 1;$$

$$\sin x = -\sqrt{\tfrac{1}{10}} = -\tfrac{1}{10}\sqrt{10};$$

$$\cos x = \tfrac{3}{10}\sqrt{10}.$$

(3.) Given $\sin(-30°) = -\frac{1}{2}$; find the other functions of $-30°$.

(4.) Given x in quadrant III and $\sin x = -\frac{1}{3}$; find all the other functions of x.

(5.) Given y in quadrant IV and $\sin y = -\frac{3}{5}$, find all the other functions of y.

(6.) Given $\cos 60° = \frac{1}{2}$; find all the other functions of $60°$.

(7.) Given $\sin 0° = 0$; find $\cos 0°$ and $\tan 0°$.

(8.) Given $\tan z = \frac{4}{3}$ and z in quadrant I; find the other functions of z.

(9.) Given $\cot 45° = 1$; find all the other functions of $45°$.

(10.) Given $\tan y = \frac{1}{2}\sqrt{5}$ and $\cos y$ negative; find all the other functions of y.

(11.) Given $\cot 30° = \sqrt{3}$; find the other functions of $30°$.

(12.) Given $2 \sin x = 1 - \cos x$ and x in quadrant II; find $\sin x$ and $\cos x$.

(13.) Given $\tan x + \cot x = 3$ and x in quadrant I; find $\sin x$.

FUNCTIONS OF AN ACUTE ANGLE OF A RIGHT TRIANGLE

14. The functions of an acute angle of a right triangle can be expressed as ratios of the sides of the triangle.

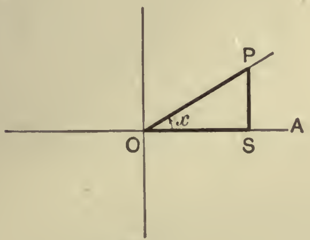


FIG. 1

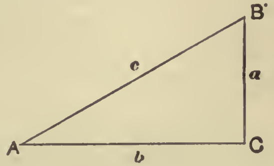


FIG. 2

Remark.—Triangles are usually lettered, as in Fig. 2, the capital letters denoting the angles, the corresponding small letters the sides opposite.

In the right triangle ABC, by § 5,

$$\mathbf{sin}\, A = \frac{BC}{AB} = \frac{a}{c} = \cos B;$$

$$\mathbf{cos}\, A = \frac{AC}{AB} = \frac{b}{c} = \sin B;$$

$$\mathbf{tan}\, A = \frac{BC}{AC} = \frac{a}{b} = \cot B;$$

$$\mathbf{cot}\, A = \frac{AC}{BC} = \frac{b}{a} = \tan B.$$

15. From § 14, for an acute angle of a right triangle, we have

$$\text{sine} = \frac{\text{side opposite angle}}{\text{hypotenuse}};$$

$$\text{cosine} = \frac{\text{side adjacent to angle}}{\text{hypotenuse}};$$

$$\text{tangent} = \frac{\text{side opposite angle}}{\text{side adjacent to angle}};$$

$$\text{cotangent} = \frac{\text{side adjacent to angle}}{\text{side opposite angle}}.$$

FUNCTIONS OF COMPLEMENTARY ANGLES

16. From § 14, we have

$$\left.\begin{aligned}\mathbf{sin}\,A&=\cos B=\mathbf{cos}(\mathbf{90^\circ}-A);\\ \mathbf{cos}\,A&=\sin B=\mathbf{sin}\,(\mathbf{90^\circ}-A);\\ \mathbf{tan}\,A&=\cot B=\mathbf{cot}(\mathbf{90^\circ}-A);\\ \mathbf{cot}\,A&=\tan B=\mathbf{tan}(\mathbf{90^\circ}-A).\end{aligned}\right\}\qquad(9)$$

Because of this relation the sine and cosine are called co-functions of each other, and the tangent and cotangent are called co-functions of each other.

The results of this article may be stated thus:

A function of an acute angle is equal to the co-function of its complementary angle.

The values of the functions of the different angles are given in "Trigonometric Tables." By the use of the principle just proved, each function of an angle between 45° and 90° can be found as a function of an angle less than 45°. Consequently, the tables need to be constructed for angles up to 45° only. The tables are so arranged that a number in them can be read either as a function of an angle less than 45° or as the co-function of the complement of this angle.

EXERCISES

17. (1.) Express as functions of an angle less than 45°:

$\sin 70^\circ$; $\cos 89^\circ\ 30'$; $\tan 63^\circ$;

$\cos 66^\circ$; $\cot 47^\circ$; $\sin 72^\circ\ 39'$.

(2.) $\cos x=\sin 2x$; find x.

(3.) $\tan x=\cot 3x$; find x.

(4.) $\sin 2x=\cos 3x$; find x.

(5.) $\cot(30^\circ-x)=\tan(30^\circ+3x)$; find x.

(6.) A, B, and C are the angles of a triangle; prove that $\cos\frac{1}{2}B=\sin\frac{1}{2}(A+C)$.

Hint.— $A+B+C=180^\circ$.

FUNCTIONS OF 0°, 90°, 180°, 270°, AND 360°

18. As the angle x decreases towards 0° (Fig. 1), $\sin x$ decreases and $\cos x$ increases. When OP comes into coincidence with OA, SP becomes 0, and OS becomes $OA (= 1)$.

Hence $\sin 0° = 0, \quad \cos 0° = 1.$

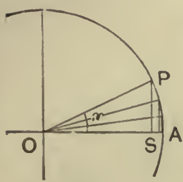


FIG. 1

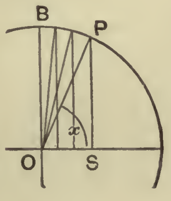


FIG. 2

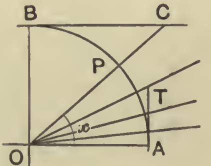


FIG. 3

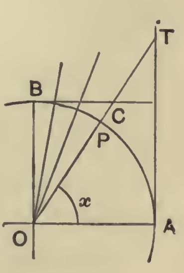


FIG. 4

As the angle x increases towards 90° (Fig. 2), $\sin x$ increases and $\cos x$ decreases. When OP comes into coincidence with OB, SP becomes $OB (= 1)$ and OS becomes 0.

Hence $\sin 90° = 1, \quad \cos 90° = 0.$

As the angle x decreases towards 0° (Fig. 3), $\tan x$ decreases and $\cot x$ increases. When OP comes into coincidence with OA, AT becomes 0 and BC has increased without limit.

Hence $\tan 0° = 0, \quad \cot 0° = \infty.$

As the angle x increases towards 90° (Fig. 4), $\tan x$ increases and $\cot x$ decreases. When OP comes into coincidence with OB, AT has increased without limit, and $BC = 0$.

Hence $\tan 90° = \infty, \quad \cot 90° = 0.$

Remark.—By $\cot 0° = \infty$ we mean that as the angle approaches indefinitely near to 0° its cotangent increases so as to become greater than any finite quantity we may choose. The symbol ∞ does not denote a definite number, but simply that the number is indefinitely great.

In every case where a trigonometric function becomes indefinitely great it is in a positive sense if the angle approaches the limiting value from one side, in a negative sense if the angle approaches the limiting value from the other side. Thus $\cot 0° = +\infty$ if the angle decreases to 0°, but $\cot 0° = -\infty$ if the angle increases from a negative angle to 0°. We shall not often need to distinguish between $+\infty$ and $-\infty$, and shall in general denote either by the symbol ∞.

By a similar method the functions of 180°, 270°, and 360° may be deduced. The results of this article are shown in the following table:

Angle	0°	90°	180°	270°	360°
sin	0	1	0	−1	0
cos	1	0	−1	0	1
tan	0	∞	0	∞	0
cot	∞	0	∞	0	∞

19. *It may now be stated that, as an angle varies, its sine and cosine can take on values from -1 to $+1$ only, its tangent and cotangent all values from $-\infty$ to $+\infty$, its secant and cosecant all values from $-\infty$ to $+\infty$, except those between -1 and $+1$.*

FUNCTIONS OF THE SUPPLEMENT OF AN ANGLE

20. Suppose the triangle OPS (Fig. 1) equal to the triangle $OP'S'$ (Fig. 2), then $SP = S'P'$ and $OS = OS'$, and the angle AOP' (Fig. 2) is equal to the supplement of AOP (Fig. 1). Also, in the triangle AOP' (Fig. 3), angle AOP' = angle AOP' (Fig. 2).

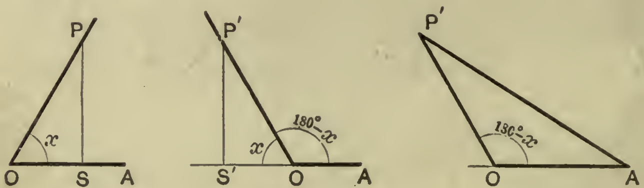


FIG. 1 FIG. 2 FIG. 3

It follows from §§ 5 and 8 that

$$\left.\begin{aligned}\sin(180^\circ - x) &= \sin x;\\ \cos(180^\circ - x) &= -\cos x;\\ \tan(180^\circ - x) &= -\tan x;\\ \cot(180^\circ - x) &= -\cot x.\end{aligned}\right\}\qquad (10)$$

The results of this article may be stated thus:

The sine of an angle is equal to the sine of its supplement, and the cosine, tangent, and cotangent are each equal to minus the same functions of its supplement.

The principle just proved is of great importance in the solution of triangles which contain an obtuse angle.

FUNCTIONS OF 45°, 30°, AND 60°

21. In the right triangle *OSP* (Fig. 1) angle $O =$ angle $P = 45°$, and $OP = 1$.

Hence $\qquad OS = SP = \frac{1}{2}\sqrt{2}.$

Therefore $\qquad \sin 45° = \cos 45° = \frac{1}{2}\sqrt{2};$ $\qquad$ §§ 14, 16

$\qquad \tan 45° = \cot 45° = 1.$

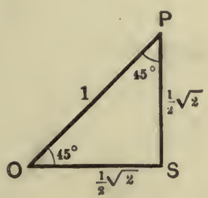


FIG. 1

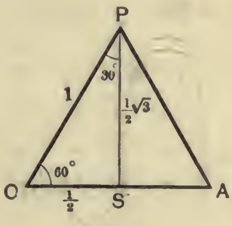


FIG. 2

In equilateral triangle *OPA* (Fig. 2) the sides are of unit length; *PS* bisects angle *OPA*, is perpendicular to *OA*, and bisects *OA*.

Hence, in the right triangle *OPS*, $OS = \frac{1}{2}$, $SP = \frac{1}{2}\sqrt{3}$.

Therefore $\qquad \sin 30° = \cos 60° = \frac{1}{2};$ $\qquad$ § 14

$$\cos 30° = \sin 60° = \tfrac{1}{2}\sqrt{3};$$
$$\tan 30° = \cot 60° = \tfrac{1}{3}\sqrt{3};$$
$$\cot 30° = \tan 60° = \sqrt{3}.$$

22. The following values should be remembered:

Angle	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

EXERCISES

Prove that if $x = 30^\circ$,

(1.) $\sin 2x = 2 \sin x \cos x$;

(2.) $\cos 3x = 4 \cos^3 x - 3 \cos x$;

(3.) $\cos 2x = \cos^2 x - \sin^2 x$;

(4.) $\sin 3x = 3 \sin x \cos^2 x - \sin^3 x$;

(5.) $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$.

(6.) Prove that the equations of exercises 1 and 3 are correct if $x = 45^\circ$.

(7) Prove that the equations of exercises (2) and (4) are correct if $x = 120^\circ$.

The following three articles, §§ 23–25, are inserted for completeness. They include the functions of $(90 - x)$ and $(180 - x)$, which, on account of their great importance, were treated separately in §§ 16 and 20.

FUNCTIONS OF $(-x)$, $(180^\circ - x)$, $(180^\circ + x)$, $(360^\circ - x)$

23. The line representing any function—as sine, cosine, etc. —of each of these angles has the same length as the line representing the same function of x.

Thus in Figs. 2 and 3, triangle $OS'P'$ = triangle OSP, hence $SP = S'P'$, and $OS = OS'$.

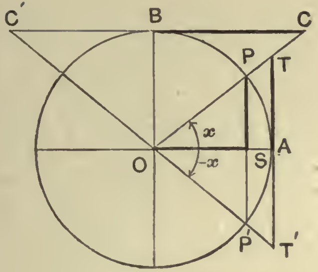


FIG. 1

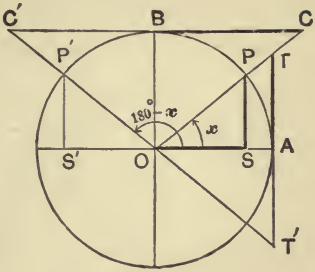


FIG. 2

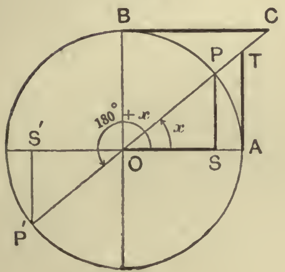


FIG. 3

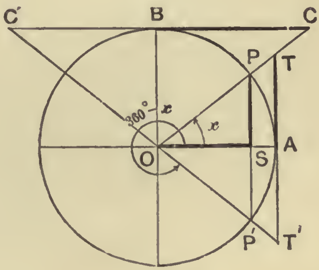


FIG. 4

In Figs. 1 and 4, triangle $OSP' =$ triangle OSP, hence $SP' = SP$.
In Figs. 1, 2, and 4, triangle $OAT' =$ triangle OAT, hence $AT' = AT$.
In Figs. 1, 2, and 4, triangle $OBC' =$ triangle OBC, hence $BC' = BC$.

Therefore any function of each of the angles $(-x)$, $(180° - x)$, $(180° + x)$, $(360° - x)$, *is equal in numerical value to the same function of* x. *Its sign, however, depends on the direction of the line representing it.*

Putting in the correct sign, we obtain the following table:

$\sin(-x) = -\sin x$	$\sin(180° - x) = \sin x$
$\cos(-x) = \cos x$	$\cos(180° - x) = -\cos x$
$\tan(-x) = -\tan x$	$\tan(180° - x) = -\tan x$
$\cot(-x) = -\cot x$	$\cot(180° - x) = -\cot x$
$\sin(180° + x) = -\sin x$	$\sin(360° - x) = -\sin x$
$\cos(180° + x) = -\cos x$	$\cos(360° - x) = \cos x$
$\tan(180° + x) = \tan x$	$\tan(360° - x) = -\tan x$
$\cot(180° + x) = \cot x$	$\cot(360° - x) = -\cot x$

FUNCTIONS OF $(90°-y)$, $(90°+y)$, $(270°-y)$, $(270°+y)$

24. The line representing the sine of each of these angles is of the same length as the line representing the cosine of y; the cosine, tangent, or cotangent, respectively, are of the same length as the sine, cotangent, and tangent of y.

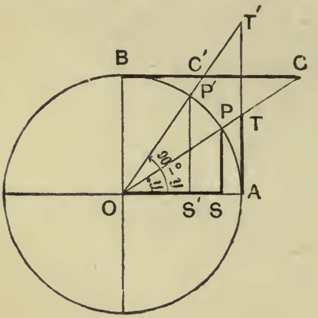


FIG. 1

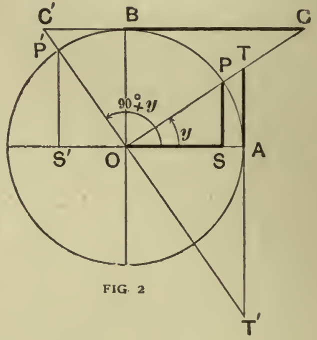


FIG. 2

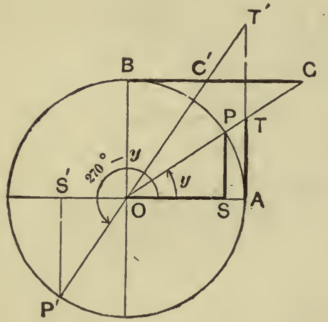


FIG. 3

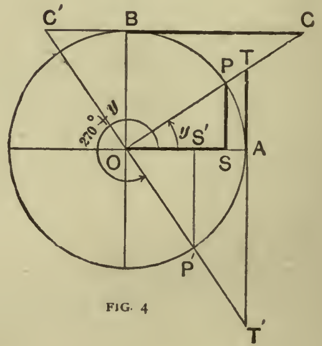


FIG. 4

For

Triangle $OS'P' =$ triangle OSP, hence $S'P' = OS$, and $OS' = SP$.
Triangle $OAT' =$ triangle OBC, hence $AT' = BC$.
Triangle $OBC' =$ triangle OAT, hence $BC' = AT$.

Therefore any function of each of the angles $(90°-y)$, $(90°+y)$, $(270°-y)$, $(270°+y)$, *is equal in numerical value to the co-function*

of y. Its sign, however, depends on the direction of the line representing it.

Putting in the correct sign, we obtain the following table :

$\sin(90^\circ - y) = \cos y$	$\sin(90^\circ + y) = \cos y$
$\cos(90^\circ - y) = \sin y$	$\cos(90^\circ + y) = -\sin y$
$\tan(90^\circ - y) = \cot y$	$\tan(90^\circ + y) = -\cot y$
$\cot(90^\circ - y) = \tan y$	$\cot(90^\circ + y) = -\tan y$
$\sin(270^\circ - y) = -\cos y$	$\sin(270^\circ + y) = -\cos y$
$\cos(270^\circ - y) = -\sin y$	$\cos(270^\circ + y) = \sin y$
$\tan(270^\circ - y) = \cot y$	$\tan(270^\circ + y) = -\cot y$
$\cot(270^\circ - y) = \tan y$	$\cot(270^\circ + y) = -\tan y$

25. Either of the two preceding articles enables us directly to express the functions of any angle, positive or negative, in terms of the functions of a positive angle less than 90°.

Thus,
$$\sin 212^\circ = \sin(180^\circ + 32^\circ) = -\sin 32^\circ;$$
$$\cos 260^\circ = \cos(270^\circ - 10^\circ) = -\sin 10^\circ.$$

EXERCISES

(1.) What angles less than 360° have the sine equal to $-\frac{1}{2}\sqrt{2}$? the tangent equal to $\sqrt{3}$?

(2.) For what values of x less than 720° is $\sin x = \frac{1}{2}\sqrt{3}$?

(3.) Find the sine and cosine of -30°; 765°; 120°; 210°.

(4.) Find the functions of 405°; 600°; 1125°; -45°; 225°.

(5.) Find the functions of -120°; -225°; -420°; 3270°.

(6.) Express as functions of an angle less than 45° the functions of 233°; -197°; 894°.

(7.) Express as functions of an angle between 45° and 90°, $\sin 267^\circ$; $\tan(-254^\circ)$; $\cos 950^\circ$.

(8.) Given $\cos 164^\circ = -.96$, find $\sin 196^\circ$.

(9.) Simplify $\cos(90^\circ + x)\cos(270^\circ - x) - \sin(180^\circ - x)\sin(360^\circ - x)$.

(10.) Simplify $\dfrac{\sin(180^\circ - x)}{\sin(270^\circ - x)}\tan(90^\circ + x) + \dfrac{1}{\sin^2(270^\circ - x)}$.

(11.) Express the functions of $(x - 90^\circ)$ in terms of functions of x.

CHAPTER II

THE RIGHT TRIANGLE

27. To solve a triangle is to find the parts not given.

A triangle can be solved if three parts, at least one of which is a side, are given. A right triangle has one angle, the right angle, always given; hence a right triangle can be solved if two sides, or one side and an acute angle, are also given.

The parts of the right triangle not given are found by the use of the following formulas:

(1) sine $=\dfrac{\text{opposite side}}{\text{hypotenuse}}$; (2) cosine $=\dfrac{\text{adjacent side}}{\text{hypotenuse}}$; § 14

(3) tangent $=\dfrac{\text{opposite side}}{\text{adjacent side}}$; (4) cotangent $=\dfrac{\text{adjacent side}}{\text{opposite side}}$;

(5) $c^2=a^2+b^2$; (6) $B=(90°-A)$. § 16

To solve, select a formula in which two given parts enter; substituting in this the given values, a third part is found. Continue this method till all the parts are found.

In a given problem there are several ways of solving the triangle; choose the shortest.

EXAMPLE

The hypotenuse of a right triangle is 47.653, a side is 21.34; find the remaining parts and the area.

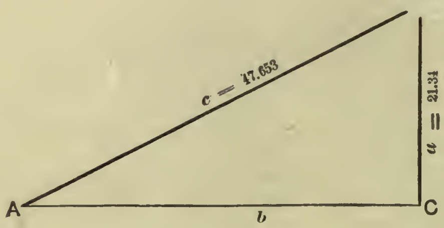


FIG. 1

SOLUTION WITHOUT LOGARITHMS

The functions of angles are given in the table of "Natural Functions."

$$\sin A = \frac{a}{c} = \frac{21.34}{47.653}$$

$$\begin{array}{r} 47.653)21.3400(.4478 \\ 190612 \\ \hline 227880 \\ 190612 \\ \hline 372680 \\ 333571 \\ \hline 391090 \\ 381224 \\ \hline 9866 \end{array}$$

$$\sin A = .4478$$
$$A = 26^\circ\ 36'$$

$$b = c \cos A$$
$$= 47.653 \times .8942$$

$$\begin{array}{r} 47.653 \\ .8942 \\ \hline 95306 \\ 190612 \\ 428877 \\ 381224 \\ \hline 42.6113126 \end{array}$$

$b = 42.61$ †

$$B = (90^\circ - 26^\circ\ 36') = 63^\circ\ 24'$$

$$\text{area} = \tfrac{1}{2}ab$$
$$= \tfrac{1}{2} \times 21.34 \times 42.61$$

$$\begin{array}{r} 21.34 \\ 42.61 \\ \hline 2134 \\ 12804 \\ [illegible] \\ \hline 2)909.2974 \\ \hline 454.6487 \end{array}$$

area $= 454.6$

SOLUTION EMPLOYING LOGARITHMS

It is usually better to solve triangles by the use of logarithms.

The *logarithms* of the functions are given in the tables of "Logarithms of Functions." *

$$\sin A = \frac{a}{c}$$

$$\log \sin A = \log a - \log c$$
$$\log 21.34 = 1.32919$$
$$\log 47.653 = 1.67809 \quad \text{sub.}$$
$$\log \sin A = 9.65110 - 10$$
$$A = 26^\circ\ 36'\ 14''$$

$$\cos A = \frac{b}{c}$$

$$\log b = \log c + \log \cos A$$
$$\log 47.653 = 1.67809$$
$$\log \cos 26^\circ\ 36'\ 14'' = 9.95140 - 10$$
$$\log b = 1.62949$$
$$b = 42.608$$

$$B = (90^\circ - 26^\circ\ 36'\ 14'') = 63^\circ\ 23'\ 46''$$

$$\text{area} = \tfrac{1}{2}ab$$
$$\log \text{area} = \log \tfrac{1}{2} + \log a + \log b$$
$$\log \tfrac{1}{2} = 9.69897 - 10$$
$$\log 21.34 = 1.32919$$
$$\log 42.608 = 1.62949$$
$$\log \text{area} = 2.65765$$
$$\text{area} = 454.62$$

* In this solution the five-place table of the "Logarithms of Functions" is used.

† No more decimal places are retained, because the figures in them are not accurate; this is due to the fact that the table of "Natural Functions" is only four-place.

CHECK ON THE CORRECTNESS OF THE WORK

$$a^2 = c^2 - b^2 = (c+b)(c-b)$$
$$= 90.263 \times 5.043$$

$$\begin{array}{r} 90.263 \\ \underline{5.043} \\ 270789 \\ 361052 \\ \underline{4513150} \\ a^2 = 455.196309 \end{array}$$

Extracting the square root, $a = 21.34$, which proves the solution correct.

$$a^2 = c^2 - b^2 = (c+b)(c-b)$$
$$= 90.261 \times 5.045$$

$$\begin{array}{rl} \log 90.261 = & 1.95550 \\ \log 5.045 = & \underline{0.70286} \\ & 2)\overline{2.65836} \\ \log 21.34 = & 1.32918 \end{array}$$

$a = 21.34$, which proves the solution correct.

Remark.—The results obtained in the solution of the preceding exercise without logarithms are less accurate than those obtained in the solution by the use of logarithms; the cause of this is that four-place tables have been used in the former method, five place in the latter.

EXERCISES

28. (1.) In a right triangle $b = 96.42$, $c = 114.81$; find a and A.

(2.) The hypotenuse of a right triangle is 28.453, a side is 18.197; find the remaining parts.

(3.) Given the hypotenuse of a right triangle $= 747.24$, an acute angle $= 23° 45'$; find the remaining parts.

(4.) Given a side of a right triangle $= 37.234$, the angle opposite $= 54° 27'$; find the remaining parts and the area.

(5.) Given a side of a right triangle $= 1.1293$, the angle adjacent $= 74° 13' 27''$; find the remaining parts and the area.

(6.) In a right triangle $A = 15° 22' 11''$, $c = .01793$; find b.

(7.) In a right triangle $B = 71° 34' 53''$, $b = 896.33$; find a.

(8.) In a right triangle $c = 3729.4$, $b = 2869.1$; find A.

(9.) In a right triangle $a = 1247$, $b = 1988$; find c.

(10.) In a right triangle $a = 8.6432$, $b = 4.7815$; find B.

The angle of **elevation** or **depression** of an object is the angle a line from the point of observation to the object makes with the horizontal.

all odd

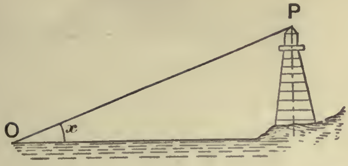


FIG. 1

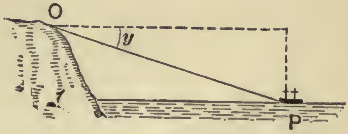


FIG. 2

Thus angle x (Fig. 1) is the angle of elevation of P if O is the point of observation; angle y (Fig. 2) is the angle of depression of P if O is the point of observation.

(11.) At a horizontal distance of 253 ft. from the base of a tower the angle of elevation of the top is $60^\circ\ 20'$; find the height of the tower.

(12.) From the top of a vertical cliff 85 ft. high the angle of depression of a buoy is $24^\circ\ 31'\ 22''$; find the distance of the buoy from the foot of the cliff.

(13.) A vertical pole 31 ft. high casts a horizontal shadow 45 ft. long; find the angle of elevation of the sun above the horizon.

(14.) From the top of a tower 115 ft. high the angle of depression of an object on a level road leading away from the tower is $22^\circ\ 13'\ 44''$; find the distance of the object from the top of the tower.

(15.) A rope 324 ft. long is attached to the top of a building, and the inclination of the rope to the horizontal, when taut, is observed to be $47^\circ\ 21'\ 17''$; find the height of the building.

(16.) A light-house is 150 ft. high. How far is an object on the surface of the water visible from the top?

[Take the radius of the earth as 3960 miles.]

(17.) Three buoys are at the vertices of a right triangle; one side of the triangle is 17,894 ft., the angle adjacent to it is $57^\circ\ 23'\ 46''$. Find the length of a course around the three buoys.

(18.) The angle of elevation of the top of a tower observed from a point at a horizontal distance of 897.3 ft. from the base is $10^\circ\ 27'\ 42''$; find the height of the tower.

(19.) A ladder $42\frac{1}{2}$ ft. long leans against the side of a building; its foot is $25\frac{1}{4}$ ft. from the building. What angle does it make with the ground?

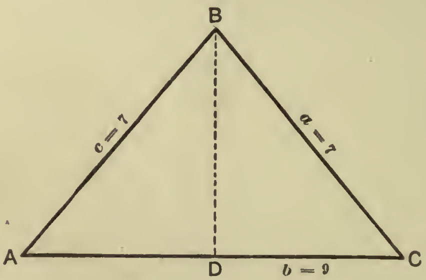
(20.) Two buildings are on opposite sides of a street 120 ft. broad.

11, 13, 15, 23

The height of the first is 55 ft.; the angle of elevation of the top of the second, observed from the edge of the roof of the first, is 26° 37′. Find the height of the second building.

(21.) A mark on a flag-pole is known to be 53 ft. 7 in. above the ground. This mark is observed from a certain point, and its angle of elevation is found to be 25° 34′. The angle of elevation of the top of the pole is then measured, and found to be 34° 17′. Find the height of the pole.

(22.) The equal sides of an isosceles triangle are each 7 in. long; the base is 9 in. long. Find the angles of the triangle.



Hint.—Draw the perpendicular BD. BD bisects the base, and also the angle ABC.

In the right triangle ABD, $AB=7$ in., $AD=4\frac{1}{2}$ in., hence ABD can be solved.

Angle $C=$angle A, angle $ABC=2$ angle ABD.

(23.) Given the equal sides of an isosceles triangle each 13.44 in., and the equal angles are each 63° 21′ 42″; find the remaining parts and the area.

(24.) The equal sides of an isosceles triangle are each 377.22 in., the angle between them is 19° 55′ 32″. Find the base and the area of the triangle.

(25.) If a chord of a circle is 18 ft. long, and it subtends at the centre an angle of 45° 31′ 10″, find the radius of the circle.

(26.) The base of a wedge is 3.92 in., and its sides are each 13.25 in. long; find the angle at its vertex.

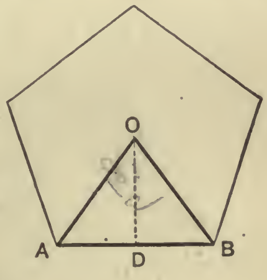
24, 25, 29, 30, 31

(27.) The angle between the legs of a pair of dividers is $64° \, 45'$, the legs are 5 in. long; find the distance between the points.

(28.) A field is in the form of an isosceles triangle, the base of the triangle is 1793.2 ft.; the angles adjacent to the base are each $53° \, 27' \, 49''$. Find the area of the field.

(29.) A house has a gable roof. The width of the house is 30 ft., the height to the eaves $25\frac{1}{2}$ ft., the height to the ridge-pole $33\frac{1}{2}$ ft. Find the length of the rafters and the area of an end of the house.

(30.) The length of one side of a regular pentagon is 29.25 in.; find the radius, the apothem, and the area of the pentagon.



Hint.—The pentagon is divided into 5 equal isosceles triangles by its radii. Let AOB be one of these triangles. $AB = 29.25$ in.; angle $AOB = \frac{1}{5}$ of $360° = 72°$. Find, by the methods previously given, OA, OD, and the area of the triangle AOB.

These are the radius of the pentagon, the apothem of the pentagon, and $\frac{1}{5}$ the area of the pentagon respectively.

(31.) The apothem of a regular dodecagon is 2; find the perimeter.

(32.) A tower is octagonal; the perimeter of the octagon is 153.7 ft. Find the area of the base of the tower.

(33.) A fence extends about a field which is in the form of a regular polygon of 7 sides; the radius of the polygon is 6283.4 ft. Find the length of the fence.

(34.) The length of a side of a regular hexagon inscribed in a circle is 3.27 ft.; find the perimeter of a regular decagon inscribed in the same circle.

(35.) The area of a field in the form of a regular polygon of 9 sides is 483930 sq. ft.; find the length of the fence about it.

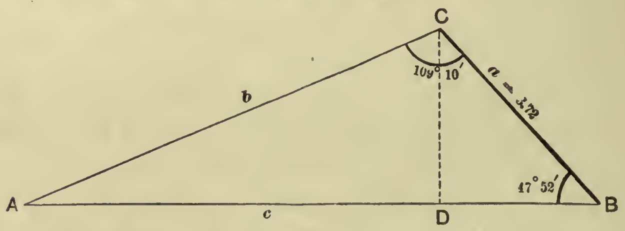
SOLUTION OF OBLIQUE TRIANGLES BY THE AID OF RIGHT TRIANGLES

29. Oblique triangles can always be solved by the aid of right triangles without the use of special formulas; the method is frequently, however, quite awkward; hence, in a later chapter, formulas are deduced which render the solution more simple.

The following exercises illustrate the solution by means of right triangles:

(1.) In an oblique triangle $a = 3.72$, $B = 47° \ 52'$, $C = 109° \ 10'$; find the remaining parts.

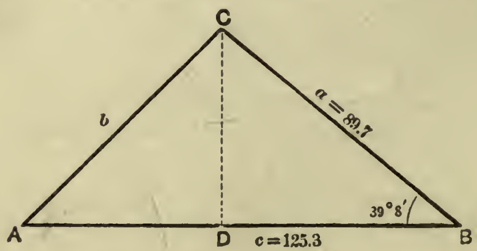
The given parts are a side and two angles.



Hint.—$A = [180° - (B + C)]$.
Draw the perpendicular CD.
Solve the right triangle BCD.
Having thus found CD, solve the right triangle ACD.

(2.) In an oblique triangle $a = 89.7$, $c = 125.3$, $B = 39° \ 8'$; find the remaining parts.

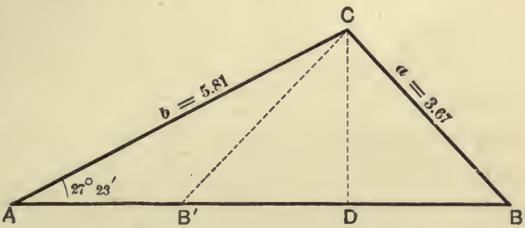
The given parts are two sides and the included angle.



Hint.—Draw the perpendicular CD.
Solve the right triangle CBD.
Having thus found CD and $AD(=c-DB)$, solve the right triangle ACD.

(3.) In an oblique triangle $a=3.67$, $b=5.81$, $A=27^\circ\ 23'$; find the remaining parts.

The given parts are two sides and an angle opposite one of them.



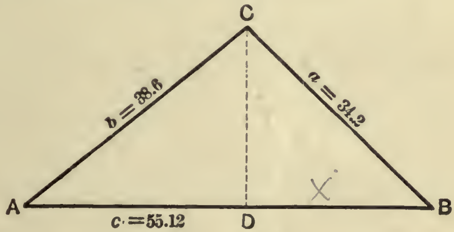
Either of the triangles ACB, ACB' contains the given parts, and is a solution.

There are two solutions when the side opposite the given angle is less than the other given side and greater than the perpendicular, CD, from the extremity of that side to the base.*

Hint.—Solve the right triangle ACD.
Having thus found CD, solve the right triangle CDB (or CDB').

(4.) The sides of an oblique triangle are $a=34.2$, $b=38.6$, $c=55.12$; find the angles.

The given parts are the three sides.



* A discussion of this case is contained in a later chapter on the solution of oblique triangles.

Hint.— Let $DB = x$,

$$a^2 - x^2 = \overline{CD}^2 = b^2 - (c - x)^2.$$

Hence

$$a^2 = b^2 - c^2 + 2cx,$$

$$x = \frac{a^2 + c^2 - b^2}{2c}.$$

In each of the right triangles ACD and BCD the hypotenuse and a side are now known; hence these triangles can be solved.

(5.) Two trees, A and B, are on opposite sides of a pond. The distance of A from a point C is 297.6 ft., the distance of B from C is 864.4 ft., the angle ACB is $87° 43' 12''$. Find the distance AB.

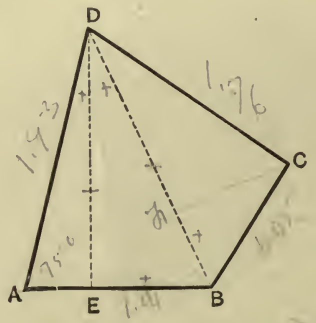
(6.) To determine the distance of a ship A from a point B on shore, a line, BC, 800 ft. long, is measured on shore; the angles, ABC and ACB, are found to be $67° 43'$ and $74° 21' 16''$ respectively. What is the distance of the ship from the point B?

(7.) A light-house 92 ft. high stands on top of a hill; the distance from its base to a point at the water's edge is 297.25 ft.; observed from this point the angle of elevation of the top is $46° 33' 15''$. Find the length of a line from the top of the light-house to the point.

(8.) The sides of a triangular field are 534 ft., 679.47 ft., 474.5 ft. What are the angles and the area of the field?

(9.) A certain point is at a horizontal distance of $117\frac{1}{4}$ ft. from a river, and is 11 ft. above the river; observed from this point the angle of depression of the farther bank is $1° 12'$. What is the width of the river?

(10.) In a quadrilateral $ABCD$, $AB = 1.41$, $BC = 1.05$, $CD = 1.76$, $DA = 1.93$, angle $A = 75° 21'$; find the other angles of the quadrilateral.



10

5 6 7 9. 10

Hint.—Draw the diagonal DB.

In the triangle ABD two sides and an included angle are given, hence the triangle can be solved.

The solution of triangle ABD gives DB.

Having found DB, there are three sides of the triangle DBC known, hence the triangle can be solved.

(11.) In a quadrilateral $ABCD$, $AB = 12.1$, $AD = 9.7$, angle $A = 47° \ 18'$, angle $B = 64° \ 49'$, angle $D = 100°$; find the remaining sides.

Hint.—Solve triangle ABD to find BD.

CHAPTER III

TRIGONOMETRIC ANALYSIS

30. In this chapter we shall prove the following fundamental formulas, and shall derive other important formulas from them:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y, \tag{11}$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y, \tag{12}$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y, \tag{13}$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y; \tag{14}$$

PROOF OF FORMULAS (11)–(14)

31. Let angle $AOQ = x$, angle $QOP = y$; then angle $AOP = (x+y)$.

The angles x and y are each acute and positive, and in Fig. 1 $(x+y)$ is less than $90°$, in Fig. 2 $(x+y)$ is greater than $90°$.

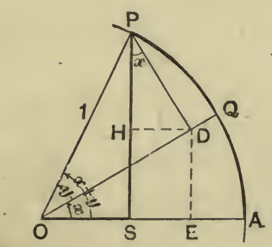


FIG. 1

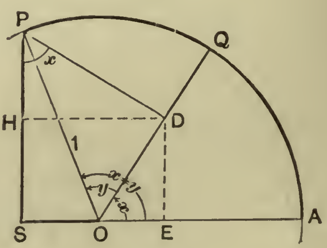


FIG. 2

In both figures the circle is a unit circle, and SP is perpendicular to OA; hence $SP = \sin(x+y)$, $OS = \cos(x+y)$.

Draw DP perpendicular to OQ;

then $$DP = \sin y, \quad OD = \cos y,$$

$$\text{angle } SPD = \text{angle } AOQ = x.$$

(Their sides being perpendicular.)

Draw DE perpendicular to OA, DH perpendicular to SP.

$$\text{Sin}\,(x+y) = SP = ED + HP.$$

$$ED = (\sin x) \times OD = \sin x \cos y.$$

(For OED being a right triangle, $\frac{ED}{OD} = \sin x$.)

$$HP = (\cos x) \times DP = \cos x \sin y.$$

(For HPD being a right triangle, $\frac{HP}{DP} = \cos x$.)

Therefore, $\mathbf{\sin (x+y) = \sin x \cos y + \cos x \sin y.}$ (11)

$$\text{Cos}\,(x+y) = OS = OE - HD.^*$$

$$OE = (\cos x) \times OD = \cos x \cos y.$$

(For OED being a right triangle, $\frac{OE}{OD} = \cos x$.)

$$HD = (\sin x) \times DP = \sin x \sin y.$$

(For PHD being a right triangle, $\frac{HD}{DP} = \sin x$.)

Therefore, $\mathbf{\cos (x+y) = \cos x \cos y - \sin x \sin y.}$ (13)

32. The preceding formulas have been proved only for the case when x and y are each acute and positive. The proof can, however, readily be extended to include all values of x and y.

Let y be acute, and let x be an angle in the second quadrant; then $x = (90° + x')$ where x' is acute.

$$\begin{aligned} \sin (x+y) &= \sin (90° + x' + y) \\ &= \cos (x' + y) && \S 24 \\ &= \cos x' \cos y - \sin x' \sin y \\ &= \sin (90° + x') \cos y + \cos (90° + x') \sin y && \S 24 \\ &= \sin x \cos y + \cos x \sin y. \end{aligned}$$

* If $(x+y)$ is greater than $90°$, OS is negative.

Thus the formula has been extended to the case where one of the angles is obtuse and less than 180°. In a similar way the formula for $\cos(x+y)$ is extended to this case.

By continuing this method both formulas are proved to be true for all positive values of x and y.

Any negative angle y is equal to a positive angle y', minus some multiple of 360°. The functions of y are equal to those of y', and the functions of $(x+y)$ are equal to those of $(x+y')$. $\sin(x+y) = \sin(x+y') = \sin x \cos y' + \cos x \sin y'$
$= \sin x \cos y + \cos x \sin y$. § 9

Therefore, the formulas being true for $(x+y')$, are true for $(x+y)$.

A repetition of this reasoning shows that the formulas are true when both angles, x and y, are negative.

33. Substituting the angle $-y$ for y in formula (11), it becomes

$$\sin(x-y) = \sin x \cos(-y) + \cos x \sin(-y).$$

But $\cos(-y) = \cos y$, and $\sin(-y) = -\sin y$. § 23

Therefore, $\mathbf{\sin(x-y) = \sin x \cos y - \cos x \sin y.}$ (12)

Substituting $(-y)$ for y in formula (13), it becomes

$$\cos(x-y) = \cos x \cos(-y) - \sin x \sin(-y),$$
$$= \cos x \cos y + \sin x \sin y.$$

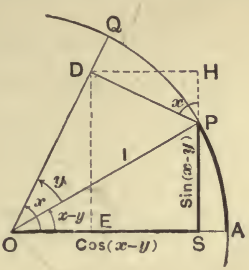
Therefore, $\mathbf{\cos(x-y) = \cos x \cos y + \sin x \sin y.}$* (14)

EXERCISES

34. (1.) Prove geometrically where x and y are acute and positive:

$$\sin(x-y) = \sin x \cos y - \cos x \sin y,$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y.$$

* Formulas (12) and (14) are proved geometrically in § 34. The geometric proof is complicated by the fact that OD and DP are functions of $-y$, while the functions of y are what we use.



Hint.—Angle $AOQ=x$, angle $POQ=y$, and angle $AOP=(x-y)$.

Draw PD perpendicular to OQ.

Then $DP=\sin(-y)=-\sin y$; but DP is negative, therefore PD taken as positive is equal to $\sin y$:

$$OD=\cos(-y)=\cos y.$$

Angle $HPD=$angle $AOQ=x$. their sides being perpendicular.

Draw DH perpendicular to SP, DE perpendicular to OA.

$$\sin(x-y)=SP=ED-PH.$$

From right triangle OED, $ED=(\sin x)\times OD=\sin x\cos y$.

From right triangle DHP, $PH=(\cos x)\times PD=\cos x\sin y$.

Therefore, $\sin(x-y)=\sin x\cos y-\cos x\sin y$.

$$\operatorname{Cos}(x-y)=OS=OE+DH.$$

From right triangle OED, $OE=(\cos x)\times OD=\cos x\cos y$.

From right triangle DHP, $DH=(\sin x)\times PD=\sin x\sin y$.

Therefore, $\cos(x-y)=\cos x\cos y+\sin x\sin y$.

(2.) Find the sine and cosine of $(45°+x)$, $(30°-x)$, $(60°+x)$, in terms of $\sin x$ and $\cos x$.

(3.) Given $\sin x=\frac{3}{5}$, $\sin y=\frac{5}{13}$, x and y acute; find $\sin(x+y)$ and $\sin(x-y)$.

(4.) Find the sine and cosine of 75° from the functions of 30° and 45°.

Hint.— $75°=(45°+30°)$.

(5.) Find the sine and cosine of 15° from the functions of 30° and 45°.

(6.) Given x and y, each in the second quadrant, $\sin x=\frac{1}{2}$, $\sin y=\frac{1}{4}$; find $\sin(x+y)$ and $\cos(x-y)$.

(7.) By means of the above formulas express the sine and cosine of $(180°-x)$, $(180°+x)$, $(270°-x)$, $(270°+x)$, in terms of $\sin x$ and $\cos x$.

(8.) Prove $\sin(60°+45°)+\cos(60°+45°)=\cos 45°$.

(9.) Given $\sin 45°=\frac{1}{2}\sqrt{2}$, $\cos 45°=\frac{1}{2}\sqrt{2}$; find $\sin 90°$ and $\cos 90°$.

(10.) Prove that $\sin(60°+x)-\sin(60°-x)=\sin x$.

odd

TANGENT OF THE SUM AND DIFFERENCE OF TWO ANGLES

35. $\text{Tan}(x+y)=\dfrac{\sin(x+y)}{\cos(x+y)}=\dfrac{\sin x \cos y+\cos x \sin y}{\cos x \cos y-\sin x \sin y}.$

Dividing each term of both numerator and denominator of the right-hand side of this equation by $\cos x \cos y$, and remembering that $\dfrac{\sin}{\cos}=\tan$, we have

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x \tan y}. \qquad (15)$$

In a similar way, dividing formula (12) by formula (14), we obtain

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x \tan y}. \qquad (16)$$

FUNCTIONS OF TWICE AN ANGLE

36. An important special case of formulas (11), (13), and (15) is when $y=x$; we then obtain the functions of $2x$ in terms of the functions of x.

From (11), $\sin(x+x)=\sin x \cos x+\cos x \sin x$.

Hence $$\sin 2x=2\sin x \cos x. \qquad (17)$$

From (13), $$\cos 2x=\cos^2 x-\sin^2 x. \qquad (18)$$

Since $\cos^2 x=1-\sin^2 x$, and $\sin^2 x=1-\cos^2 x$,

we derive from equation (18),

$$\cos 2x=1-2\sin^2 x, \qquad (19)$$

and $$\cos 2x=2\cos^2 x-1. \qquad (20)$$

From (15), $$\tan 2x=\frac{2\tan x}{1-\tan^2 x}. \qquad (21)$$

FUNCTIONS OF HALF AN ANGLE

37. Equations (19) and (20) are true for any angle; therefore for the angle $\frac{1}{2}x$.

From (19), $\cos x=1-2\sin^2\frac{1}{2}x;$

or $$\sin^2 \tfrac{1}{2}x = \frac{1 - \cos x}{2};$$

therefore, $$\mathbf{\sin \tfrac{1}{2}x = \pm\sqrt{\frac{1 - \cos x}{2}}}. \tag{22}$$

From (20), $$\cos x = 2\cos^2 \tfrac{1}{2}x - 1;$$

or $$\cos^2 \tfrac{1}{2}x = \frac{1 + \cos x}{2};$$

therefore, $$\mathbf{\cos \tfrac{1}{2}x = \pm\sqrt{\frac{1 + \cos x}{2}}}. \tag{23}$$

Dividing (22) by (23), we obtain

$$\mathbf{\tan \tfrac{1}{2}x = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}}. \tag{24}$$

FORMULAS FOR SUMS AND DIFFERENCES OF FUNCTIONS

38. From formulas (11)–(14), we obtain

$$\sin(x+y) + \sin(x-y) = 2\sin x \cos y;$$
$$\sin(x+y) - \sin(x-y) = 2\cos x \sin y;$$
$$\cos(x+y) + \cos(x-y) = 2\cos x \cos y;$$
$$\cos(x+y) - \cos(x-y) = -2\sin x \sin y.$$

Let $$u = (x+y) \text{ and } v = (x-y);$$

then $$x = \tfrac{1}{2}(u+v),\ y = \tfrac{1}{2}(u-v).$$

Substituting in the above equations, we obtain

$$\mathbf{\sin u + \sin v = 2\sin\tfrac{1}{2}(u+v)\cos\tfrac{1}{2}(u-v);} \tag{25}$$
$$\mathbf{\sin u - \sin v = 2\cos\tfrac{1}{2}(u+v)\sin\tfrac{1}{2}(u-v);} \tag{26}$$
$$\mathbf{\cos u + \cos v = 2\cos\tfrac{1}{2}(u+v)\cos\tfrac{1}{2}(u-v);} \tag{27}$$
$$\mathbf{\cos u - \cos v = -2\sin\tfrac{1}{2}(u+v)\sin\tfrac{1}{2}(u-v).} \tag{28}$$

Dividing (25) by (26),

$$\mathbf{\frac{\sin u + \sin v}{\sin u - \sin v} = \frac{\tan\tfrac{1}{2}(u+v)}{\tan\tfrac{1}{2}(u-v)}}. \tag{29}$$

EXERCISES

39. Express in terms of functions of x, by means of the formulas of this chapter,

(1.) $\tan(180^\circ - x)$; $\tan(180^\circ + x)$.

(2.) The functions of $(x - 180^\circ)$.

(3.) $\sin(x - 90^\circ)$ and $\cos(x - 90^\circ)$.

(4.) $\sin(x - 270^\circ)$, and $\cos(x - 270^\circ)$.

(5.) The sine and cosine of $(45^\circ - x)$; of $(45^\circ + x)$.

(6.) Given $\tan 45^\circ = 1$, $\tan 30^\circ = \frac{1}{3}\sqrt{3}$; find $\tan 75^\circ$; $\tan 15^\circ$.

(7.) Prove $\mathbf{cot}\ (x+y) = \dfrac{\cot x \cot y - 1}{\cot y + \cot x}$. (30)

Hint.—Divide formula (13) by formula (11).

(8.) Prove $\mathbf{cot}\ (x-y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$. (31)

(9.) Prove $\cos(30 + y) - \cos(30^\circ - y) = -\sin y$.

(10.) Prove $\sin 3x = 3 \sin x - 4 \sin^3 x$.

Hint.—$\sin 3x = \sin(x + 2x)$.

(11.) Prove $\cos 3x = 4 \cos^3 x - 3 \cos x$.

(12.) If x and y are acute and $\tan x = \frac{1}{2}$, $\tan y = \frac{1}{3}$, prove that $(x + y) = 45^\circ$.

(13.) Prove that $\tan(x + 45^\circ) = \dfrac{1 + \tan x}{1 - \tan x}$.

(14.) Given $\sin y = \frac{2}{3}$ and y acute; find $\sin \frac{1}{2}y$, $\cos \frac{1}{2}y$, and $\tan \frac{1}{2}y$.

(15.) Given $\cos x = -\frac{3}{5}$ and x in quadrant II; find $\sin 2x$ and $\cos 2x$.

(16.) Given $\cos 45^\circ = \frac{1}{2}\sqrt{2}$; find the functions of $22\frac{1}{2}^\circ$.

(17.) Given $\tan x = 2$ and x acute; find $\tan \frac{1}{2}x$.

(18.) Given $\cos 30^\circ = \frac{1}{2}\sqrt{3}$; find the functions of 15°.

(19.) Given $\cos 90^\circ = 0$; find the functions of 45°.

(20.) Find $\sin 5x$ in terms of $\sin x$.

(21.) Find $\cos 5x$ in terms of $\cos x$.

(22.) Prove $\sin(x + y + z) = \sin x \cos y \cos z + \cos x \sin y \cos z + \cos x \cos y \sin z - \sin x \sin y \sin z$.

Hint.—$\sin(x+y+z) = \sin(x+y) \cos z + \cos(x+y) \sin z$.

(23.) Given $\tan 2x = 3 \tan x$; find x.

(24.) Prove $\sin 32^\circ + \sin 28^\circ = \cos 2^\circ$.

(25.) Prove $\tan x + \cot x = 2 \csc 2x$.

(26.) Prove $(\sin \frac{1}{2}x + \cos \frac{1}{2}x)^2 = 1 + \sin x$.

(27.) Prove $(\sin \frac{1}{2}x - \cos \frac{1}{2}x)^2 = 1 - \sin x$.

(28.) Prove $\cos 2x = \cos^4 x - \sin^4 x$.

(29.) Prove $\tan(45° + x) + \tan(45° - x) = 2 \sec 2x$.

(30.) Prove $\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$.

(31.) Prove $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$.

(32.) Prove $\frac{1 + \sin 2x}{1 - \sin 2x} = \left(\frac{\tan x + 1}{\tan x - 1}\right)^2$.

(33.) Prove $\tan \frac{1}{2} x = \frac{\sin x}{1 + \cos x}$.

(34.) Prove $\cot \frac{1}{2} x = \frac{\sin x}{1 - \cos x}$.

(35.) Express as a product $\frac{\cos x - \cos y}{\cos x + \cos y}$.*

Hint.—
$$\frac{\cos x - \cos y}{\cos x + \cos y} = \frac{-2 \sin \frac{1}{2}(x+y) \sin \frac{1}{2}(x-y)}{2 \cos \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y)}$$
$$= -\tan \frac{1}{2}(x+y) \tan \frac{1}{2}(x-y).$$

(36.) Express as a product $\frac{\tan x + \tan y}{\cot x + \cot y}$.

(37.) Prove $1 - \tan x \tan y = \frac{\cos(x+y)}{\cos x \cos y}$.

THE INVERSE TRIGONOMETRIC FUNCTIONS

40. *Def.*—The expressions $\sin^{-1} a$, $\cos^{-1} a$, $\tan^{-1} a$, etc., denote respectively an angle whose sine is a, an angle whose cosine is a, an angle whose tangent is a, etc. They are called the **inverse sine** of a, the **inverse cosine** of a, the **inverse tangent** of a, etc., and are the **inverse trigonometric functions.**

$\sin^{-1} a$ is an angle whose sine is equal to a, and hence denotes, not a single definite angle, but each and every angle whose sine is a.

* Since quantities cannot be added or subtracted by the ordinary operations with logarithms, an expression must be reduced to a form in which no addition or subtraction is required, to be convenient for logarithmic computation.

Thus, if $\sin x = \frac{1}{2}$, $x = 30°$, $150°$, $(30° + 360°)$, etc.,
and $\sin^{-1}\frac{1}{2} = 30°$, $150°$, $(30° + 360°)$, etc.

Remark.—The sine or cosine of an angle cannot be less than -1 or greater than $+1$; hence $\sin^{-1}a$ and $\cos^{-1}a$ have no meaning unless a is between -1 and $+1$. In a similar manner we see that $\sec^{-1}a$ and $\csc^{-1}a$ have no meaning if a is between -1 and $+1$.

EXERCISES

41. (1.) Find the following angles in degrees:

$\sin^{-1}\frac{1}{2}\sqrt{2}$, $\tan^{-1}(-1)$, $\sin^{-1}(-\frac{1}{2})$.
$\cos^{-1}\frac{1}{2}$, $\cos^{-1}1$,

(2.) If $x = \cot^{-1}\frac{1}{8}$, find $\tan x$.

(3.) If $x = \sin^{-1}\frac{3}{5}$, find $\cos x$ and $\tan x$.

(4.) Find $\sin(\tan^{-1}\frac{1}{3}\sqrt{3})$.

(5.) Find $\sin(\cos^{-1}\frac{4}{5})$.

(6.) Find $\cot(\tan^{-1}\frac{1}{17})$.

(7.) Given $\sin^{-1}a = 2\cos^{-1}a$, and both angles acute; find a.

(8.) Given $\sin^{-1}a = \cos^{-1}a$; find the values of $\sin^{-1}a$ less than $360°$.

(9.) Given $\tan^{-1}1 = \frac{1}{4}\tan^{-1}0$, and both angles less than $360°$; find the angles.

(10.) Given $\sin^{-1}a = \cos^{-1}a$ and $\sin^{-1}a + \cos^{-1}a = 450°$; find $\sin^{-1}a$.

(11.) Prove $\sin(\cos^{-1}a) = \pm\sqrt{1-a^2}$.

Hint.— Let $x = \cos^{-1}a$; then $a = \cos x$,
$\sin x = \pm\sqrt{1-\cos^2 x} = \pm\sqrt{1-a^2}$.

(12.) Prove $\tan(\tan^{-1}a + \tan^{-1}b) = \dfrac{a+b}{1-ab}$.

(13.) Prove $\tan(\tan^{-1}a - \tan^{-1}b) = \dfrac{a-b}{1+ab}$.

(14.) Prove $\cos(2\cos^{-1}a) = 2a^2 - 1$.

(15.) Prove $\sin(2\cos^{-1}a) = \pm 2a\sqrt{1-a^2}$.

(16.) Prove $\tan(2\tan^{-1}a) = \dfrac{2a}{1-a^2}$.

(17.) Prove $\cos(2\tan^{-1}a) = \dfrac{1-a^2}{1+a^2}$.

(18.) Prove $\sin(\sin^{-1}a + \cos^{-1}b) = ab \pm \sqrt{(1-a^2)(1-b^2)}$.

CHAPTER IV

THE OBLIQUE TRIANGLE

DERIVATION OF FORMULAS

42. The formulas derived in this and the succeeding articles reduce the solution of the oblique triangle to its simplest form.

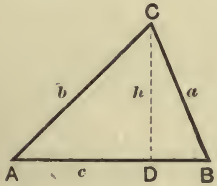


FIG. 1

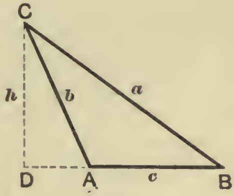


FIG. 2

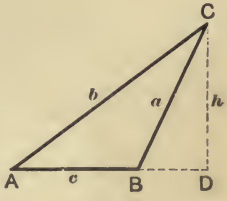


FIG. 3

Draw the perpendicular CD. Let $CD = h$,

Then $$\frac{h}{b} = \sin A;$$

$$\left(\text{In Fig. 2 } \frac{h}{b} = \sin(180° - A) = \sin A\right)$$

and $$\frac{h}{a} = \sin B.$$

$$\left(\text{In Fig. 3 } \frac{h}{a} = \sin(180° - B) = \sin B.\right)$$

By division we obtain,

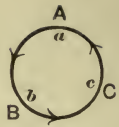
$$\frac{a}{b} = \frac{\sin A}{\sin B}. \tag{32}$$

Remark.—This formula expresses the fact that the ratio of two sides of an oblique triangle is equal to the ratio of the sines of the angles opposite, and does not in any respect depend upon which side has been taken as the base. Hence if the letters are advanced one step, as shown in the figure, we obtain, as another form of the same formula,

$$\frac{b}{c}=\frac{\sin B}{\sin C}.$$

Repeating the process, we obtain

$$\frac{c}{a}=\frac{\sin C}{\sin A}.$$



The same procedure may be applied to all the formulas for the solution of oblique triangles. Henceforth only one expression of each formula will be given.

Formula (32) *is used for the solution of triangles in which a side and two angles, or two sides and an angle, opposite one of them are given.*

43. We obtain from formula (32) by division and composition,

$$\frac{a-b}{a+b}=\frac{\sin A-\sin B}{\sin A+\sin B}.$$

By formula (29), denoting the angles by A and B, instead of u and v,

$$\frac{\sin A-\sin B}{\sin A+\sin B}=\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}.$$

Therefore,
$$\boldsymbol{\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}}. \qquad (33)$$

This formula is used for the solution of triangles in which two sides and the included angle are given.

44. Whether A is acute or obtuse, we have

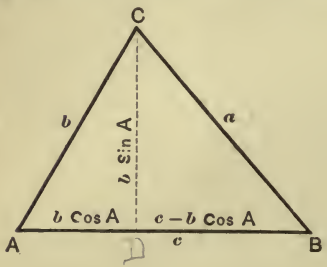


FIG. 1

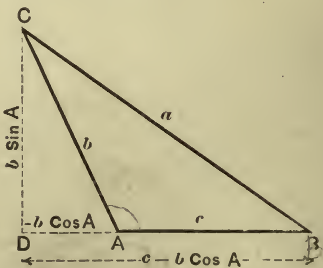


FIG. 2

(If A is acute (Fig. 1), $AD=b\cos A$, $DB=AB-AD=c-b\cos A$, $CD=b\sin A$. If A is obtuse (Fig. 2), $AD=b\cos(180°-A)=-b\cos A$, $DB=AB+AD=c-b\cos A$, $CD=b\sin(180°-A)=b\sin A$.)

$$a^2=(c-b\cos A)^2+(b\sin A)^2,$$
$$=c^2-2\,bc\cos A+b^2(\cos^2 A+\sin^2 A).$$

Therefore, $\boldsymbol{a^2=b^2+c^2-2bc\cos A.}$ (34)

This formula is used in deriving formula (37).

It is also used in the solution without logarithms of triangles of which two sides and the included angle or three sides are given.

45. From formula (34), $\cos A=\dfrac{b^2+c^2-a^2}{2bc}.$

From formula (22), § 37,

$$2\sin^2\tfrac{1}{2}A=1-\cos A=1-\frac{b^2+c^2-a^2}{2bc}.$$

Hence $$2\sin^2\tfrac{1}{2}A=\frac{2bc+a^2-b^2-c^2}{2bc},$$
$$=\frac{a^2-(b-c)^2}{2bc},$$
$$=\frac{(a-b+c)(a+b-c)}{2bc}.$$

Let $s=\dfrac{a+b+c}{2}$, then $(a-b+c)=2(s-b)$, and $(a+b-c)=2(s-c)$.

Substituting, $2\sin^2\tfrac{1}{2}A=\dfrac{2(s-b)(s-c)}{bc}.$

Hence $$\sin\tfrac{1}{2}A=\sqrt{\frac{(s-b)(s-c)}{bc}}.\text{*} \qquad (35)$$

From formula (23), § 37,

$$2\cos^2\tfrac{1}{2}A=1+\cos A=\frac{2bc+b^2+c^2-a^2}{2bc},$$
$$=\frac{2s(s-a)}{bc}.$$

* In extracting the root the plus sign is chosen because it is known that $\sin\frac{1}{2}A$ is positive.

Hence $$\cos\tfrac{1}{2}A=\sqrt{\frac{s(s-a)}{bc}}. \qquad (36)$$

Dividing (35) by (36), we obtain

$$\mathbf{\tan\tfrac{1}{2}A=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}}, \qquad (37)$$

$$=\sqrt{\frac{(s-a)(s-b)(s-c)}{s(s-a)^2}},$$

$$=\frac{1}{s-a}\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

Let $$K=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}},$$

$$\mathbf{\tan\tfrac{1}{2}A=\frac{K}{s-a}}. \qquad (38)$$

Formulas (37) *and* (38) *are used to find the angles of a triangle when the three sides are given.*

FORMULAS FOR THE AREA OF A TRIANGLE

46. Denote the area by S.

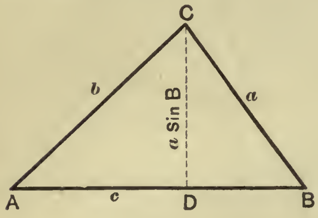


FIG. 1

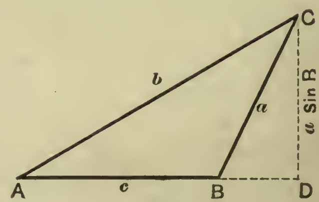


FIG. 2

(In Fig. 1, $CD=a\sin B$; in Fig. 2, $CD=a\sin(180°-B)=a\sin B$.)

In Figs. 1 and 2, $$S=\tfrac{1}{2}c.CD.$$

Hence $$\mathbf{S=\tfrac{1}{2}ac\sin B}. \qquad (39)$$

From formula (17),

$$\sin B=2\sin\tfrac{1}{2}B\cos\tfrac{1}{2}B.$$

Substituting for $\sin\frac{1}{2}B$ and $\cos\frac{1}{2}B$ the values found in formulas (35) and (36), we obtain

$$\sin B = \frac{2}{ac}\sqrt{s(s-a)(s-b)(s-c)}.$$

Therefore, $$\boldsymbol{S=\sqrt{s(s-a)(s-b)(s-c)}}. \qquad (40)$$

This formula may also be written,

$$\boldsymbol{S=sK}. \qquad (41)$$

Formula (39) *is used to find the area of a triangle when two sides and the included angle are known; formula* (40) *or formula* (41), *when the three sides are known.*

THE AMBIGUOUS CASE

47. The given parts are two sides, and the angle opposite one of them.

Let these parts be denoted by a, b, A.

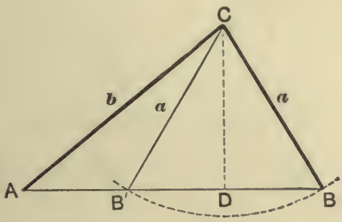


FIG. 1

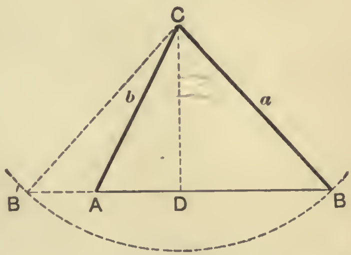


FIG. 2

If a is less than b and greater than the perpendicular CD (Fig. 1), there are the two triangles ACB and ACB', which contain the given parts, or, in other words, there are two solutions.

If a is greater than b (Fig. 2), there is one solution.

If a is equal to the perpendicular CD, there is one solution, the right triangle ACD.

If the given value of a is less than CD, evidently there can be no triangle containing the given parts.

Since $CD = b \sin A$, there is no solution when $a < b \sin A$; there is one solution, the right triangle ACD when $a = b \sin A$; there are two solutions when $a < b$ and $> b \sin A$.

48. Case I.—*Given a side and two angles.*

EXAMPLE

Given $a = 36.738$, $A = 36° \; 55' \; 54''$, $B = 72° \; 5' \; 56''$,

$$C = 180° - (A + B) = 180° - 109° \; 1' \; 50'' = 70° \; 58' \; 10''.$$

To find b.

$$\frac{b}{a} = \frac{\sin B}{\sin A}$$

$$\begin{aligned} \log a &= 1.56512 \\ \log \sin B &= 9.97845 - 10 \\ \operatorname{colog} \sin A &= 0.22123 \\ \hline \log b &= 1.76480 \\ b &= 58.184 \end{aligned}$$

To find c.

$$\frac{c}{a} = \frac{\sin C}{\sin A}$$

$$\begin{aligned} \log a &= 1.56512 \\ \log \sin C &= 9.97559 - 10 \\ \operatorname{colog} \sin A &= 0.22123 \\ \hline \log c &= 1.76194 \\ c &= 57.80 \end{aligned}$$

Check.

Determine b from c, C, and B by the formula

$$\frac{b-a}{b+a} = \frac{\tan \frac{1}{2}(B-A)}{\tan \frac{1}{2}(B+A)}.$$

This check is long, but is quite certain to reveal an error. A check which is shorter, but less sure, is

$$\frac{b}{c} = \frac{\sin B}{\sin C}.$$

Solve the following triangles:

(1.) Given $a = 567.25$, $A = 11° \; 15'$, $B = 47° \; 12'$.

(2.) Given $a = 783.29$, $A = 81° \; 52'$, $B = 42° \; 27'$.

(3.) Given $c = 1125.2$, $A = 79° \; 15'$, $B = 55° \; 11'$.

(4.) Given $b = 15.346$, $B = 15° \; 51'$, $C = 58° \; 10'$.

(5.) Given $a = 5301.5$, $A = 69° \; 44'$, $C = 41° \; 18'$.

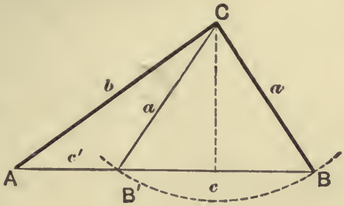
(6.) Given $b = 1002.1$, $A = 48° \; 59'$, $C = 76° \; 3'$.

49. Case II.—*Given two sides of a triangle and the angle opposite one of them.*

3, 4

EXAMPLE

Given $a = 23.203$, $b = 35.121$, $A = 36° \; 8' \; 10''$.



To find B and B'.

$$\frac{\sin B}{\sin A} = \frac{b}{a}$$

$$\begin{aligned} \log b &= 1.54556 \\ \log \sin A &= 9.77064 - 10 \\ \text{colog}\, a &= 8.63445 - 10 \\ \hline \log \sin B &= 9.95065 - 10 \\ B &= 63° \; 12' \\ B' = 180° - B &= 116° \; 48' \end{aligned}$$

To find C and C'.

$$C = 180° - (A + B) = 80° \; 39' \; 50''$$

$$C' = 180° - (A + B') = 27° \; 3' \; 50''$$

To find c and c'.

$$\frac{c}{a} = \frac{\sin C}{\sin A}$$

$$\begin{aligned} \log a &= 1.36555 \\ \log \sin C &= 9.99421 - 10 \\ \text{colog} \sin A &= 0.22936 \\ \hline \log c &= 1.58912 \\ c &= 38.825 \end{aligned}$$

$$\begin{aligned} \log a &= 1.36555 \\ \log \sin C' &= 9.65800 - 10 \\ \text{colog} \sin A &= 0.22936 \\ \hline \log c' &= 1.25291 \\ c' &= 17.902 \end{aligned}$$

Check.

Determine b from c, C, and B by the formula

$$\frac{b-a}{b+a} = \frac{\tan \frac{1}{2}(B-A)}{\tan \frac{1}{2}(B+A)}.$$

This check is long, but is quite certain to reveal an error. A check which is shorter, but less sure, is

$$\frac{b}{c} = \frac{\sin B}{\sin C}.$$

(1.) How many solutions are there in each of the following?

(1.) $A = 30°$, $a = 15$, $b = 20$;

(2.) $A = 30°$, $a = 10$, $b = 20$;

(3.) $B = 30°$, $a = 8$, $b = 20$;

(4.) $B = 37° \; 23'$, $a = 9.1$, $b = 7.5$. 2

v13

Solve the following triangles, finding all possible solutions:

(2.) Given $A = 147° \ 12'$, $a = 0.63735$, $b = 0.34312$.

(3.) Given $A = 24° \ 31'$, $a = 1.7424$, $b = 0.96245$.

(4.) Given $A = 21° \ 21'$, $a = 45.693$, $b = 56.723$. $c' = 12.069$

(5.) Given $A = 61° \ 16'$, $a = 9.5124$, $b = 12.752$.

(6.) Given $C = 22° \ 32'$, $a = 0.78727$, $c = 0.47311$.

50. CASE III.—*Given two sides and the included angle.*

EXAMPLE

Given $a = 41.003$, $b = 48.718$, $C = 68° \ 33' \ 58''$; find the remaining parts and the area.

To find A and B.

$$\frac{\tan\frac{1}{2}(B-A)}{\tan\frac{1}{2}(B+A)} = \frac{b-a}{b+a}.$$

$$\begin{aligned} b - a &= 7.715 \\ b + a &= 89.721 \\ \tfrac{1}{2}(B+A) &= 55° \ 43' \ 1''. \end{aligned}$$

$$\begin{aligned} \log(b-a) &= 0.88734 \\ \operatorname{colog}(b+a) &= 8.04710 - 10 \\ \log\tan\tfrac{1}{2}(B+A) &= 0.16639 \\ \hline \log\tan\tfrac{1}{2}(B-A) &= 9.10083 - 10 \\ \tfrac{1}{2}(B-A) &= 7° \ 11' \ 20'' \\ \tfrac{1}{2}(B+A) &= 55° \ 43' \ 1'' \\ \hline B &= 62° \ 54' \ 21'' \\ A &= 48° \ 31' \ 41'' \end{aligned}$$

To find c.

$$\frac{c}{a} = \frac{\sin C}{\sin A}.$$

$$\begin{aligned} \log a &= 1.61281 \\ \log\sin C &= 9.96888 - 10 \\ \operatorname{colog}\sin A &= 0.12535 \\ \hline \log c &= 1.70704 \\ c &= 50.938 \end{aligned}$$

To find the area.

$$S = \tfrac{1}{2}ab\sin C$$

$$\begin{aligned} \log\tfrac{1}{2} &= 9.69897 - 10 \\ \log a &= 1.61281 \\ \log b &= 1.68769 \\ \log\sin C &= 9.96888 - 10 \\ \hline \log S &= 2.96835 \\ S &= 929.72 \end{aligned}$$

Check.

$$\frac{\sin C}{\sin B} = \frac{c}{b}.$$

$$\begin{aligned} \log\sin B &= 9.94951 - 10 \\ \log c &= 1.70704 \\ \operatorname{colog} b &= 8.31231 - 10 \\ \hline \log\sin C &= 9.96886 - 10 \end{aligned}$$

Solve the following triangles, and also find their areas:

(1.) Given $A = 41° 15'$, $b=0.14726$, $c=0.10971$. (1)

(2.) Given $C = 58° 47'$, $b=11.726$, $a=16.147$.

(3.) Given $B = 49° 50'$, $a=103.74$, $c=99.975$.

(4.) Given $A = 33° 31'$, $b=0.32041$, $c=0.9203$.

(5.) Given $C=128° 7'$, $b=17.738$, $a=60.571$.

51. CASE IV.—*Given the three sides.*

EXAMPLE

Given $a=32.456$, $b=41.724$, $c=53.987$; find the angles and area.

$$s=64.084$$
$$(s-a)=31.628$$
$$(s-b)=22.360$$
$$(s-c)=10.097$$

$$K=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

$$\log(s-a)=1.50007$$
$$\log(s-b)=1.34947$$
$$\log(s-c)=1.00419$$
$$\text{colog } s=8.19325-10$$
$$2)\underline{2.04698}$$
$$\log K=1.02349$$

To find A.

$$\tan\tfrac{1}{2}A=\frac{K}{s-a}.$$

$$\log K=1.02349$$
$$\log(s-a)=1.50007 \quad \text{sub.}$$
$$\log\tan\tfrac{1}{2}A=9.52342-10$$
$$\tfrac{1}{2}A=18° 27' 23''$$
$$A=36° 54' 46''$$

To find B.

$$\tan\tfrac{1}{2}B=\frac{K}{s-b}.$$

$$\log K=1.02349$$
$$\log(s-b)=1.34947 \quad \text{sub.}$$
$$\log\tan\tfrac{1}{2}B=9.67402-10$$
$$\tfrac{1}{2}B=25° 16' 16''$$
$$B=50° 32' 32''$$

*To find C.**

$$\tan\tfrac{1}{2}C=\frac{K}{s-c}.$$

$$\log K=1.02349$$
$$\log(s-c)=1.00419 \quad \text{sub.}$$
$$\log\tan\tfrac{1}{2}C=0.01930$$
$$\tfrac{1}{2}C=46° 16' 22''$$
$$C=92° 32' 44''$$

Check.

$$(A+B+C)=180° 0' 2''.$$

Find the angles and areas of the following triangles:

(1.) Given $a=38.516$, $b=44.873$, $c=14.517$. (1)

(2.) Given $a=2.1158$, $b=3.5854$, $c=3.5679$.

* C could be found from $(A+B)=(180°-C)$, but for the sake of the check it is worked out independently.

1 50   1, 4 p. 48

1 (3)   1, 3, 3, 5, 7

(3.) Given $a=82.818$, $b=99.871$, $c=36.363$.

(4.) Given $a=36.789$, $b=11.698$, $c=33.328$.

(5.) Given $a=113.03$, $b=131.17$, $c=114.29$.

(6.) Given $a=\ .9763$, $b=1.2489$, $c=1.6543$.

EXERCISES

52. (1.) A tree, A, is observed from two points, B and C, 1863 ft. apart on a straight road. The angle BCA is $36° \, 43'$, and the angle CBA is $57° \, 21'$. Find the distance of the tree from the nearer point.

(2.) Two houses, A and B, are 3876 yards apart. How far is a third house, C, from A, if the angles ABC and BAC are $49° \, 17'$ and $58° \, 18'$ respectively?

(3.) A triangular lot has one side 285.4 ft. long. The angles adjacent to this side are $41° \, 22'$ and $31° \, 19'$. Find the length of a fence around it, and its area.

(4.) The two diagonals of a parallelogram are 8 and 10, and the angle between them is $53° \, 8'$; find the sides of the parallelogram.

(5.) Two mountains, A and B, are 9 and 13 miles from a town, C; the angle ACB is $71° \, 36' \, 37''$. Find the distance between the mountains.

(6.) Two buoys are 2789 ft. apart, and a boat is 4325 ft. from the nearer buoy. The angle between the lines from the buoys to the boat is $16° \, 13'$. How far is the boat from the farther buoy? Are there two solutions?

(7.) Given $a=64.256$, $c=19.278$, $C=16° \, 19' \, 11''$; find the difference in the areas of the two triangles which have these parts.

(8.) A prop 13 ft. long is placed 6 ft. from the base of an embankment, and reaches 8 ft. up its face; find the slope of the embankment.

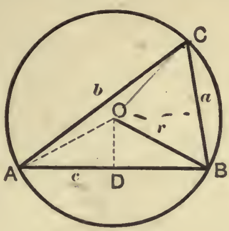
(9.) The bounding lines of a township form a triangle of which the sides are 8.943 miles, 7.2415 miles, and 10.817 miles; find the area of the township.

(10.) Prove that the diameter of a circle circumscribed about a triangle is equal to any side of the triangle divided by the sine of the angle opposite.

$a = .01235$

$b = .10037$

4. 10 and 15



Hint.—By Geometry, angle $AOB = 2C$.

Draw OD perpendicular to AB.

Angle $DOB = \frac{1}{2} AOB = C$.

$DB = r \sin DOB = r \sin C$.

Hence $$c = 2r \sin C,$$

or $$2r = \frac{c}{\sin C}.$$

(11.) The distances AB, BC, and AC, between three cities, A, B, and C, are 12 miles, 14 miles, and 17 miles respectively. Straight railroads run from A to B and C. What angle do they make?

(12.) A balloon is directly over a straight road, and between two points on the road from which it is observed. The points are 15847 ft. apart, and the angles of elevation are found to be 49° 12′ and 53° 29′ respectively. Find the distance of the balloon from each of the points.

(13.) To find the distance from a point A to a point B on the opposite side of a river, a line, AC, and the angles CAB and ACB were measured and found to be 315.32 ft., 58° 43′, and 57° 13′ respectively. Find the distance AB. 222.1

(14.) A building 50 ft. high is situated on the slope of a hill. From a point 200 ft. away the building subtends an angle of 12° 13′. Find the distance from this point to the top of the building.

(15.) Prove that the area of a quadrilateral is equal to one-half the product of the diagonals by the sine of the angle between them.

(16.) From points A and B, at the bow and stern of a ship respectively, the foremast, C, of another ship is observed. The points A and B are 300 ft. apart; the angles ABC and BAC are found to be

65° 31′ and 110° 46′ respectively. What is the distance between the points *A* and *C* of the two ships?

(17.) Two steamers leave the same port at the same time; one sails, directly northwest, 12 miles an hour; the other 17 miles an hour, in a direction 67° south of west. How far apart will they be at the end of three hours?

(18.) Two stakes, *A* and *B*, are on opposite sides of a stream; a third stake, *C*, is set 62 ft. from *A*; the angles *ACB* and *CAB* are found to be 50° 3′ 5″ and 61° 18′ 20″ respectively. How long is a rope connecting *A* and *B*?

(19.) To find the distance between two inaccessible mountain-tops, *A* and *B*, of practically the same height, two points, *C* and *D*, are taken one mile apart. The angle *CDA* is found to be 88° 34′, the angle *DCA* is 63° 8′, the angle *CDB* is 64° 27′, the angle *DCB* is 87° 9′. What is the distance?

(20.) Two islands, *B* and *C*, are distant 5 and 3 miles respectively from a light-house, *A*, and the angle *BAC* is 33° ~~14′~~; find the distance between the islands. 35′

(21.) Two points, *A* and *B*, are visible from a third point *C*, but not from each other; the distances *AC*, *BC*, and the angle *ACB* were measured, and found to be 1321 ft., 1287 ft., and 61° 22′ respectively. Find the distance *AB*. 1331.2

(22.) Of three mountains, *A*, *B*, and *C*, *B* is directly north of *C* 5 miles, *A* is 8 miles from *C* and 11 from *B*. How far is *A* south of *B*?

(23.) From a position 215.75 ft. from one end of a building and 198.25 ft. from the other end, the building subtends an angle of 53° 37′ 28″; find its length.

(24.) If the sides of a triangle are 372.15, 427.82, and 404.17; find the cosine of the smallest angle.

(25.) From a point 3 miles from one end of an island and 7 miles from the other end, the island subtends an angle of 33° 55′ 15″; find the length of the island.

(26.) A point is 13581 in. from one end of a wall 12342 in. long, and 10025 in. from the other end. What angle does the wall subtend at this point?

(27.) A straight road ascends a hill a distance of 213.2 ft., and is in-

20

clined 12° 2′ to the horizontal; a tree at the bottom of the hill subtends at the top an angle of 10° 5′ 16″. Find the height of the tree.

(28.) Two straight roads cross at an angle of 37° 50′ at the point *A*; ~~3~~ 4 miles distant on one road is the town *B*, and 5 miles distant on the other is the town *C*. How far are *B* and *C* apart?

(29.) Two stations, *A* and *B*, on opposite sides of a mountain, are both visible from a third station, *C*; $AC = 11.5$ miles, $BC = 9.4$ miles, and the angle $ACB = 59°$ ~~31~~ 41′. Find the distance from *A* to *B*.

(30.) To obtain the distance of a battery, *A*, from a point, *B*, of the enemy's lines, a point, *C*, 372.7 yards distant from *A* is taken; the angles *ACB* and *CAB* are measured and found to be ~~79°~~ 58° 53′ and 74° 35′ respectively. What is the distance *AB*?

(31.) A town, *B*, is 14 miles due west of another town, *A*. A third town, *C*, is 19 miles from *A* and 17 miles from *B*. How far is *C* west of *A*?

(32.) Two towns, *A* and *B*, are on opposite sides of a lake. *A* is 18 miles from a third town, *C*, and *B* is 13 miles from *C*; the angle *ACB* is 13° 17′. Find the distance between the towns *A* and *B*.

(33.) At a point in a level plane the angle of elevation of the top of a hill is 39° 51′, and at a point in the same direct line from the hill, but 217.2 feet farther away, the angle of elevation is ~~26°~~ 27° 53′. Find the height of the hill above the plane.

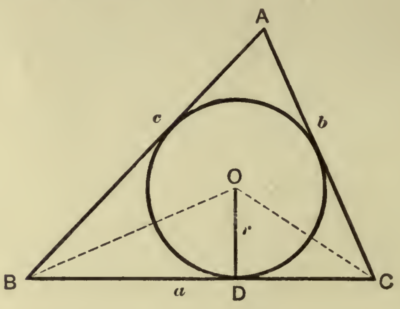
(34.) It is required to find the distance between two inaccessible points, *A* and *B*. Two stations, *C* and *D*, 2547 ft. apart, are chosen and the angles are measured; they are $ACB = 27° 21′$, $BCD = 33° 14′$, $BDA = 18° 17′$, and $ADC = 51° 23′$. Find the distance from *A* to *B*.

(35.) Two trains leave the same station at the same time on straight tracks inclined to each other 21° 12′. If their average speeds are 40 and ~~50~~ 46 miles an hour, how far apart will they be at the end of the first fifteen minutes?

(36.) A ship, *A*, is seen from a light-house, *B*; to determine its distance a point, *C*, 300 ft. from the light-house is taken and the angles *BCA* and *CBA* measured. If $BCA = 108° 34′$ and $CBA = 65° 27′$, what is the distance of the ship from the light-house?

28, 29, 30

(37.) Prove that the radius of the inscribed circle of a triangle is equal to $a \sin\frac{1}{2}B \sin\frac{1}{2}C \sec\frac{1}{2}A$.



Hint.—Draw OB, OC, and the perpendicular OD.

OB and OC bisect the angles B and C respectively, and $OD=r$.

$$a=BD+DC=r(\cot\tfrac{1}{2}B+\cot\tfrac{1}{2}C).$$

$$\cot\tfrac{1}{2}B+\cot\tfrac{1}{2}C=\frac{\sin\frac{1}{2}C\cos\frac{1}{2}B+\cos\frac{1}{2}C\sin\frac{1}{2}B}{\sin\frac{1}{2}B\sin\frac{1}{2}C},$$

$$=\frac{\sin\frac{1}{2}(B+C)}{\sin\frac{1}{2}B\sin\frac{1}{2}C}=\frac{\cos\frac{1}{2}A}{\sin\frac{1}{2}B\sin\frac{1}{2}C}.$$

Hence $$r=a\frac{\sin\frac{1}{2}B\sin\frac{1}{2}C}{\cos\frac{1}{2}A}=a\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C\sec\tfrac{1}{2}A.$$

CHAPTER V

CIRCULAR MEASURE—GRAPHICAL REPRESENTATION

CIRCULAR MEASURE

53. The length of the semicircumference of a circle is πR ($\pi = 3.14159+$); the angle the semicircumference subtends at the centre of the circle is $180°$. Hence an arc whose length is equal to the radius will subtend the angle $\frac{180°}{\pi}$; this angle is the unit angle of **circular measure**, and is called a **radian.**

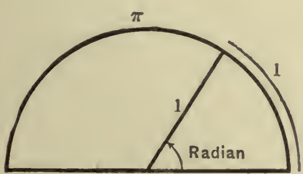


FIG. 1

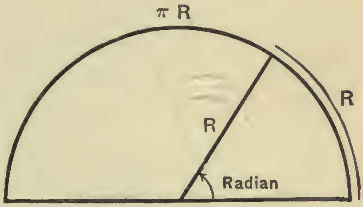


FIG. 2

If the radius of the circle is unity, an arc of *unit* length subtends a radian; hence in the *unit* circle the length of an arc represents the circular measure of the angle it subtends.

Thus, if the length of an arc is $\frac{\pi}{2}$, it subtends the angle $\frac{\pi}{2}$ radians.

Since one radian $= \frac{180°}{\pi}$, we have

$$90° = \frac{\pi}{2} \text{ radians,}$$

$$180° = \pi \text{ radians,}$$

$$270^\circ = \frac{3\pi}{2} \text{ radians,}$$

$$360^\circ = 2\pi \text{ radians, etc.}$$

The value of a radian in degrees and of a degree in radians are:

$$1 \text{ radian} = 57.29578^\circ,$$
$$= 57^\circ\ 17'\ 45''.$$
$$1^\circ = .0174533 \text{ radian.}$$

In the use of the circular measure it is customary to omit the word radian; thus we write $\frac{\pi}{2}$, π, etc., denoting $\frac{\pi}{2}$ radians, π radians, etc. On the other hand, the symbols $^\circ$ $'$ $''$ are always printed if an angle is measured in degrees, minutes, and seconds; hence there is no confusion between the systems.

EXERCISES

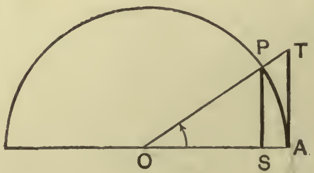
(1.) Express in circular measure 30°, 45°, 60°, 120°, 135°, 720°, 990°. (Take $\pi = 3.1416$.)

(2.) Express in degrees, minutes, and seconds the angles $\frac{\pi}{8}, \frac{\pi}{10}, \frac{1}{2}, \frac{7}{4}$.

(3.) What is the circular measure of the angle subtended by an arc of length 2.7 in., if the radius of the circle is 2 in.? if the radius is 5 in.?

54. The following important relations exist between the circular measure x of an angle and the sine and tangent of the angle.

(1.) *If x is less than $\frac{\pi}{2}$, $\sin x < x < \tan x$.*



Draw a circle of unit radius.

By Geometry, $SP < \text{arc } AP < AT$.

Hence $\sin x < x < \tan x$.

(2.) *As x approaches the limit* 0, $\frac{\sin x}{x}$ *and* $\frac{\tan x}{x}$ *approach the limit* 1.

Dividing $\sin x < x < \tan x$ by $\sin x$, we obtain

$$1 < \frac{x}{\sin x} < \frac{1}{\cos x}.$$

Inverting,
$$1 > \frac{\sin x}{x} > \frac{\cos x}{1}.$$

As x approaches the limit 0, $\cos x$ approaches the length of the radius, that is, 1, as a limit.

Therefore, $\frac{\sin x}{x}$ approaches the limit 1.

Dividing $1 > \frac{\sin x}{x} > \cos x$ by $\cos x$, we obtain

$$\frac{1}{\cos x} > \frac{\tan x}{x} > 1.$$

As x approaches the limit 0, $\cos x$ approaches the limit 1;

hence $\frac{1}{\cos x}$ approaches the limit 1.

Therefore, $\frac{\tan x}{x}$ approaches the limit 1.

PERIODICITY OF THE TRIGONOMETRIC FUNCTIONS

55. The sine of an angle x is the same as the sine of $(x+360°)$, $(x+720°)$, etc.—that is, of $(x+2n\pi)$, where n is any integer.

The sine is therefore said to be a periodic* function, having the period 360°, or 2π.

The same is true of the cosine, secant, and cosecant.

* If a function, denoted by $f(x)$, of a variable x, is such that $f(x+k)=f(x)$ for every value of x, k being a constant, the function $f(x)$ is periodic; if k is the least constant which possesses this property, k is the period of $f(x)$.

The tangent of an angle x is the same as the tangent of $(x+180°)$, $(x+360°)$, etc.—that is, of $(x+n\pi)$, where n is any integer.

The tangent is therefore a periodic function, having the period $180°$, or π.

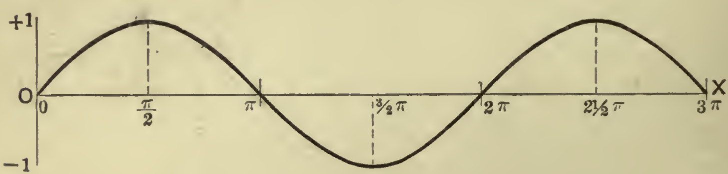
The same is true of the cotangent.

GRAPHICAL REPRESENTATION

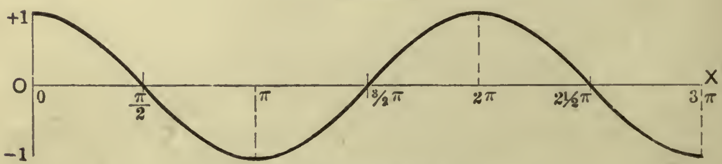
56. On the line OX lay off the distance $OA(=x)$ to represent the circular measure of the angle x. At the point A erect a perpendicular equal to $\sin x$. If perpendiculars are thus erected for each value of x, the curve passing through their extremities is called the sine curve.

If $\sin x$ is negative, the perpendicular is drawn downward.

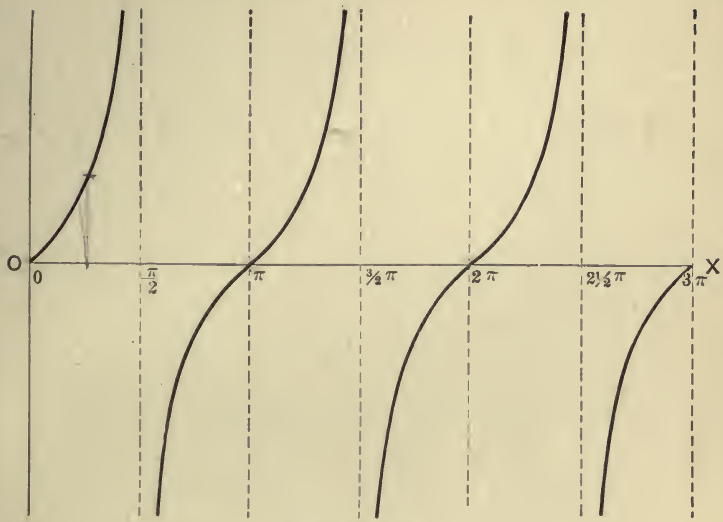
In a similar manner the cosine, tangent, cotangent, secant, and cosecant curves can be constructed.



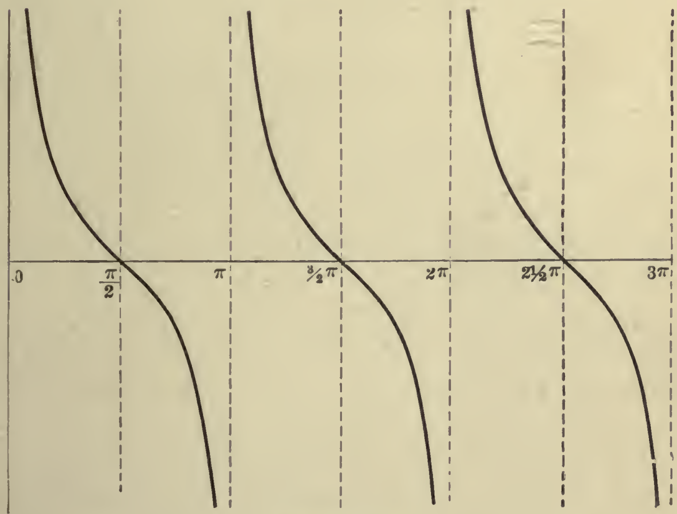
Sine Curve



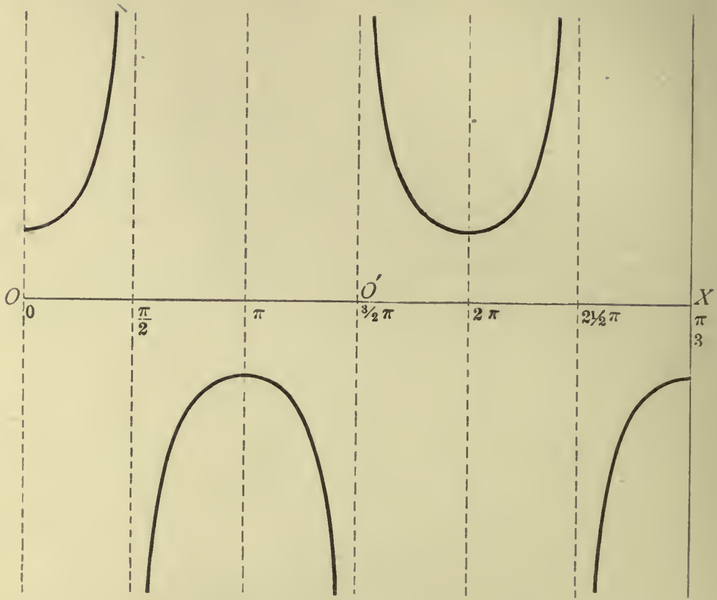
Cosine Curve



Tangent Curve



Cotangent Curve

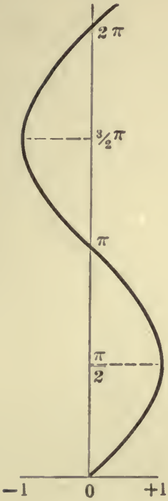


SECANT CURVE

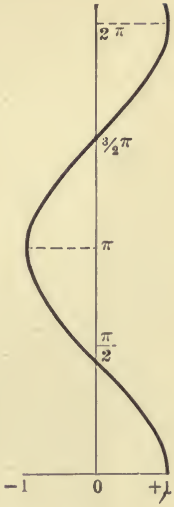
If the distances on OX are measured from O' instead of O, we obtain from the secant curve the cosecant curve.

In the construction of the inverse curves the number is represented by the distance to the right or left from O; the circular measure of the angle by the length of the perpendicular erected.

All of the preceding curves, except the tangent and cotangent curves, have a period of 2π along the line OX; that is, the curve extended in either direction is of the same form in each case between 2π and 4π, 4π and 6π,—2π and 0, etc., as between 0 and 2π, while the corresponding inverse curves repeat along the vertical line in the same period. The period of the tangent and cotangent curves is π.

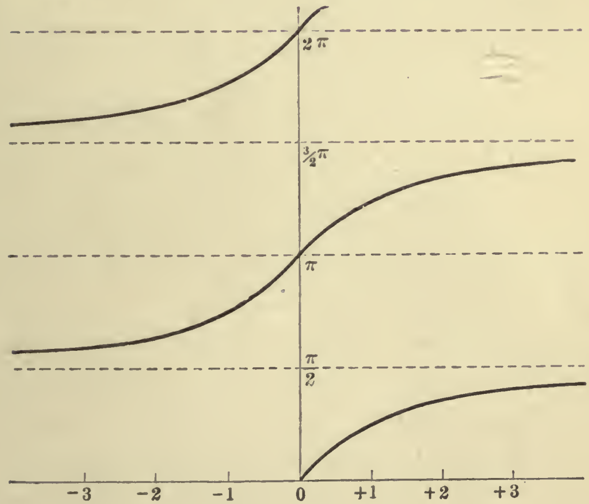


INVERSE SINE CURVE

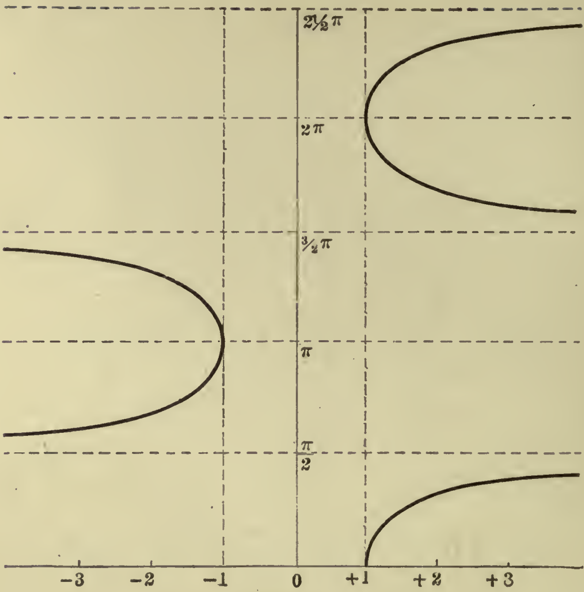


INVERSE COSINE CURVE

$\alpha = \cos^{-1} x$



INVERSE TANGENT CURVE



INVERSE SECANT

CHAPTER VI

COMPUTATION OF LOGARITHMS AND OF THE TRIGONOMETRIC FUNCTIONS—DE MOIVRE'S THEOREM —HYPERBOLIC FUNCTIONS

57. A convenient method of calculating logarithms and the trigonometric functions is to use infinite series. In works on the Differential Calculus it is shown that

$$\log_e(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots \qquad (1)$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots * \qquad (2)$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots \qquad (3)$$

Another development which we shall use later is

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots \qquad (4)$$

where $e=2.7182818\ldots$ is the base of the Naperian system of logarithms.

58. The series (1) converges only for values of x which satisfy the inequality $-1<x\leqq 1$. The series (2), (3), and (4) converge for all finite values of x.

It is to be noted that the logarithm in (1) is the Naperian, and the angle x in (2) and (3) is expressed in circular measure.

* $3!$ denotes $1\times2\times3$; $4!$ denotes $1\times2\times3\times4$, etc.

COMPUTATION OF LOGARITHMS

59. We first recall from Algebra the definition and some of the principal theorems of logarithms.

The logarithm to the base a of the number m is the number x which satisfies the equation,

$$a^x = m.$$

This is written $x = \log_a m$.

The logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers.

Thus $$\log_a mn = \log_a m + \log_a n.$$

The logarithm of the quotient of two numbers is equal to the logarithm of the dividend minus the logarithm of the divisor.

Thus $$\log_a \frac{m}{n} = \log_a m - \log_a n.$$

The logarithm of the power of a number is equal to the logarithm of the number multiplied by the exponent.

Thus $$\log_a m^p = p \log_a m.$$

To obtain the logarithm of a number to any base a from its Naperian logarithm, we have

$$\log_a m = \frac{\log_e m}{\log_e a} = M_a \log_e m,$$

where $M_a = \frac{1}{\log_e a}$; M_a is called the modulus of the system.

60. We proceed now to the computation of logarithms. The series (1) enables us to compute directly the Naperian logarithms of positive numbers not greater than 2.

Example.—To compute $\log_e \frac{3}{2}$ to five places of decimals.

Substitute $\frac{1}{2}$ for x in (1):

$$\log_e \frac{3}{2} = \log_e \left(1 + \frac{1}{2}\right) = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2^2} + \frac{1}{3} \cdot \frac{1}{2^3} - \frac{1}{4} \cdot \frac{1}{2^4} + \dots$$

If the result is to be correct to five places of decimals, we must take enough terms so that the remainder shall not affect the fifth decimal place. Now we

know by Algebra that in a series of which the terms are each less in numerical value than the preceding, and are also alternately positive and negative, the remainder is less in numerical value than its first term. Hence we need to take enough terms to know that the first term neglected would not affect the fifth place.

Positive terms		*Negative terms*	
$\frac{1}{2}$	$=0.5000000$	$\frac{1}{2}\cdot\frac{1}{2^2}$	$=0.1250000$
$\frac{1}{3}\cdot\frac{1}{2^3}$	$=.0416667$	$\frac{1}{4}\cdot\frac{1}{2^4}$	$=.0156250$
$\frac{1}{5}\cdot\frac{1}{2^5}$	$=.0062500$	$\frac{1}{6}\cdot\frac{1}{2^6}$	$=.0026042$
$\frac{1}{7}\cdot\frac{1}{2^7}$	$=.0011161$	$\frac{1}{8}\cdot\frac{1}{2^8}$	$=.0004883$
$\frac{1}{9}\cdot\frac{1}{2^9}$	$=.0002170$	$\frac{1}{10}\cdot\frac{1}{2^{10}}$	$=.0000977$
$\frac{1}{11}\cdot\frac{1}{2^{11}}$	$=.0000444$	$\frac{1}{12}\cdot\frac{1}{2^{12}}$	$=.0000203$
$\frac{1}{13}\cdot\frac{1}{2^{13}}$	$=.0000094$	$\frac{1}{14}\cdot\frac{1}{2^{14}}$	$=.0000044$
	$.5493036$		$.1438399$

Subtracting the sum of the negative from the sum of the positive terms, we obtain

$$\log_e\frac{3}{2}=.4054637.$$

Denote the sum of the remaining terms of the series by R. Then, by Algebra,

$$R<\frac{1}{15}\cdot\frac{1}{2^{15}}$$
$$<.0000021.$$

The error caused by retaining no more decimal places in the computation is less than .0000006. Hence the total error is less than .0000027. Therefore the result is correct to five decimal places.

61. As remarked, the series (1) does not enable us to calculate directly the logarithms of numbers greater than 2, but it can be readily transformed into a series which gives us the logarithm of any positive number.

Replacing x by $-x$ in (1), we obtain

$$\log_e (1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \dots$$

This series converges for $-1 \leqq x < 1$.

Subtracting this from (1), we obtain

$$\log_e (1+x) - \log_e (1-x) = \log_e \left(\frac{1+x}{1-x}\right)$$

$$= 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots\right), \quad (5)$$

$= \frac{m-n}{m+n}$

$x = \frac{y-1}{y+1}$, i.e. $y = \frac{1+x}{1-x}$

$x = \frac{1}{2y+1}$

which converges for $-1 < x < 1$.

Putting $y = \left(\frac{1+x}{1-x}\right)$, we see that y passes from 0 to ∞ as x passes from -1 to $+1$; hence, if we make this substitution in (5), we get a series

$$\log_e y = 2\left[\left(\frac{y-1}{y+1}\right) + \frac{1}{3}\left(\frac{y-1}{y+1}\right)^3 + \frac{1}{5}\left(\frac{y-1}{y+1}\right)^5 + \dots\right], \quad (6)$$

which converges for all positive values of y, and therefore enables us to compute the Naperian logarithm of any number.

From (5) we can get another series which is useful: put $x = \frac{1}{2y+1}$; then, as $\frac{1+x}{1-x} = \frac{y+1}{y}$, equation (5) gives us

$$\log_e \left(\frac{y+1}{y}\right) = 2\left(\frac{1}{2y+1} + \frac{1}{3}\cdot\frac{1}{(2y+1)^3} + \frac{1}{5}\cdot\frac{1}{(2y+1)^5} + \dots\right),$$

which converges for all positive values of y. Hence,

$$\log_e(y+1) = \log_e y + 2\left(\frac{1}{2y+1} + \frac{1}{3}\cdot\frac{1}{(2y+1)^3} + \frac{1}{5}\cdot\frac{1}{(2y+1)^5} + \dots\right). \quad (7)$$

This series gives us $\log_e(y+1)$, when $\log_e y$ is known. It converges more rapidly than (6), when y is greater than 2, and hence should be used under these circumstances.

62. To construct a table we need to compute directly only the logarithms of prime numbers, since the others can be obtained by the relation

$$\log xy = \log x + \log y.$$

Nat Log 5 = 1.6094379 | log 11 = 2.3978
6 = 1.7917595 | log 12 = 2.4849

Thus, to obtain the logarithms of the integers up to 10, we need to compute by series only the logarithms of the numbers 2, 3, 5, and 7.

(For $4=2^2$, $6=2.3$, $8=2^3$, $9=3^2$, $10=2.5$, and $\log 1=0$.)

In this case we are computing the logarithms of successive integers, and should therefore use (7).

63. *Example.*—Compute the Naperian logarithms of 2, 3, 4, and 5.

$$\log_e 2=2\left(\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{3^3}+\frac{1}{5}\cdot\frac{1}{3^5}+\frac{1}{7}\cdot\frac{1}{3^7}+\frac{1}{9}\cdot\frac{1}{3^9}+\ldots\right).$$

$$\begin{aligned}
\frac{1}{3}&=.3333333\\
\frac{1}{3}\cdot\frac{1}{3^3}&=.0123457\\
\frac{1}{5}\cdot\frac{1}{3^5}&=.0008230\\
\frac{1}{7}\cdot\frac{1}{3^7}&=.0000653\\
\frac{1}{9}\cdot\frac{1}{3^9}&=.0000056\\
&\overline{.3465729}\\
&\quad\quad\quad\;\; 2\\
\log_e 2&=\overline{.6931458}
\end{aligned}$$

Denote the sum of the remaining terms of this series by R.

Then, by Algebra,

$$R<\frac{1}{11}\cdot\frac{1}{3^{11}}\cdot\frac{1}{1-\frac{1}{9}},$$

or $\quad R<.000000573.$

The error caused by not retaining more places of decimals in the preceding column is less than .0000005.

Hence, the total error is less than .00000165.

Remark.—We should get the same series if we were to use (6).

$$\log_e 3=\log_e 2+2\left(\frac{1}{5}+\frac{1}{3}\cdot\frac{1}{5^3}+\frac{1}{5}\cdot\frac{1}{5^5}+\frac{1}{7}\cdot\frac{1}{5^7}+\ldots\right).$$

$$\begin{aligned}
\frac{1}{5}&=.2000000\\
\frac{1}{3}\cdot\frac{1}{5^3}&=.0026667\\
\frac{1}{5}\cdot\frac{1}{5^5}&=.0000640\\
\frac{1}{7}\cdot\frac{1}{5^7}&=.0000018\\
&\overline{.2027325}\\
&\quad\quad\quad\;\; 2\\
&\overline{.4054650}\\
\text{Add }\log_e 2&=\;\;.6931458\\
\log_e 3&=1.0986108
\end{aligned}$$

$$R<\frac{1}{9}\cdot\frac{1}{5^9}\cdot\frac{1}{1-\frac{1}{25}},$$

or $\quad R<.00000006.$

Noting the errors in the preceding column and in $\log_e 2$, we see that the total error is less than .00000217.

Remark.—If we were to use (6) to compute $\log_e 3$, we should have

$$\log_e 3 = 2\left[\frac{1}{2} + \frac{1}{3}\left(\frac{1}{2}\right)^3 + \frac{1}{5}\left(\frac{1}{2}\right)^5 + \frac{1}{7}\left(\frac{1}{2}\right)^7 \cdots\right].$$

This series converges much more slowly than the above, since its terms are multiples of powers of $\frac{1}{2}$, while the terms of the above are the same multiples of powers of $\frac{1}{5}$. Thus, we should be obliged to use eight instead of four terms to have the result correct to five places.

$$\log_e 4 = 2 \log_e 2 = 1.3862916.$$

$$\log_e 5 = \log_e 4 + 2\left(\frac{1}{9} + \frac{1}{3}\cdot\frac{1}{9^3} + \frac{1}{5}\cdot\frac{1}{9^5} + \ldots\right),$$

or $\log_e 5 = 1.60944$.

64. Proceeding in like manner, we may calculate any number of logarithms.

The following table gives the Naperian logarithms of the first ten integers:

$\log_e 1 = .00000$	$\log_e 6 = 1.79176$
$\log_e 2 = .69315$	$\log_e 7 = 1.94591$
$\log_e 3 = 1.09861$	$\log_e 8 = 2.07944$
$\log_e 4 = 1.38629$	$\log_e 9 = 2.19722$
$\log_e 5 = 1.60944$	$\log_e 10 = 2.30259$

The common logarithm of any number may be found by multiplying its Naperian logarithm by $M_{10} = .43429448$. § 59

Thus $\qquad \log_{10} 5 = \log_e 5 \times .43429448 = .69897.$

65. *Remark.*—If a table of logarithms were to be computed, the theory of interpolation and other special devices would be employed.

COMPUTATION OF TRIGONOMETRIC FUNCTIONS

66. Since $\tan x = \dfrac{\sin x}{\cos x}$, $\cot x = \dfrac{\cos x}{\sin x}$, etc., the computation of all the trigonometric functions depends upon that of the sine and cosine; thus the developments (2) and (3) suffice for all the trigonometric functions. Further, since the

log 13 = 2.5649426

sine or cosine of any angle is a sine or cosine of an angle $\overline{\underline{<}}\frac{\pi}{4}$, it is never necessary to take x greater than $\frac{\pi}{4}$ in the series (2) and (3). § 16

Since $\frac{\pi}{4}=0.785398\ldots<\frac{8}{10}$, these series converge rapidly; in fact, $\frac{1}{9!}=.000003$ does not affect the fifth decimal place, and $\frac{1}{11!}$ the seventh.

67. *Remark.*—In the systematic computation of tables we should not calculate the functions of each angle from the series independently. We should rather make use of the formulas (25) and (27) of § 38, thus obtaining

$$\sin nx = 2\cos x \sin(n-1)x - \sin(n-2)x,$$
$$\cos nx = 2\cos x \cos(n-1)x - \cos(n-2)x.$$

If our tables are to be at intervals of $1'$, we should calculate the sine and cosine of $1'$ by the series. The above expressions then enable us to find successively the sine and cosine of $2'$, $3'$, $4'$, etc., till we have the sine and cosine of all angles up to $30°$ at intervals of $1'$.

To obtain the sine and cosine of angles from $30°$ to $45°$ we should make use of these results by means of the formulas

$$\sin(30°+y)=\cos y-\sin(30°-y),$$
$$\cos(30°+y)=\cos(30°-y)-\sin y.$$

68. To employ series (2) and (3) in computing the sine and cosine we must first convert the angle into circular measure.

To do this we recall that

$$1°=.017453293,\quad 1'=.0002908882,\quad 1''=.000004848137.$$

Example.—To compute the sine and cosine of $12°\ 15'\ 39''$.

$$\begin{aligned} 12° &= .209439516 \\ 15' &= .004363323 \\ 39'' &= \underline{.000189076} \\ 12°\ 15'\ 39'' &= .213991915 \text{ in circular measure.} \end{aligned}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} -$$

$$x = .2139919$$

$$\frac{x^5}{5!} = .0000037$$

$$.2139956$$

$$\text{subtract } \frac{x^3}{3!} = .0016332$$

$$\sin x = .2123624$$

Correct to five decimal places.

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} -$$

$$1 = 1.0000000$$

$$\frac{x^4}{4!} = .0000874$$

$$1.0000874$$

$$\text{subtract } \frac{x^2}{2!} = .0228963$$

$$\cos x = .9771911$$

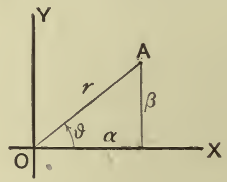
Correct to five decimal places.

DE MOIVRE'S THEOREM

69. In Algebra we learn that the complex number

$$a = \alpha + \beta\sqrt{-1} = \alpha + \beta i \tag{8}$$

may be represented graphically thus:



Take two lines, OX and OY, at right angles to each other. To the number a will correspond the point A, whose distances from the two lines of reference are β and α respectively.

This geometrical representation shows at once that we can also write a in the form

$$\boldsymbol{a = r(\cos\vartheta + i\sin\vartheta)}. \tag{9}$$

70. From Algebra we recall the definition of the sum of the complex numbers $a = \alpha + i\beta$ and $b = \gamma + i\delta$; namely

$$a + b = \alpha + \gamma + i(\beta + \delta).$$

Subtraction is defined as the inverse of addition, so that

$$a - b = \alpha - \gamma + i(\beta - \delta).$$

Multiplication is most conveniently defined when a and b are written in form (9). If

$$a=r\,(\cos\vartheta+i\sin\vartheta) \text{ and } b=s\,(\cos\phi+i\sin\phi),$$

their product is defined by the equation

$$ab=rs\,[\cos(\vartheta+\phi)+i\sin(\vartheta+\phi)]. \qquad (10)$$

Division is defined as the inverse of multiplication, so that

$$\frac{a}{b}=\frac{r}{s}\,[\cos(\vartheta-\phi)+i\sin(\vartheta-\phi)].$$

Finally, we recall that in an equation between complex numbers,

$$\alpha+i\beta=\gamma+i\delta,$$

we have

$$\alpha=\gamma, \quad \beta=\delta. \qquad (11)$$

71. Consider the different powers of the complex number

$$x=\cos\vartheta+i\sin\vartheta.$$

By (10) we have

$$x^2=(\cos\vartheta+i\sin\vartheta)\,(\cos\vartheta+i\sin\vartheta),$$
$$=\cos 2\vartheta+i\sin 2\vartheta.$$
$$x^3=x^2\,.\,x=(\cos 2\vartheta+i\sin 2\vartheta)\,(\cos\vartheta+i\sin\vartheta),$$
$$=\cos 3\vartheta+i\sin 3\vartheta.$$

And, in general, for any integer n,

$$x^n=(\cos\vartheta+i\sin\vartheta)^n=\cos n\vartheta+i\sin n\vartheta.$$

From this equation we have De Moivre's Theorem, which is expressed by the formula

$$(\cos\vartheta+i\sin\vartheta)^n=(\cos n\vartheta+i\sin n\vartheta). \qquad (12)$$

72. An interesting application of De Moivre's Theorem is the expansion of $\sin nx$ and $\cos nx$ in terms of $\sin x$ and $\cos x$. Expanding the left-hand side of (12) by the binomial theorem, and substituting x for ϑ, we have

$$\cos nx+i\sin nx=\cos^n x+n\cos^{n-1}x\,(i\sin x)+\frac{n(n-1)}{2\,!}\cos^{n-2}x$$
$$(i\sin x)^2+\frac{n\,.\,(n-1)\,(n-2)}{3\,!}\cos^{n-3}x\,(i\sin x)^3+\ldots$$

or

$$\cos nx + i \sin nx = \left(\cos^n x - \frac{n(n-1)}{2!}\cos^{n-2} x \sin^2 x + \ldots\right)$$
$$+ i\left[n \cos^{n-1} x \sin x - \frac{n(n-1)(n-2)}{3!}\cos^{n-3} x \sin^3 x + \ldots\right].$$

Equating real and imaginary parts, as in (11), we have

$$\cos nx = \cos^n x - \frac{n(n-1)}{2!}\cos^{n-2} x \sin^2 x + \ldots \tag{13}$$

$$\sin nx = n \cos^{n-1} x \sin x - \frac{n(n-1)(n-2)}{3!}\cos^{n-3} x \sin^3 x + \ldots \tag{14}$$

Example.—$n=5$.

$$\cos 5x = \cos^5 x - 10 \cos^3 x \sin^2 x + 5 \cos x \sin^4 x.$$
$$\sin 5x = 5 \cos^4 x \sin x - 10 \cos^2 x \sin^3 x + \sin^5 x.$$

THE ROOTS OF UNITY

73. We find another application of De Moivre's Theorem in obtaining the roots of unity. The n^{th} roots of unity are by definition the roots of the equation

$$x^n = 1.$$

Every equation has n roots and no more; hence, if we can find n distinct numbers which satisfy this equation we shall have all the n^{th} roots of unity.

Consider the n numbers

$$x_r = \cos \frac{2\pi r}{n} + i \sin \frac{2\pi r}{n},$$
$$r = 0, 1, 2, \ldots n-1.$$

Geometrically these numbers are represented by the n vertices of a regular polygon. They are, therefore, all different. We shall see now that they are precisely the n^{th} roots of unity.

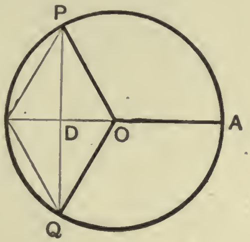
In fact, we have by (12),

$$x_r^n = \left(\cos \frac{2\pi r}{n} + i \sin \frac{2\pi r}{n}\right)^n,$$

$$=\cos\left(n\,.\,\frac{2\pi r}{n}\right)+i\sin\left(n\,.\,\frac{2\pi r}{n}\right),$$
$$=\cos 2\pi r+i\sin 2\pi r,$$
$$=1+i\,.\,0=1.$$

Therefore x_r is one of the roots of unity.

Thus the cube roots of unity are represented by the points A, P, and Q of the following figure. In the figure $OA=1$, angle $AOP=\frac{2\pi}{3}=120°$, angle $AOQ=\frac{4\pi}{3}=240°$; that is, the circumference is divided into three equal parts by the points A, P, and Q. Then $OD=\frac{1}{2}$, and $DP=DQ=\frac{1}{2}\sqrt{3}$. Hence we see from the method of representing a complex number given above that A represents $+1$, P represents $-\frac{1}{2}+i\,\frac{1}{2}\sqrt{3}$, Q represents $-\frac{1}{2}-i\,\frac{1}{2}\sqrt{3}$.



EXERCISES

74. (1.) Express $\sin 4x$ and $\cos 4x$ in terms of $\sin x$ and $\cos x$.

(2.) Express $\sin 6x$ and $\cos 6x$ in terms of $\sin x$ and $\cos x$.

(3.) Find the six 6th roots of unity.

(4.) Find the five 5th roots of unity.

THE HYPERBOLIC FUNCTIONS

75. The hyperbolic functions are defined by the equations

$$\sinh x=\frac{e^x-e^{-x}}{2}, \tag{15}$$

$$\cosh x=\frac{e^x+e^{-x}}{2}, \tag{16}$$

in which $\sinh x$ and $\cosh x$ denote the hyperbolic sine and

hyperbolic cosine of x respectively. These functions are called the hyperbolic sine and cosine on account of their relation to the hyperbola analogous to the relation of the sine and cosine to the circle. A natural and convenient way to arrive at the hyperbolic functions and to study their properties is by using complex numbers in the following manner. The series (2), (3), and (4) give the value of $\sin x$, $\cos x$, and e^x for every real value of x. These series also serve to *define* $\sin x$, $\cos x$, and e^x for *complex* values of x. In the more advanced parts of Algebra it is shown that the following fundamental formulas which we have proved only for a real variable,

$$\sin(x+y)=\sin x\cos y+\cos x\sin y, \tag{17}$$

$$\cos(x+y)=\cos x\cos y-\sin x\sin y, \tag{18}$$

$$e^{x+y}=e^x e^y, \tag{19}$$

hold unchanged when the variable is complex.

This fact enables us to calculate with ease $\sin x$, $\cos x$, and e^x for any complex value of the variable.

In so doing we are led directly to the hyperbolic functions. At the same time a relation between the trigonometric and hyperbolic functions is established by means of which the formulas of Chapter III. can be converted into corresponding formulas for the hyperbolic functions.

Taking x and y real and replacing y in (17), (18), and (19) by iy, we get

$$\sin(x+iy)=\sin x\cos iy+\cos x\sin iy,$$

$$\cos(x+iy)=\cos x\cos iy-\sin x\sin iy,$$

$$e^{x+iy}=e^x e^{iy}.$$

Thus the calculation of these functions when the variable is complex is made to depend upon the case where the variable is a pure imaginary.

If we replace x by ix in series (4) we obtain

$$e^{ix} = 1 + ix + \frac{(ix)^2}{2\,!} + \frac{(ix)^3}{3\,!} + \frac{(ix)^4}{4\,!} + \ldots$$
$$= \left(1 - \frac{x^2}{2\,!} + \frac{x^4}{4\,!} - \frac{x^6}{6\,!} + \ldots\right),$$
$$+ i\left(x - \frac{x^3}{3\,!} + \frac{x^5}{5\,!} - \frac{x^7}{7\,!} + \ldots\right).$$

A comparison with series (2) and (3) shows that these two series are $\cos x$ and $\sin x$ respectively; hence the important formula due to Euler—

$$\boldsymbol{e^{ix} = \cos x + i \sin x.} \tag{20}$$

This enables us to calculate e^{ix} from $\sin x$ and $\cos x$ when ix is a pure imaginary; that is, when x is real.

To find $\sin ix$ and $\cos ix$ replace x in (20) by ix; we obtain

$$e^{-x} = \cos ix + i \sin ix. \tag{21}$$

Again replacing x by $-ix$ in (20), we obtain

$$e^{x} = \cos ix - i \sin ix. \tag{22}$$

The sum and difference of (21) and (22) give

$$\boldsymbol{\cos ix = \frac{e^x + e^{-x}}{2} = \cosh x,} \tag{23}$$

$$\boldsymbol{\sin ix = \frac{i(e^x - e^{-x})}{2} = i \sinh x.} \tag{24}$$

If we compute the value of e^x by the aid of series (4) for a succession of values of x, we find that $\sinh x$ and $\cosh x$ are represented by the curves on page 76.

The system of formulas belonging to the hyperbolic functions is obtained from those of the trigonometric functions by using (23) and (24). This shows that for every formula in analytic trigonometry there exists a corresponding formula in hyperbolic trigonometry which we get by this sub-

stitution. In the examples which follow, this method is used to obtain important formulas in hyperbolic trigonometry.

Replacing x by $-ix$ in (23) and (24), we get

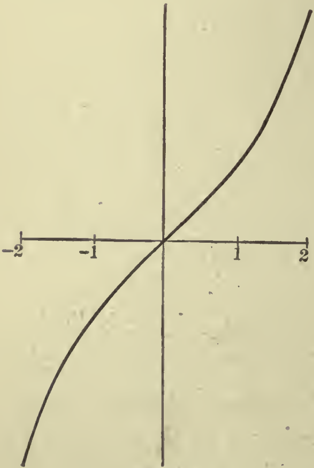
$$\cos x = \frac{e^{ix}+e^{-ix}}{2}, \tag{25}$$

$$\sin x = \frac{e^{ix}-e^{-ix}}{2i}, \tag{26}$$

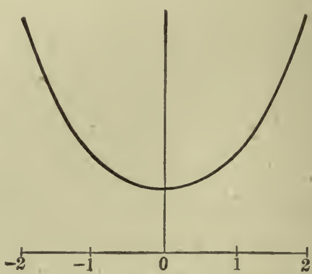
which are formulas frequently used.

Example.—$\sinh(x+y) = -i \sin i(x+y),$
$= -i[\sin ix \cos iy + \cos ix \sin iy],$
$= -i[i \sinh x \cosh y + i \cosh x \sinh y],$
$= \sinh x \cosh y + \cosh x \sinh y.$

Example.—$\sinh x + \sinh y = -i(\sin ix + \sin iy),$
$= -i\, 2 \sin \frac{1}{2} i(x+y) \cos \frac{1}{2} i(x-y),$
$= 2 \sinh \frac{1}{2}(x+y) \cosh \frac{1}{2}(x-y).$



sinh x



cosh x

EXERCISES

76. (1.) Prove $\sinh 0 = 0, \quad \cosh 0 = 1.$

(2.) Prove $\sinh \frac{1}{2}\pi i = i, \quad \cosh \frac{1}{2}\pi i = 0.$

(3.) Prove $\sinh \pi i = 0, \quad \cosh \pi i = -1.$

Prove that

(4.) $\sin(-ix) = -\sin ix.$

(5.) $\cos(-ix) = \cos ix.$

(6.) $\sinh(-x) = -\sinh x.$

(7.) $\cosh(-x) = \cosh x.$

Remark.—The hyperbolic tangent, cotangent, secant, and cosecant are defined by

$$\tanh x = \frac{\sinh x}{\cosh x}, \qquad \coth x = \frac{\cosh x}{\sinh x},$$

$$\operatorname{sech} x = \frac{1}{\cosh x}, \qquad \operatorname{csch} x = \frac{1}{\sinh x}.$$

Prove that

(8.) $\tan(ix) = i \tanh x.$

(9.) $\coth(-x) = -\coth x.$

(10.) $\operatorname{sech}(-x) = \operatorname{sech} x.$

(11.) $\cosh^2 x - \sinh^2 x = 1.$

(12.) $\operatorname{sech}^2 x + \tanh^2 x = 1.$

(13.) $\coth^2 x - \operatorname{csch}^2 x = 1.$

(14.) $\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y.$

(15.) $\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y.$

(16.) $\cosh \frac{1}{2} x = \sqrt{\frac{1+\cosh x}{2}}.$

(17.) $\sinh u - \sinh v = 2 \cosh \frac{1}{2}(u+v) \sinh \frac{1}{2}(u-v).$

(18.) $\cosh u + \cosh v = 2 \cosh \frac{1}{2}(u+v) \cosh \frac{1}{2}(u-v).$

(19.) $\cosh u - \cosh v = 2 \sinh \frac{1}{2}(u+v) \sinh \frac{1}{2}(u-v).$

CHAPTER VII

MISCELLANEOUS EXERCISES

RELATION OF FUNCTIONS

77. Prove the following:

(1.) $\cos x = \sin x \cot x.$

(2.) $\csc x \tan x = \sec x.$

(3.) $(\tan x + \cot x) \sin x \cos x = 1.$

(4.) $(\sec y - \tan y)(\sec y + \tan y) = 1.$

(5.) $(\csc z - \cot z)(\csc z + \cot z) = 1.$

(6.) $\cos^2 y + (\tan y - \cot y) \sin y \cos y = \sin^2 y.$

(7.) $\cos^4 x - \sin^4 x + 1 = 2 \cos^2 x.$

(8.) $(\sin y - \cos y)^2 = 1 - 2 \sin y \cos y.$

(9.) $\sin^3 x + \cos^3 x = (\sin x + \cos x)(1 - \sin x \cos x).$

(10.) $\dfrac{\cot x + \tan y}{\tan x + \cot y} = \cot x \tan y.$

(11.) $\cos^2 y - \sin^2 y = 2 \cos^2 y - 1.$

(12.) $1 - \tan^4 x = 2 \sec^2 x - \sec^4 x.$

(13.) $\dfrac{\cos x}{\sin x \cot^2 x} = \tan x.$

(14.) $\sec^2 y \csc^2 y = \tan^2 y + \cot^2 y + 2.$

(15.) $\cot y - \csc y \sec y (1 - 2 \sin^2 y) = \tan y.$

(16.) $\left(\dfrac{1}{\sin z} - \cot z\right)^2 = \dfrac{1 - \cos z}{1 + \cos z}.$

(17.) $\dfrac{\sec y}{1 + \cos y} = \dfrac{\tan y - \sin y}{\sin^3 y}.$

(18.) $1 + \dfrac{2 \sin x}{\sec x} = (\sin x + \cos x)^2.$

(19.) $\dfrac{1}{\sec^3 x} - \sin^3 x = (\cos x - \sin x)(1 + \sin x \cos x).$

(20.) $(\sin x \cos y + \cos x \sin y)^2 + (\cos x \cos y - \sin x \sin y)^2 = 1.$

(21.) $(a \cos x - b \sin x)^2 + (a \sin x + b \cos x)^2 = a^2 + b^2$.

(22.) $\dfrac{1}{(\cos^2 y - \sin^2 y)^2} = 1 + \dfrac{4 \tan^2 y}{(1 - \tan^2 y)^2}$.

Find an angle not greater than 90° which satisfies each of the following equations:

(23.) $4 \cos x = 3 \sec x$.

(24.) $\sin y = \csc y - \frac{3}{2}$.

(25.) $\sqrt{2} \sin x - \tan x = 0$.

(26.) $2 \cos x - \sqrt{3} \cot x = 0$.

(27.) $\tan y + \cot y - 2 = 0$.

(28.) $2 \sin^2 y - 2 = -\sqrt{2} \cos y$.

(29.) $3 \tan^2 x - 1 = 4 \sin^2 x$.

(30.) $\cos^2 x + 2 \sin^2 x - \frac{5}{2} \sin x = 0$.

(31.) $\csc x = \frac{2}{3} \tan x$.

(32.) $\sec x + \tan x = \pm \sqrt{3}$.

(33.) $\tan x + 2 \sqrt{3} \cos x = 0$.

(34.) $3 \sin x - 2 \cos^2 x = 0$.

Express the following in terms of the functions of angles less than 45°:

(35.) sin 92°.

(36.) cos 127°.

(37.) tan 320°.

(38.) cot 350°.

(39.) sin 265°.

(40.) tan 171°.

(41.) Given $\sin x = \frac{4}{7}$ and x in quadrant II; find all the other functions of x.

(42.) Given $\cos x = -\frac{3}{8}$ and x in quadrant III; find all the other functions of x.

(43.) Given $\tan x = \frac{3}{2}$ and x in quadrant III; find all the other functions of x.

(44.) Given $\cot x = -\frac{7}{5}$ and x in quadrant IV; find all the other functions of x.

In what quadrants must the angles lie which satisfy each of the following equations:

(45.) $\sin x \cos x = \frac{1}{4}\sqrt{3}$.

(46.) $\sec x \tan x = 2\sqrt{3}$. 1, 2

(47.) $\tan y + \sqrt{20} \cos y = 0$. 3.

(48.) $\cos x \cot x = \frac{5}{6}$. 1, 2

Find all the values of y less than 360° which will satisfy the following equations:

(49.) $\tan y + 2 \sin y = 0$.

(50.) $(1 + \tan x)(1 - 2 \sin x) = 0$.

(51.) $\sin x \cos x (1 + 2 \cos x) = 0$.

Prove the following:

(52.) $\cos 780° = \frac{1}{2}$.

(53.) $\sin 1485° = \frac{1}{2}\sqrt{2}$.

(54.) $\cos 2550° = \frac{1}{2}\sqrt{3}$.

(55.) $\sin(-3000°) = -\cos 30°$.

(56.) $\cos 1300° = -\cos 40°$.

(57.) Find the value of $a \sin 90° + b \tan 0° + a \cos 180°$.

(58.) Find the value of $a \sin 30° + b \tan 45° + a \cos 60° + b \tan 135°$.

(59.) Find the value of $(a - b) \tan 225° + b \cos 180° - a \sin 270°$.

(60.) Find the value of $(a \sin 45° + b \cos 45°)(a \sin 135° + b \sin 225°)$.

RIGHT TRIANGLES

78. In the following problems the planes on which distances are measured are understood to be horizontal unless otherwise stated.

(1.) The angle of elevation of the top of the tower from a point 1121 ft. from its base is observed to be 15° 17′; find the height of the tower.

(2.) A tree, 77 ft. high, stands on the bank of a river; at a point on the other bank just opposite the tree the angle of elevation of the top of the tree is found to be 5° 17′ 37″. Find the breadth of the river.

(3.) What angle will a ladder 42 ft. long make with the ground if its foot is 25 ft. from the base of the building against which it is placed?

(4.) When the altitude of the sun is $33° 22'$, what is the height of a tree which casts a shadow 75 ft.?

(5.) Two towns are 3 miles apart. The angle of depression of one, from a balloon directly above the other, is observed to be $8° 15'$. How high is the balloon?

(6.) From a point 197 ft. from the base of a tower the angle of elevation was found to be $46° 45' 54''$; find the height of the tower.

(7.) A man 5 ft. 10 in. high stands at a distance of 4 ft. 7 in. from a lamp-post, and casts a shadow 18 ft. long; find the height of the lamp-post.

(8.) The shadow of a building 101.3 ft. high is found to be 131.5 ft. long; find the elevation of the sun at that time.

(9.) A rope 112 ft. long is attached to the top of a building and reaches the ground, making an angle of $77° 20'$ with the ground; find the height of the building.

(10.) A house is 130 ft. above the water, on the banks of a river; from a point just opposite on the other bank the angle of elevation of the house is $14° 30' 21''$. Find the width of the river.

(11.) From the top of a headland, 1217.8 ft. above the level of the sea, the angle of depression of a dock was observed to be $10° 9' 13''$; find the distance from the foot of the headland to the dock.

(12.) 1121.5 ft. from the base of a tower its angle of elevation is found to be $11° 3' 5''$; find the height of the tower.

(13.) One bank of a river is 94.73 ft. vertically above the water, and subtends an angle of $10° 54' 13''$ from a point directly opposite at the water's edge; find the width of the river.

(14.) The shadow of a vertical cliff 113 ft. high just reaches a boat on the sea 93 ft. from its base; find the altitude of the sun.

(15.) A rope, 38 ft. long, just reached the ground when fastened to the top of a tree 29 ft. high. What angle does it make with the ground?

(16.) A tree is broken by the wind. Its top strikes the ground 15 ft. from the foot of the tree, and makes an angle of $42° 28'$ with the ground. Find the height of the tree before it was broken.

(17.) The pole of a circular tent is 18 ft. high, and the ropes reaching from its top to stakes in the ground are 37 ft. long; find the distance from the foot of the pole to one of the stakes, and the angle between the ground and the ropes.

(18.) A ship is sailing southwest at the rate of 8 miles an hour. At what rate is it moving south?

(19.) A building is 121 ft. high. From a point directly across the street its angle of elevation is $65° 3'$. Find the width of the street.

(20.) From the top of a building 52 ft. high the angle of elevation of another building 112 ft. high is $30° 12'$. How far are the buildings apart?

(21.) A window in a house is 24 ft. from the ground. What is the inclination of a ladder placed 8 ft. from the side of the building and reaching the window?

(22.) Given that the sun's distance from the earth is 92,000,000 miles, and its apparent semidiameter is $16' 2''$; find its diameter.

(23.) Given that the radius of the earth is 3963 miles, and that it subtends an angle of $57' 2''$ at the moon; find the distance of the moon from the earth.

(24.) Given that when the moon's distance from the earth is 238885 miles, its apparent semidiameter is $15' 34''$; find its diameter in miles.

(25.) Given that the radius of the earth is 3963 miles, and that it subtends an angle of $9''$ at the sun; find the distance of the sun from the earth.

(26.) A light-house is 57 ft. high; the angles of elevation of the top and bottom of it, as seen from a ship, are $5° 3' 20''$ and $4° 28' 8''$. Find the distance of its base above the sea-level.

(27.) At a certain point the angle of elevation of a tower was observed to be $53° 51' 16''$, and at a point 302 ft. farther away in the same straight line it was $9° 52' 10''$; find the height of the tower.

(28.) A tree stands at a distance from a straight road and between two mile-stones. At one mile-stone the line to the tree is observed to make an angle of $25° 15'$ with the road, and at the other an angle of $45° 17'$. Find the distance of the tree from the road.

(29.) From the top of a light-house, 225 ft. above the level of the sea, the angle of depression of two ships are $17° 21' 50''$ and $13° 50' 22''$,

and the line joining the ships passes directly beneath the light-house; find the distance between the two ships.

ISOSCELES TRIANGLES AND REGULAR POLYGONS

79. (1.) The area of a regular dodecagon is 37.52 sq. ft.; find its apothem.

(2.) The perimeter of a regular polygon of 11 sides is 23.47 ft.; find the radius of the circumscribing circle.

(3.) A regular decagon is circumscribed about a circle whose radius is 3.147 ft.; find its perimeter.

(4.) The side of a regular decagon is 23.41 ft.; find the radius of the inscribed circle.

(5.) The perimeter of an equilateral triangle is 17.2 ft.; find the area of the inscribed circle.

(6.) The area of a regular octagon is 2478 sq. in.; find its perimeter.

(7.) The area of a regular pentagon is 32.57 sq. ft.; find the radius of the inscribed circle.

(8.) The angle between the legs of a pair of dividers is 43°, and the legs are 7 in. long; find the distance between the points.

(9.) A building is 37.54 ft. wide, and the slope of the roof is 43° 36′; find the length of the rafters.

(10.) The radius of a circle is 12732, and the length of a chord is 18321; find the angle the chord subtends at the centre.

(11.) If the radius of a circle is taken as unity, what is the length of a chord which subtends an angle of 77° 17′ 40″?

(12.) What angle at the centre of a circle does a chord which is $\frac{4}{7}$ of the radius subtend?

(13.) What is the radius of a circle if a chord 11223 ft. subtends an angle of 59° 50′ 52″?

(14.) Two light-houses at the mouth of a harbor are each 2 miles from the wharf. A person on the wharf finds the angle between the lines to the light-houses to be 17° 32′. Find the distance between the two light-houses.

(15.) The side of a regular pentagon is 2; find the radius of the inscribed circle.

(16.) The perimeter of a regular heptagon inscribed in a circle is 12; find the radius of the circle.

(17.) The radius of a circle inscribed in an octagon is 3; find the perimeter of the octagon.

(18.) A regular polygon of 9 sides is inscribed in a circle of unit radius; find the radius of the inscribed circle.

(19.) Find the perimeter of a regular decagon circumscribed about a unit circle.

(20.) Find the area of a regular hexagon circumscribed about a unit circle.

(21.) Find the perimeter of a polygon of 11 sides inscribed in a unit circle.

(22.) The perimeter of a dodecagon is 30; find its area.

(23.) The area of a regular polygon of 11 sides is 18; find its perimeter.

TRIGONOMETRIC IDENTITIES AND EQUATIONS

80. Prove the following:

(1.) $\sin \frac{1}{2} y \pm \cos \frac{1}{2} y = \sqrt{1 \pm \sin y}$.

(2.) $\dfrac{\cos x - \cos y}{\cos x + \cos y} = -\tan \frac{1}{2}(x+y) \tan \frac{1}{2}(x-y)$.

(3.) $\dfrac{\sin 2x + \sin 4x}{\cos 2x + \cos 4x} = \tan 3x$.

(4.) $\cos^2 y \tan^2 y + \sin^2 y \cot^2 y = 1$.

(5.) $\dfrac{\cos(x+y+z)}{\sin x \sin y \sin z} = \cot x \cot y \cot z - \cot x - \cot y - \cot z$.

(6.) $\cos^2(x-y) - \sin^2(x+y) = \cos 2x \cos 2y$.

(7.) $\dfrac{\sin x + \sin y}{\cos x - \cos y} = -\cot \frac{1}{2}(x-y)$.

(8.) $\dfrac{\cos x - \sec x}{\sec x} = 4 \cos^2 \frac{1}{2} x (\cos^2 \frac{1}{2} x - 1)$.

(9.) $\cot x = \dfrac{\sin 2x}{1 - \cos 2x}$.

(10.) $\tan^2 y = \dfrac{1 - \cos 2y}{1 + \cos 2y}$.

(11.) $\cot x - \tan x = 2 \cot 2x$.

P.

2

(12.) $\tan \frac{1}{2} x + 2 \sin^2 \frac{1}{2} x \cot x = \sin x.$

(13.) $\dfrac{\tan x \pm \tan y}{\cot x \pm \cot y} = \pm \sin x \sec x \tan y.$

(14.) $\sin x - 2 \sin^3 x = \sin x \cos 2x.$

(15.) $4 \sin y \sin (60^\circ - y) \sin (60^\circ + y) = \sin 3y.$

(16.) $\dfrac{\sin y (1 - \tan^2 y)}{\sec^2 y} \left(\dfrac{1}{\cos y - \sin y} + \dfrac{1}{\cos y + \sin y} \right) = \sin 2y.$

(17.) $1 + \tan y \tan \frac{1}{2} y = \sec y.$

(18.) $\sin 4x = 4 \sin x \cos^3 x - 4 \cos x \sin^3 x.$

(19.) $\sec 2x + \tan 2x + 1 = \dfrac{2}{1 - \tan x}.$

(20.) $\tan 50^\circ + \cot 50^\circ = 2 \sec 10^\circ.$

(21.) $\cos (x + 45^\circ) + \sin (x - 45^\circ) = 0.$

(22.) $\dfrac{\tan x}{1 - \cot 2x \tan x} = \sin 2x.$

(23.) $(1 - \tan^2 x) \sin x \cos x = \cos 2x \sqrt{\dfrac{1 - \cos 2x}{1 + \cos 2x}}.$

(24.) $\dfrac{\cos y + \sin y}{\cos y - \sin y} = \tan 2y + \sec 2y.$

(25.) $\sin (x + y) \cos x - \cos (x + y) \sin x = \sin y.$

(26.) $\cos (x - y) \sin y + \sin (x - y) \cos y = \sin x.$

(27.) $\dfrac{\sin (x - y)}{\cos x \cos y} + \dfrac{\sin (y - z)}{\cos y \cos z} + \dfrac{\sin (z - x)}{\cos z \cos x} = 0.$

(28.) $\dfrac{\sin x + \sin 2x}{\cos x - \cos 2x} = \cot \frac{1}{2} x.$

(29.) $2 \sin^2 x \sin^2 y + 2 \cos^2 x \cos^2 y = 1 + \cos 2x \cos 2y.$

(30.) $\sin 60^\circ + \sin 30^\circ = 2 \sin 45^\circ \cos 15^\circ.$

(31.) $\dfrac{\tan (x - y) + \tan y}{1 - \tan (x - y) \tan y} = \tan x.$

(32.) $\dfrac{2}{\sin y \tan \frac{1}{2} y} = 1 + \cot^2 \frac{1}{2} y.$

(33.) $\sin 4x + \sin 2x = 2 \sin 3x \cos x.$

(34.) $\dfrac{\sin x + \sin y}{\cos x - \cos y} = \dfrac{\cos x + \cos y}{\sin y - \sin x}.$

(35.) $\sin 75^\circ = \dfrac{\sqrt{3} + 1}{2 \sqrt{2}}.$

(36.) $2 \tan 2y = \tan (45^\circ + y) - \tan (45^\circ - y).$

(37.) $\dfrac{\tan 2x + \tan x}{\tan 2x - \tan x} = \dfrac{\sin 3x}{\sin x}$.

(38.) $\tan 3y = \dfrac{3 \tan y - \tan^3 y}{1 - 3 \tan^2 y}$.

(39.) $\sin 60° + \sin 20° = 2 \sin 40° \cos 20°$.

(40.) $\sin 40° - \sin 10° = 2 \cos 25° \sin 15°$.

(41.) $\cos 2x - \cos 4x = 2 \sin 3x \sin x$.

(42.) $\tan 15° = 2 - \sqrt{3}$.

(43.) $(\sqrt{1 + \sin x} - \sqrt{1 - \sin x})^2 = 4 \sin^2 \frac{1}{2}x$.

(44.) $(\sqrt{1 + \sin x} + \sqrt{1 - \sin x})^2 = 4 \cos^2 \frac{1}{2}x$.

(45.) $\dfrac{\sin(2x + y)}{\sin x} - 2 \cos(x + y) = \dfrac{\sin y}{\sin x}$.

(46.) $\dfrac{\sin 4x}{\sin 2x} = 2 \cos 2x$.

(47.) $\sin 50° - \sin 70° + \sin 10° = 0$.

(48.) $\cos\dfrac{\pi}{3} - \cos\dfrac{\pi}{2} = 2 \sin \dfrac{5\pi}{12} \sin \dfrac{\pi}{12}$.

(49.) $\dfrac{1 - \tan^2(45° - x)}{1 + \tan^2(45° - x)} = \sin 2x$.

(50.) $\dfrac{\sin 75° - \sin 15°}{\cos 75° + \cos 15°} = \sqrt{\dfrac{1}{3}}$.

(51.) $\tan^3 \frac{1}{2}x(1 + \cot^2 \frac{1}{2}x)^3 = \dfrac{8}{\sin^3 x}$,

(52.) $\tan 75° = 2 + \sqrt{3}$.

(53.) $\sin 3x + \sin 5x = 2 \sin 4x \cos x$.

(54.) $\cos 5x + \cos 9x = 2 \cos 7x \cos 2x$.

(55.) $\sin 15° = \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$.

(56.) $\dfrac{\sin 3x - \sin x}{\cos 3x + \cos x} = \tan x$.

(57.) $\sin 5y = 5 \sin y - 20 \sin^3 y + 16 \sin^5 y$.

(58.) $\cos 5y = 5 \cos y - 20 \cos^3 y + 16 \cos^5 y$.

(59.) $\sin 4x = \dfrac{4 \tan x(1 - \tan^2 x)}{(1 + \tan^2 x)^2}$.

(60.) $\cos(45° + x) + (\cos 45° - x) = \sqrt{2} \cos x$.

(61.) $\cos 3x + \cos 5x + \cos 7x + \cos 15x = 4 \cos 4x \cos 5x \cos 6x$.

(62.) $\sin^2 \frac{1}{2}x(\cot \frac{1}{2}x - 1)^2 = 1 - \sin x.$

(63.) $\dfrac{3\sin x - \sin 3x}{\cos 3x + 3\cos x} = \tan^3 x.$

(64.) $\sin x(1+\tan x) + \cos x(1+\cot x) = \csc x + \sec x.$

(65.) $\dfrac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = \dfrac{2+\sin 2x}{2}.$

(66.) $\cos y + \cos(120 - y) + \cos(120 + y) = 0.$

(67.) $\dfrac{\sin 3x}{\sin x} = 2\cos 2x + 1.$

(68.) $\dfrac{(\cos y - \cos 3y)(\sin 8y + \sin 2y)}{(\sin 5y - \sin y)(\cos 4y - \cos 6y)} = 1.$

(69.) $\left(\dfrac{\sin x}{1+\cos x}\right)^2 = \dfrac{1-\cos x}{1+\cos x}.$

(70.) $\dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x} = 2.$

(71.) $\dfrac{1+\sin x+\cos x}{1+\sin x-\cos x} = \cot \frac{1}{2}x.$

(72.) $\dfrac{\sin(4x-2y)+\sin(4y-2x)}{\cos(4x-2y)+\cos(4y-2x)} = \tan(x+y).$

(73.) $\dfrac{\sin x+\sin 3x+\sin 5x+\sin 7x}{\cos x+\cos 3x+\cos 5x+\cos 7x} = \tan 4x.$

If A, B, and C are the angles of a triangle, prove the following:

(74.) $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C.$

(75.) $\sin 2A + \sin 2B - \sin 2C = 4\cos A \cos B \sin C.$

(76.) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cos B \cos C.$

(77.) $\tan A + \tan B + \tan C = \tan A \tan B \tan C.$

Solve the following equations for values of x less than 360°.

(78.) $\cos 2x + \cos x = -1.$

(79.) $\sin x + \sin 7x = \sin 4x.$

(80.) $\cos x - \sin 2x - \cos 3x = 0.$

(81.) $\cos x - \sin 3x - \cos 2x = 0.$

(82.) $\sin 4x - 2\sin 2x = 0.$

(83.) $\sin 2x - \cos 2x - \sin x + \cos x = 0.$

(84.) $\sin(60° - x) - \sin(60° + x) = +\frac{1}{2}\sqrt{3}.$

(85.) $\sin(30° + x) - \cos(60° + x) = -\frac{1}{2}\sqrt{3}.$

(86.) $\csc x = 1 + \cot x$.

(87.) $\cos 2x = \cos^2 x$.

(88.) $2 \sin y = \sin 2y$.

(89.) $\sin 3y + \sin 2y + \sin y = 0$.

(90.) $\sin^2 x + 5 \cos^2 x = 3$.

(91.) $\tan(45° - x) + \cot(45° - x) = 4$.

OBLIQUE TRIANGLES

81. (1.) It is required to find the distance between two points, A and B, on opposite sides of a river. A line, AC, and the angles BAC and ACB are measured and found to be 2483 ft., 61° 25′, and 52° 17′ respectively.

(2.) A straight road leads from a town A to a town B, 12 miles distant; another road, making an angle of 77° with the first, goes from A to a town C, 7 miles distant. How far are the towns B and C apart?

(3.) In order to determine the distance of a fort, A, from a battery, B, a line, BC, one-half mile long, is measured, and the angles ABC and ACB are observed to be 75° 18′ and 78° 21′ respectively. Find the distance AB.

(4.) Two houses, A and B, are 1728 ft. apart. Find the distance of a third house, C, from A if $BAC = 47° 51'$ and $ABC = 57° 23'$.

(5.) In order to determine the distance of a bluff, A, from a house, B, in a plane, a line, BC, was measured and found to be 1281 yards, also the angles ABC and BCA 65° 31′ and 70° 2′ respectively. Find the distance AB.

(6.) Two towns, 3 miles apart, are on opposite sides of a balloon. The angles of elevation of the balloon are found to be 13° 19′ and 20° 3′. Find the distance of the balloon from the nearer town.

(7.) It is required to find the distance between two posts, A and B, which are separated by a swamp. A point C is 1272.5 ft. from A, and 2012.4 ft. from B. The angle ACB is 41° 9′ 11″.

(8.) Two stakes, A and B, are on opposite sides of a stream; a third point, C, is so situated that the distances AC and BC can be found, and are 431.27 yards and 601.72 yards respectively. The angle ACB is 39° 53′ 13″. Find the distance between the stakes A and B.

(9.) Two light-houses, A and B, are 11 miles apart. A ship, C, is observed from them to make the angles $BAC = 31^\circ\ 13'\ 31''$ and $ABC = 21^\circ\ 46'\ 8''$. Find the distance of the ship from A.

(10.) Two islands, A and B, are 6103 ft. apart. Find the distance from A to a ship, C, if the angle ABC is $37^\circ\ 25'$ and BAC is $40^\circ\ 32'$.

(11.) In ascending a cliff towards a light-house at its summit, the light-house subtends at one point an angle of $21^\circ\ 22'$. At a point 55 ft. farther up it subtends an angle of $40^\circ\ 27'$. If the light-house is 58 ft. high, how far is this last point from its foot?

(12.) The distances of two islands from a buoy are 3 and 4 miles respectively. The islands are 2 miles apart. Find the angle subtended by the islands at the buoy.

(13.) The sides of a triangle are 151.45, 191.32, and 250.91. Find the length of the perpendicular from the largest angle upon the opposite side.

(14.) A tree stands on a hill, and the angle between the slope of the hill and the tree is $110^\circ\ 23'$. At a point $85.\dot{6}$ ft. down the hill the tree subtends an angle of $22^\circ\ 22'$. Find the height of the tree.

(15.) A light-house 54 ft. high is built upon a rock. From the top of the light-house the angle of depression of a boat is $19^\circ\ 10'$, and from its base the angle of depression of the boat is $12^\circ\ 22'$. Find the height of the rock on which the light-house stands.

(16.) Three towns, A, B, and C, are connected by straight roads. $AB = 4$ miles, $BC = 5$ miles, and $AC = 7$ miles. Find the angle made by the roads AB and BC.

(17.) Two buoys, A and B, are one-half mile apart. Find the distance from A to a point C on the shore if the angles ABC and BAC are $77^\circ\ 7'$ and $67^\circ\ 17'$ respectively.

(18.) The top of a tower is 175 ft. above the level of a bay. From its top the angles of depression of the shores of the bay in a certain direction are $57^\circ\ 16'$ and $15^\circ\ 2'$. Find the distance across the bay.

(19.) The lengths of two sides of a triangle are $\sqrt{2}$ and $\sqrt{3}$. The angle between them is 45°. Find the remaining side.

(20.) The sides of a parallelogram are 172.43 and 101.31, and the angle included by them is $61^\circ\ 16'$. Find the two diagonals.

(21.) A tree 41 ft. high stands at the top of a hill which slopes

$10° 12'$ to the horizontal. At a certain point down the hill the tree subtends an angle of $28° 29'$. Find the distance from this point to the foot of the tree.

(22.) A plane is inclined to the horizontal at an angle of $7° 33'$. At a certain point on the plane a flag-pole subtends an angle $20° 3'$, and at a point 50 ft. nearer the pole an angle of $40° 35'$. Find the height of the pole.

(23.) The angle of elevation of an inaccessible tower, situated in a plane, is $53° 19'$. At a point 227 ft. farther from the tower the angle of elevation is $22° 41'$. Find the height of the tower.

(24.) A house stands on a hill which slopes $12° 18'$ to the horizontal. 75 ft. from the house down the hill the house subtends an angle of $32° 5'$. Find the height of the house.

(25.) From one bank of a river the angle of elevation of a tree on the opposite bank is $28° 31'$. From a point 139.4 ft. farther away in a direct line its angle of elevation is $19° 10'$. Find the width of the river.

(26.) From the foot of a hill in a plane the angle of elevation of the top of the hill is $21° 7'$. After going directly away 211 ft. farther, the angle of elevation is $18° 37'$. Find the height of the hill.

(27.) A monument at the top of a hill is 153.2 ft. high. At a point 321.4 ft. down the hill the monument subtends an angle of $11° 13'$. Find the distance from this point to the top of the monument.

(28.) A building is situated on the top of a hill which is inclined $10° 12'$ to the horizontal. At a certain distance up the hill the angle of elevation of the top of the building is $20° 55'$, and 115.3 ft. farther down the hill the angle of elevation is $15° 10'$. Find the height of the building.

(29.) A cloud, C, is observed from two points, A and B, 2874 ft. apart, the line AB being directly beneath the cloud. At A, the angle of elevation of the cloud is $77° 19'$, and the angle CAB is $51° 18'$. The angle ABC is found to be $60° 45'$. Find the height of the cloud above A.

(30.) Two observers, A and B, are on a straight road, 675.4 ft. apart, directly beneath a balloon, C. The angles ABC and BAC are $34° 42'$ and $41° 15'$ respectively. Find the distance of the balloon from the first observer.

(31.) A man on the opposite side of a river from two objects, A and B, wishes to obtain their distance apart. He measures the distance $CD = 357$ ft., and the angles $ACB = 29° 33'$, $BCD = 38° 52'$, $ADB = 54° 10'$, and $ADC = 34° 11'$. Find the distance AB.

(32.) A cliff is 327 ft. above the sea-level. From the top of the cliff the angles of depression of two ships are 15° 11′ and 13° 13′. From the bottom of the cliff the angle subtended by the ships are 122° 39′. How far are the ships apart?

(33.) A man standing on an inclined plane 112 ft. from the bottom observed the angle subtended by a building at the bottom to be 33° 52′. The inclination of the plane to the horizontal is 18° 51′. Find the height of the building.

(34) Two boats, A and B, are 451.35 ft. apart. The angle of elevation of the top of a light-house, as observed from A, is 33° 17′. The base of the light-house, C, is level with the water; the angles ABC and CAB are 12° 31′ and 137° 22′ respectively. Find the height of the light-house.

(35.) From a window directly opposite the bottom of a steeple the angle of elevation of the top of the steeple is 29° 21′. From another window, 20 ft. vertically below the first, the angle of elevation is 39° 3′. Find the height of the steeple.

(36.) A dock is 1 mile from one end of a breakwater, and $1\frac{1}{8}$ miles from the other end. At the dock the breakwater subtends an angle of 31° 11′. Find the length of the breakwater in feet.

(37.) A straight road ascending a hill is 1022 ft. long. The hill rises 1 ft. in every 4. A tower at the top of the hill subtends an angle of 7° 19′ at the bottom. Find the height of the tower.

(38.) A tower, 192 ft. high, rises vertically from one corner of a triangular yard. From its top the angles of depression of the other corners are 58° 4′ and 17° 49′. The side opposite the tower subtends from the top of the tower an angle of 75° 15′. Find the length of this side.

(39.) There are two columns left standing upright in a certain ruins; the one is 66 ft. above the plain, and the other 48. In a straight line between them stands an ancient statue, the head of which is 100 ft. from the summit of the higher, and 84 ft. from the top of the lower

column, the base of which measures just 74 ft. to the centre of the figure's base. Required the distance between the tops of the two columns.

(40.) Two sides of a triangle are in the ratio of 11 to 9, and the opposite angles have the ratio of 3 to 1. What are these angles?

(41.) The diagonals of a parallelogram are 12432 and 8413, and the angle between them is $78° 44'$; find its area.

(42.) One side of a triangle is 1012.6 and two angles are $52° 21'$ and $57° 32'$; find its area.

(43.) Two sides of a triangle are 218.12 and 123.72, and the included angle is $59° 10'$; find its area.

(44.) Two angles of a triangle are $35° 15'$ and $47° 18'$, and one side is 2104.7; find its area.

(45.) The three sides of a triangle are 1.2371, 1.4713, and 2.0721; find the area.

(46.) Two sides of a triangle are 168.12 and 179.21, and the included angle is $41° 14'$; find its area.

(47.) The three sides of a triangle are 51 ft., 48.12 ft., and 32.2 ft.; find the area.

(48.) Two sides of a triangle are 111.18 and 121.21, and the included angle is $27° 50'$; find its area.

(49.) The diagonals of a parallelogram are 37 and 51, and they form an angle of $65°$; find its area.

(50.) If the diagonals of a quadrilateral are 34 and 56, and if they intersect at an angle of $67°$, what is the area?

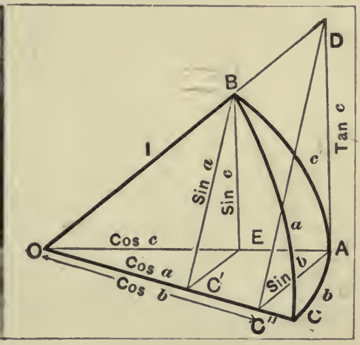
SPHERICAL TRIGONOMETRY

CHAPTER VIII

RIGHT AND QUADRANTAL TRIANGLES

RIGHT TRIANGLES

82. Let O be the centre of a sphere of unit radius, and ABC a right spherical triangle, right angled at A, formed by the intersection of the three planes AOC, AOB, and BOC



with the surface of the sphere. Suppose the planes DAC'' and BEC' passed through the points A and B respectively, and perpendicular to the line OC. The plane angles $DC''A$ and $BC'E$ each measure the angle C of the spherical triangle, and the sides of the spherical triangle a, b, c have the same numerical measure as BOC, AOC, and AOB respectively

tively, then, $AD = \tan c$, $BE = \sin c$, $BC' = \sin a$, $OC' = \cos a$, $OC'' = \cos b$, $OE = \cos c$, $AC'' = \sin b$.

In the two similar triangles OEC' and OAC'',

$$\frac{\cos c}{OA} = \frac{\cos c}{1} = \frac{\cos a}{\cos b}, \text{ or } \cos a = \cos b \cos c. \qquad (1)$$

In the triangle $BC'E$,

$$\sin C = \frac{BE}{BC'}, \text{ or } \sin C = \frac{\sin c}{\sin a}. \qquad (2)$$

In the triangle DAC'',

$$\tan C = \frac{DA}{C''A}, \text{ or } \tan C = \frac{\tan c}{\sin b}. \qquad (3)$$

Combining formulas (2) and (3) with (1),

$$\cos C = \frac{\tan b}{\tan a}. \qquad (4)$$

Again, if AB were made the base of the right spherical triangle ABC, we should have

$$\sin B = \frac{\sin b}{\sin a}. \qquad (5)$$

$$\tan B = \frac{\tan b}{\sin c}. \qquad (6)$$

$$\cos B = \frac{\tan c}{\tan a}. \qquad (7)$$

From the foregoing equations we may also obtain by combinations,

$$\cos B = \sin C \cos b. \qquad (8)$$

$$\cos C = \sin B \cos c. \qquad (9)$$

$$\cos a = \cot B \cot C. \qquad (10)$$

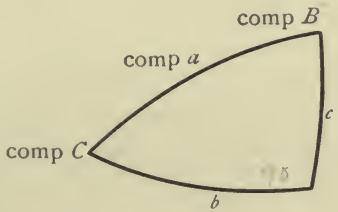
NAPIER'S RULES OF CIRCULAR PARTS

83. The above ten formulas are sufficient to solve all cases of right spherical triangles. They may, however, be

expressed as two simple rules, called, after their inventor, Napier's rules.

The two sides adjacent to the right angle, the complement of the hypotenuse, and the complements of the oblique angles are called the **circular parts.**

The right angle is not one of the circular parts.



Thus there are *five* circular parts—namely, b, c, comp a, comp B, comp C. Any one of the five parts may be called the *middle* part, then the two parts next to it are called *adjacent* parts, and the remaining two parts are called the *opposite* parts.

Thus if c is taken for the *middle* part, comp B and b are *adjacent* parts, and comp a and comp C are *opposite* parts.

The ten formulas may be written and grouped as follows:

1st Group.	*2d Group.*
sin comp C = tan comp a tan b.	sin comp a = cos b cos c.
sin comp B = tan comp a tan c.	sin b = cos comp a cos comp B.
sin comp a = tan comp B tan comp C.	sin c = cos comp a cos comp C.
sin c = tan comp B tan b.	sin comp B = cos comp C cos b.
sin b = tan comp C tan c.	sin comp C = cos comp B cos c.

Napier's rules may be stated:

I. *The sine of the middle part is equal to the product of the tangents of the adjacent parts.*

II. *The sine of the middle part is equal to the product of the cosines of the opposite parts.*

84. In the right spherical triangles considered in this work, each side is taken less than a semicircumference, and each angle less than two right angles.

In the solution of the triangles, it is to be observed,

(1.) If the two sides about the right angle are both less or both greater than 90°, the hypotenuse is less than 90°; if one side is less and the other greater than 90°, the hypotenuse is greater than 90°.

(2.) An angle and the side opposite are either both less or both greater than 90°.

EXAMPLE

85. Given $a=63°\ 56'$, $b=40°\ 0'$, to find c, B, and C.

To find c.

comp a is the middle part.

c and b are the opposite parts.

$$\sin \text{comp}\, a=\cos b \cos c,$$

or

$$\cos a=\cos b \cos c.$$

$$\cos c=\frac{\cos a}{\cos b}.$$

$$\begin{aligned} \log \cos a &= 9.64288 \\ \text{colog} \cos b &= 0.11575 \\ \hline \log \cos c &= 9.75863 \\ c &= 54°\ 59'\ 47'' \end{aligned}$$

To find B.

b is the middle part.

comp a and comp B are the opposite parts.

$$\sin b=\cos \text{comp}\, a \cos \text{comp}\, B,$$

or

$$\sin b=\sin a \sin B.$$

$$\sin B=\frac{\sin b}{\sin a}.$$

$$\begin{aligned} \log \sin b &= 9.80807 \\ \text{colog} \sin a &= 0.04659 \\ \hline \log \sin B &= 9.85466 \\ B &= 45°\ 41'\ 28'' \end{aligned}$$

To find C.

comp C is the middle part.

comp a, and b are adjacent parts.

$$\sin \text{comp}\, C=\tan \text{comp}\, a \tan b,$$

$$\cos C=\cot a \tan b.$$

$$\begin{aligned} \log \cot a &= 9.68946 \\ \log \tan b &= 9\ 92381 \\ \hline & 9.61327 \\ C &= 65°\ 45'\ 58'' \end{aligned}$$

Check.

Use the three parts originally required.

comp C is the middle part.

comp B and c are opposite parts.

$$\sin \text{comp}\, C=\cos c \cos \text{comp}\, B,$$

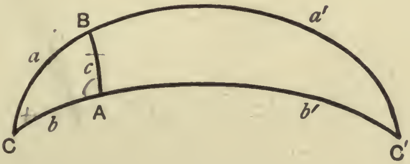
or

$$\cos C=\cos c \sin B.$$

$$\begin{aligned} \log \cos c &= 9.75863 \\ \log \sin B &= 9.85466 \\ \hline \log \cos C &= 9.61329 \\ C &= 65°\ 45'\ 54'' \end{aligned}$$

AMBIGUOUS CASE

86. When a side about the right angle and the angle opposite this side are given, there are two solutions, as illustrated by the following figure. Since the solution gives the values of each part in terms of the sine, the results are not only the values of a, b, B, but $180^\circ - a$, $180^\circ - b$, $180^\circ - B$.



Given $c = 26^\circ\ 4'$.
$C = 36^\circ\ 0'$.

To find a, a', b, b' and B, B', using Napier's rules.

To find B and B'.

$\sin \text{comp } C = \cos \text{comp } B \cos c,$

or $\cos C = \sin B \cos c,$

or $\sin B = \dfrac{\cos C}{\cos c}.$

$\log \cos C = 9.90796$
$\text{colog} \cos c = 0.04659$
$\log \sin B = 9.95455$
$B = 64^\circ\ 14'\ 30''$
$B' = 180^\circ - B = 115^\circ\ 45'\ 30''$

To find b and b'.

$\sin b = \tan c \tan \text{comp } C,$

or $\sin b = \tan c \cot C.$

$\log \tan c = 9.68946$
$\log \cot C = 0.13874$
$\log \sin b = 9.82820$
$b = 42^\circ\ 19'\ 17''$
$b' = 180^\circ - b = 137^\circ\ 40'\ 43''$

To find a and a'.

$\sin c = \cos \text{comp } a \cos \text{Comp } C,$

or $\sin c = \sin a \sin C,$

or $\sin a = \dfrac{\sin c}{\sin C}.$

$\log \sin c = 9.64288$
$\text{colog} \sin C = 0.23078$
$\log \sin a = 9.87366$
$a = 48^\circ\ 22'\ 55''-$
$a' = 180^\circ - a = 131^\circ\ 37'\ 5''+$

(Discrepancy due to omitted decimals.)

Check.

$\sin b = \cos \text{comp } a \cos \text{comp } B,$

or $\sin b = \sin a \sin B.$

$\log \sin a \text{ or } a' = 9.87366$
$\log \sin B \text{ or } B' = 9.95455$
$\log \sin b = 9.82821$
$b = 42^\circ\ 19'\ 21''$
$b' = 180^\circ - b = 137^\circ\ 40'\ 39''$

QUADRANTAL TRIANGLES

87. *Def.*—A quadrantal triangle is a spherical triangle one side of which is a quadrant.

A quadrantal triangle may be solved by Napier's rules for right spherical triangles as follows:

By making use of the polar triangle where

$$A = 180^\circ - a' \qquad a = 180^\circ - A'$$
$$B = 180^\circ - b' \qquad b = 180^\circ - B'$$
$$C = 180^\circ - c' \qquad c = 180^\circ - C'$$

we see that the polar triangle of the quadrantal triangle is a right triangle which can be solved by Napier's rules. Whence we may at once derive the required parts of the quadrantal triangle.

EXAMPLE

Given $A = 136^\circ\ 4'$. $\quad B = 140^\circ\ 0'$. $\quad a = 90^\circ\ 0'$.

The corresponding parts of the polar triangle are

$$a' = 63^\circ\ 56', \qquad b' = 40^\circ\ 0', \qquad A' = 90^\circ.$$

By Napier's rules we find

$$B' = 45^\circ\ 41'\ 28'', \qquad C' = 65^\circ\ 45'\ 58'', \qquad c = 54^\circ\ 59'\ 47'';$$

whence, by applying to these parts the rule of polar triangles, we obtain

$$b = 134^\circ\ 18'\ 32'', \qquad c = 114^\circ\ 14'\ 2'', \qquad C = 125^\circ\ 0'\ 13''.$$

EXERCISES

88. (1.) In the right-angled spherical triangle ABC, the side $a = 63^\circ\ 56'$, and the side $b = 40^\circ$. Required the other side, c, and the angles B and C.

(2.) In a right-angled triangle ABC, the hypotenuse $a = 91^\circ\ 42'$, and the angle $B = 95^\circ\ 6'$. Required the remaining parts.

(3.) In the right-angled triangle ABC, the side $b = 26^\circ\ 4'$, and the angle $B = 36^\circ$. Required the remaining parts.

(4.) In the right-angled spherical triangle ABC, the side $c = 54^\circ\ 30'$, and the angle $B = 44^\circ\ 50'$. Required the remaining parts.

Why is not the result ambiguous in this case?

(5.) In the right-angled spherical triangle ABC, the side $b = 55° \ 28'$, and the side $c = 63° \ 15'$. Required the remaining parts.

(6.) In the right-angled spherical triangle ABC, the angle $B = 69° \ 20'$, and the angle $C = 58° \ 16'$. Required the remaining parts.

(7.) In the spherical triangle ABC, the side $a = 90°$, the angle $C = 42° \ 10'$, and the angle $A = 115° \ 20'$. Required the remaining parts.

Hint.—The angle A of the polar triangle is a right angle.

(8.) In the spherical triangle ABC, the side $b = 90°$, the angle $C = 69° \ 13' \ 46''$, and the angle $A = 72° \ 12' \ 4''$. Required the remaining parts.

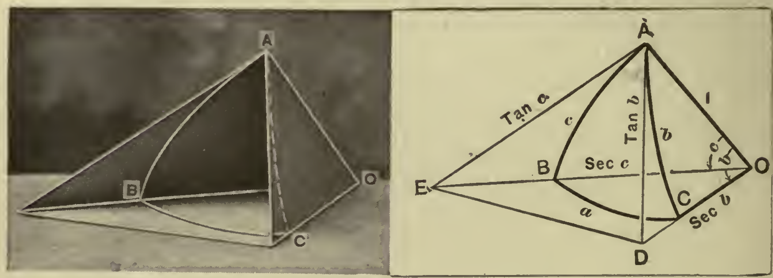
(9.) In the right-angled spherical triangle ABC, the angle $C = 23° \ 27' \ 42''$, and the side $b = 10° \ 39' \ 40''$. Required the angle B and the sides a and c.

(10.) In the right spherical triangle ABC, the angle $B = 47° \ 54' \ 20''$, and the angle $C = 61° \ 50' \ 29''$. Required the sides.

CHAPTER IX

OBLIQUE-ANGLED TRIANGLES

89. Let O be the centre of a sphere of unit radius, and ABC an oblique-angled spherical triangle formed by the three planes AOB, BOC, and AOC. Suppose the plane



AED passed through the point A perpendicular to AO, intersecting the planes AOB, BOC, and AOC, in AE, ED, and AD respectively. Then $AD = \tan b$, $AE = \tan c$, $OD = \sec b$, $OE = \sec c$.

In the triangle EOD,

$$ED^2 = \sec^2 b + \sec^2 c - 2 \sec b \sec c \cos a.$$

In the triangle AED,

$$ED^2 = \tan^2 b + \tan^2 c - 2 \tan b \tan c \cos A.$$

Subtracting these two equations and remembering that

$$\sec^2 b - \tan^2 b = 1, \text{ we have}$$

$$0 = 2 - 2 \sec b \sec c \cos a + 2 \tan b \tan c \cos A.$$

Reducing, we have

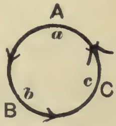
$$\mathbf{\cos a = \cos b \cos c + \sin b \sin c \cos A.} \qquad (1)$$

If we make b and c in turn the base of the triangle, we obtain in a similar way,

$$\cos b = \cos c \cos a + \sin c \sin a \cos B,$$

and $$\cos c = \cos a \cos b + \sin a \sin b \cos C.$$

Remark.—In this group of formulas the second may be obtained from the first, and the third from the second, by advancing one letter in the cycle as shown in the figure; thus, writing b for a, c for b, a for c, B for A, C for B, and A for C. The same principle will apply in all the formulas of Oblique-Angled Spherical Triangles, and only the first one of each group will be given in the text.



90. By making use of the polar triangle where

$$a = 180° - A' \qquad A = 180° - a'$$
$$b = 180° - B' \qquad B = 180° - b'$$
$$c = 180° - C' \qquad C = 180° - c'$$

we may obtain a second group of formulas.

Substituting these values of a, b, c, and A in (1), and remembering that $\cos(180° - A) = -\cos A$ and $\sin(180° - A) = \sin A$, we have

$$\cos A' = -\cos B' \cos C' + \sin B' \sin C' \cos a'.$$

Since this is true for any triangle, we may omit the accents and write,

$$\mathbf{\cos A = -\cos B \cos C + \sin B \sin C \cos a.} \qquad (2)$$

FORMULAS FOR LOGARITHMIC COMPUTATION

91. Formula (1), $\cos a = \cos b \cos c + \sin b \sin c \cos A$,

gives $$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

By § 36, $$\cos A = 1 - 2 \sin^2 \tfrac{1}{2}A$$

Whence $$1 - 2\sin^2\tfrac{1}{2}A = \frac{\cos a - \cos b \cos c}{\sin b \sin c},$$

or $$\sin^2 \tfrac{1}{2} A = \frac{\cos b \cos c + \sin b \sin c - \cos a}{2 \sin b \sin c},$$

$$=\frac{\cos(b-c)-\cos a}{2\sin b\sin c},$$

$$=\frac{\sin\dfrac{a+b-c}{2}\,\sin\dfrac{a-b+c}{2}}{\sin b\,\sin c}. \qquad (38)$$

Putting

$$\frac{a+b+c}{2}=s,\ \text{then}\ \frac{a+b-c}{2}=s-c,\ \text{and}\ \frac{a-b+c}{2}=s-b,$$

we have $$\sin\tfrac{1}{2}A=\sqrt{\frac{\sin(s-b)\,\sin(s-c)}{\sin b\,\sin c}}.$$

Since, also, $\cos A = 1 + 2\cos^2\frac{1}{2}A$,
we have, similarly,

$$\cos\tfrac{1}{2}A=\sqrt{\frac{\sin s\,\sin(s-a)}{\sin b\,\sin c}}.$$

Hence $$\boldsymbol{\tan\tfrac{1}{2}A=\sqrt{\frac{\sin(s-b)\,\sin(s-c)}{\sin s\,\sin(s-a)}}}. \qquad \text{(I)}$$

By a like process, formula (2) reduces to

$$\boldsymbol{\tan\tfrac{1}{2}a=\sqrt{\frac{-\cos S\,\cos(S-A)}{\cos(S-B)\,\cos(S-C)}}}. \qquad \text{(II)}$$

92. If, in formula I, we advance one letter, we have

$$\tan\tfrac{1}{2}B=\sqrt{\frac{\sin(s-c)\,\sin(s-a)}{\sin s\,\sin(s-b)}}.$$

And dividing $\tan\frac{1}{2}A$ by $\tan\frac{1}{2}B$, and reducing, we obtain

$$\frac{\tan\frac{1}{2}A}{\tan\frac{1}{2}B}=\frac{\sin(s-b)}{\sin(s-a)}.$$

By composition and division,

$$\frac{\tan\frac{1}{2}A+\tan\frac{1}{2}B}{\tan\frac{1}{2}A-\tan\frac{1}{2}B}=\frac{\sin(s-b)+\sin(s-a)}{\sin(s-b)-\sin(s-a)}.$$

By §§ 30, 38, this becomes

$$\boldsymbol{\frac{\sin\frac{1}{2}(A+B)}{\sin\frac{1}{2}(A-B)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a-b)}}. \qquad \text{(III)}$$

Multiplying $\tan\frac{1}{2}A$ by $\tan\frac{1}{2}B$, and reducing, we obtain

$$\frac{\tan\frac{1}{2}A\,\tan\frac{1}{2}B}{1}=\frac{\sin(s-c)}{\sin s}.$$

By division and composition, and by §§ 30, 38, this becomes

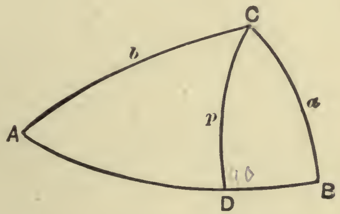
$$\frac{\cos\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A-B)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a+b)}. \qquad \text{(IV)}$$

Proceeding in a similar way with formula II, we obtain

$$\frac{\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}(a-b)}=\frac{\cot\frac{1}{2}C}{\tan\frac{1}{2}(A-B)}. \qquad \text{(V)}$$

And
$$\frac{\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}(a-b)}=\frac{\cot\frac{1}{2}C}{\tan\frac{1}{2}(A+B)}. \qquad \text{(VI)}$$

93. In the spherical triangle ABC, suppose CD drawn perpendicularly to AB, then, by the formulas for right spherical triangles,



In triangle ACD, $\sin p=\sin b \sin A$.
In triangle BCD, $\sin p=\sin a \sin B$.
Whence $\sin a \sin B=\sin b \sin A$,

or
$$\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}. \qquad \text{(VII)}$$

Remark.—If $(A+B)>180°$, then $(a+b)>180°$, and if $(A+B)<180°$, then $(a+b)<180°$.

94. All cases of oblique-angled triangles may be solved by applying one or more of the formulas I, II, III, IV, V, VI, VII, as shown in the following cases.

CASES

(1.) Given three sides, to find the angles.
Apply formula I. Check: apply V or VI.

(2.) Given three angles, to find the sides.
Apply formula II. Check: apply III or IV.

(3.) Given two sides and the included angle.
Apply V and VI, and VII. Check: apply III or IV.

(4.) Given two angles and included side.
Apply III and IV, and VII. Check: apply V or VI.

(5.) Given two angles and an opposite side.
Apply VII, V, and III. Check: apply IV.

(6.) Given two sides and an opposite angle.
Apply VII, V, and IV. Check: apply III.

EXAMPLE—CASE (1)

95. Given $a=81°\ 10'$ $\qquad b=60°\ 20'$ $\qquad c=112°\ 25'$

To find A, B, and C.

$$\begin{aligned}
a &= 81°\ 10' \\
b &= 60°\ 20' \\
c &= 112°\ 25' \\
\hline
2s &= 253°\ 55' \\
s &= 126°\ 57'\ 30'' \\
s-a &= 45°\ 47'\ 30'' \\
s-b &= 66°\ 37'\ 30'' \\
s-c &= 14°\ 32'\ 30'' \\
\log \sin s &= 9.90259 \\
\log \sin (s-a) &= 9.85540 \\
\log \sin (s-b) &= 9.96281 \\
\log \sin (s-c) &= 9.39982
\end{aligned}$$

To find A.

$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}}.$$

$$\begin{aligned}
\log \sin (s-b) &= 9.96281 \\
\log \sin (s-c) &= 9.39982 \\
\text{colog} \sin s &= 0.14460 \\
\text{colog} \sin (s-a) &= 0.09741 \\
2\,)\,&\underline{19.60464} \\
\log \tan \tfrac{1}{2} A &= 9.80232 \\
\tfrac{1}{2} A &= 32°\ 23'\ 19'' \\
A &= 64°\ 46'\ 38''
\end{aligned}$$

Napier's analogies, Wentworth, p. 155.

To find B.

$$\tan\tfrac{1}{2}B=\sqrt{\frac{\sin(s-a)\sin(s-c)}{\sin s\sin(s-b)}}.$$

$\log\sin(s-a)=9.85540$
$\log\sin(s-c)=9.39982$
$\text{colog}\sin s=0.09741$
$\text{colog}\sin(s-b)=0.03719$
$2\overline{)19.38982}$
$\log\tan\tfrac{1}{2}B=\quad 9.69491$
$\tfrac{1}{2}B=26^\circ\ 21'\ 6''$
$B=52^\circ\ 42'\ 12''$

To find C.

$$\tan\tfrac{1}{2}C=\sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s\sin(s-c)}}.$$

$\log\sin(s-a)=9.85540$
$\log\sin(s-b)=9.96281$
$\text{colog}\sin s=0.09741$
$\text{colog}\sin(s-c)=0.60018$
$2\overline{)20.51580}$
$\log\tan\tfrac{1}{2}C=\quad 10.25790$
$\tfrac{1}{2}C=61^\circ\ 5'\ 32''$
$C=122^\circ\ 11'\ 4''$

Check.

Formula V, $\cot\tfrac{1}{2}C=\dfrac{\tan\frac{1}{2}(A-B)\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}(a-b)}.$

$A=64^\circ\ 46'\ 38''$
$B=52^\circ\ 42'\ 12''$
$A-B=12^\circ\ 4'\ 26''$
$\tfrac{1}{2}(A-B)=6^\circ\ 2'\ 13''$

$a=81^\circ\ 10'$
$b=60^\circ\ 20'$
$a+b=141^\circ\ 30'$; $\tfrac{1}{2}(a+b)=70^\circ\ 45'$
$a-b=20^\circ\ 50'$; $\tfrac{1}{2}(a-b)=10^\circ\ 25'$

$\log\tan\tfrac{1}{2}(A-B)=9.02430$
$\log\sin\tfrac{1}{2}(a+b)=9.97501$
$\text{colog}\sin\tfrac{1}{2}(a-b)=0.74279$
$\cot\tfrac{1}{2}C=9.74210$
$\tfrac{1}{2}C=61^\circ\ 5'\ 32''$
$C=122^\circ\ 11'\ 4''$

EXAMPLE—CASE (3)

96. Given $a=78^\circ\ 15'$ $\quad b=56^\circ\ 20'$ $\quad C=120^\circ$

To find A, B, and c.

$\tfrac{1}{2}(a+b)=67^\circ\ 17'\ 30''$
$\tfrac{1}{2}(a-b)=10^\circ\ 57'\ 30''$
$\tfrac{1}{2}C=60^\circ$

$\log\sin\tfrac{1}{2}(a+b)=9.96498$
$\log\cos\tfrac{1}{2}(a+b)=9.58663$
$\log\sin\tfrac{1}{2}(a-b)=9.27897$
$\log\cos\tfrac{1}{2}(a-b)=9.99201$
$\log\cot\tfrac{1}{2}C=9.76144$

To find $\tfrac{1}{2}(A+B)$.

Formula V may be written

$$\tan\tfrac{1}{2}(A+B)=\frac{\cos\frac{1}{2}(a-b)\cot\frac{1}{2}C}{\cos\frac{1}{2}(a+b)}.$$

$\log\cos\tfrac{1}{2}(a-b)=\ 9.99201$
$\log\cot\tfrac{1}{2}C=\ 9.76144$
$\text{colog}\cos\tfrac{1}{2}(a+b)=\ 0.41337$
$\log\tan\tfrac{1}{2}(A+B)=10.16682$
$\tfrac{1}{2}(A+B)=55^\circ\ 44'\ 36''-$
$\tfrac{1}{2}(A-B)=\ 6^\circ\ 47'\ 4''$
$A=62^\circ\ 31'\ 40''$
$B=48^\circ\ 57'\ 32''-$

To find $\tfrac{1}{2}(A-B)$.

Formula VI may be written

$$\tan\tfrac{1}{2}(A-B)=\frac{\sin\frac{1}{2}(a-b)\cot\frac{1}{2}C}{\sin\frac{1}{2}(a+b)}.$$

$\log\sin\tfrac{1}{2}(a-b)=9.27897$
$\log\cot\tfrac{1}{2}C=9.76144$
$\text{colog}\sin\tfrac{1}{2}(a+b)=0.03502$
$\tfrac{1}{2}(A-B)=6^\circ\ 57'\ 4'$

To find c.

From Formula VII, $\sin c = \dfrac{\sin b \sin C}{\sin B}$.

$$\begin{aligned} \log \sin b &= 9.92027 \\ \log \sin C &= 9.93753 \\ \operatorname{colog} \sin B &= \underline{0.12249} \\ \log \sin c &= 9.98029 \\ c &= 107^\circ\ 8' \end{aligned}$$

Check.

Formula III may be written

$$\tan \tfrac{1}{2} c = \frac{\sin \frac{1}{2}(A+B) \tan \frac{1}{2}(a-b)}{\sin \frac{1}{2}(A-B)}.$$

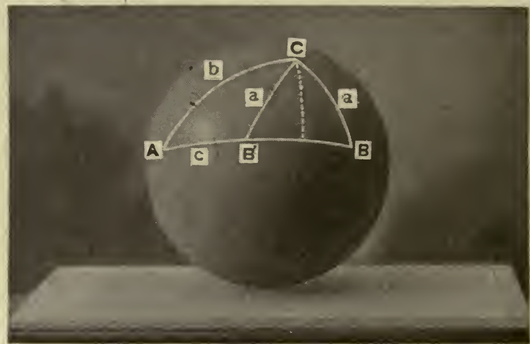
$$\begin{aligned} \log \sin \tfrac{1}{2}(A+B) &= 9.91725 \\ \log \tan \tfrac{1}{2}(a-b) &= 9.28696 \\ \operatorname{colog} \sin \tfrac{1}{2}(A-B) &= \underline{0.92762} \\ \log \tan \tfrac{1}{2} c &= 10.13183 \\ \tfrac{1}{2} c &= 53^\circ\ 33'\ 56''- \\ c &= 107^\circ\ 7'\ 51''- \end{aligned}$$

(Discrepancy due to omitted decimals.)

AMBIGUOUS CASES

97. (1.) Two sides and an angle opposite one of them are the given parts.

If the side opposite the given angle differs from 90° more than the other given side, the given angle and the side opposite being either both less or both greater than 90°, there are two solutions.



(2.) Two angles and a side opposite one of them are the given parts.

If the angle opposite the given side differs from 90° more than the other given angle, the given side and the angle opposite being either both less or both greater than 90°, there are two solutions.

Remark.—There is no solution if, in either of the formulas,

$$\sin B = \frac{\sin A \sin b}{\sin a}, \qquad \sin a = \frac{\sin b \sin A}{\sin B},$$

the numerator of the fraction is greater than the denominator.

EXAMPLE—CASE (6)

98. Given $a=40° 16'$ $b=47° 44'$ $A=52° 30'$

To find B, B', C, C', and c, c'.

To find B and B'.

Formula VII may be written

$$\sin B=\frac{\sin A \sin b}{\sin a}.$$

$$\begin{aligned} \log \sin A &= 9.89947 \\ \log \sin b &= 9.86924 \\ \text{colog} \sin a &= 0.18953 \\ \hline \log \sin B &= 9.95824 \\ B &= 65° 16' 30'' \\ B' &= 114° 43' 30'' \end{aligned}$$

To find c.

Formula IV may be written

$$\tan \tfrac{1}{2} c=\frac{\cos \frac{1}{2}(A+B) \tan \frac{1}{2}(a+b)}{\cos \frac{1}{2}(A-B)}.$$

$$\begin{aligned} \log \cos \tfrac{1}{2}(A+B) &= 9.71326 \\ \log \tan \tfrac{1}{2}(a+b) &= 9.98484 \\ \text{colog} \cos \tfrac{1}{2}(A-B) &= 0.00270 \\ \hline \log \tan \tfrac{1}{2} c &= 9.70080 \\ \tfrac{1}{2} c &= 26° 39' 42'' \\ c &= 53° 19' 24'' \end{aligned}$$

To find c'.

$$\begin{aligned} \log \cos \tfrac{1}{2}(A+B') &= 9.04631 \\ \log \tan \tfrac{1}{2}(a+b) &= 9.98484 \\ \text{colog} \cos \tfrac{1}{2}(A-B') &= 0.06745 \\ \hline \log \tan \tfrac{1}{2} c' &= 9.09860 \\ \tfrac{1}{2} c' &= 7° \ 9' \ 9'' \\ c' &= 14° 18' 18'' \end{aligned}$$

To find C.

Formula V may be written

$$\cot \tfrac{1}{2} C=\frac{\sin \frac{1}{2}(a+b) \tan \frac{1}{2}(A-B)}{\sin \frac{1}{2}(a-b)}.$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(a+b) &= 9.84177 \\ \log \tan \tfrac{1}{2}(A-B) &= 9.04901 \text{ n} \\ \text{colog} \sin \tfrac{1}{2}(a-b) &= 1.18633 \text{ n} \\ \hline \log \cot \tfrac{1}{2} C &= 10.07711 \\ \tfrac{1}{2} C &= 39° 56' 24'' \\ C &= 79° 52' 48'' \end{aligned}$$

To find C'.

$$\begin{aligned} \log \sin \tfrac{1}{2}(a+b) &= 9.84177 \\ \log \tan \tfrac{1}{2}(A-B') &= 9.78153 \text{ n} \\ \text{colog} \sin \tfrac{1}{2}(a-b) &= 1.18633 \text{ n} \\ \hline \log \cot \tfrac{1}{2} C' &= 10.80963 \\ \tfrac{1}{2} C' &= 8° 48' 41'' \\ C' &= 17° 37' 22'' \end{aligned}$$

Check.

Formula III may be written

$$\sin b=\frac{\sin B \sin c}{\sin C}.$$

$$\begin{aligned} \log \sin B &= 9.95824 \\ \log \sin c &= 9.90418 \\ \text{colog} \sin C &= 0.00682 \\ \hline \log \sin b &= 9.86924 \\ b &= 47° 44' \end{aligned}$$

EXERCISES

99. (1.) In the spherical triangle ABC, the side $a=124° 53'$, the side $b=31° 19'$, and the angle $A=16° 26'$. Find the other parts.

(2.) In the oblique-angled spherical triangle ABC, angle $A=128° 45'$, angle $C=30° 35'$, and the angle $B=68° 50'$. Find the other parts.

* The letter "n" indicates that these quantities are negative.

(3.) In the spherical triangle ABC, the side $c = 78° \ 15'$, $b = 56° \ 20'$, and $A = 120°$. Required the other parts.

(4.) In the spherical triangle ABC, the angle $A = 125° \ 20'$, the angle $C = 48° \ 30'$, and the side $b = 83° \ 13'$. Required the remaining parts.

(5.) In the spherical triangle ABC, the side $c = 40° \ 35'$, $b = 39° \ 10'$, and $a = 71° \ 15'$. Required the angles.

(6.) In the spherical triangle ABC, the angle $A = 109° \ 55'$, $B = 116° \ 38'$, and $C = 120° \ 43'$. Required the sides.

(7.) In the spherical triangle ABC, the angle $A = 130° \ 5' \ 22''$, the angle $C = 36° \ 45' \ 28''$, and the side $b = 44° \ 13' \ 45''$. Required the remaining parts.

(8.) In the spherical triangle ABC, the angle $A = 33° \ 15' \ 7''$, $B = 31° \ 34' \ 38''$, and $C = 161° \ 25' \ 17''$. Required the sides.

(9.) In the spherical triangle ABC, the side $c = 112° \ 22' \ 58''$, $b = 52° \ 39' \ 4''$, and $a = 89° \ 16' \ 53''$. Required the angles.

(10.) In the spherical triangle ABC, the side $c = 76° \ 35' \ 36''$, $b = 50° \ 10' \ 30''$, and the angle $A = 34° \ 15' \ 3''$. Required the remaining parts.

AREA OF THE SPHERICAL TRIANGLE

100. It is proved in geometry that the area of a spherical triangle is equal to its spherical excess, that is,

area $= (A + B + C - 2$ rt. angles$) \times$ area of the tri-rectangular triangle,

where A, B, and C are the angles of the spherical triangle. Hence

$$\frac{\text{area}}{\text{surface of sphere}} = \frac{A + B + C - 180°}{720°}.$$

The surface of the sphere is $4\pi R^2$, therefore

$$\textbf{area} = \pi R^2 \left(\frac{A + B + C - 180°}{180°} \right)$$

The following formula, called Lhuilier's theorem, simplifies the derivation of $(A + B + C - 180°)$ where the three

sides of the spherical triangle are given; in it a, b, and c denote the sides of the triangle, and $2s=a+b+c$.

$$\tan\left(\frac{A+B+C-180^\circ}{4}\right)=\sqrt{\tan\tfrac{1}{2}s\tan\tfrac{1}{2}(s-a)\tan\tfrac{1}{2}(s-b)\tan\tfrac{1}{2}(s-c)}.$$

EXERCISES

(1.) The angles of a spherical triangle are, $A=63^\circ$, $B=84^\circ\ 21'$, $C=79^\circ$; the radius of the sphere is 10 in. What is the area of the triangle?

(2.) The sides of a spherical triangle are, $a=6.47$ in., $b=8.39$ in., $c=9.43$ in.; the radius of the sphere is 25 in. What is the area of the triangle?

(3.) In a spherical triangle, $A=75^\circ\ 16'$, $B=39^\circ\ 20'$, $c=26$ in.; the radius of the sphere is 14 in. Find the area of the triangle.

(4.) In a spherical triangle, $a=441$ miles, $b=287$ miles, $C=38^\circ\ 21'$; the radius of the sphere is 3960 miles. Find the area of the triangle.

CHAPTER X

APPLICATIONS TO THE CELESTIAL AND TERRESTRIAL SPHERES

ASTRONOMICAL PROBLEMS

101. An observer at any place on the earth's surface finds himself seemingly at the centre of a sphere, one-half of which is the sky above him. This sphere is called the celestial sphere, and upon its surface appear all the heavenly bodies. The entire sphere seems to turn completely around once in 23 hours and 56 minutes, as on an axis. The imaginary axis is the axis of the earth indefinitely produced. The points in which it pierces the celestial sphere appear stationary, and are called the north and south poles of the heavens. The North Star (Polaris) marks very nearly (within $1^\circ\ 16'$) the position of the north pole. As the observer travels towards the north he finds that the north pole of the heavens appears higher and higher up in the sky, and that its height above the horizon, measured in degrees, corresponds to the latitude of the place of observation.

The fixed stars and nebulæ preserve the same relative positions to each other. The sun, moon, planets, and comets change their positions with respect to the fixed stars continually, the sun appearing to move eastward among the stars about a degree a day, and the moon about thirteen times as far.

The **zenith** is the point on the celestial sphere directly overhead.

The **horizon** is the great circle everywhere 90° from the zenith.

The **celestial equator** is the great circle in which the plane of the earth's equator if extended would cut the celestial sphere.

The **ecliptic** is the path on the celestial sphere described by the sun in its apparent eastward motion among the stars. The ecliptic is a great circle inclined to the plane of the equator at an angle of approximately $23\frac{1}{2}°$.

The **poles of the equator** are the points where the axis of the earth if produced would pierce the celestial sphere, and are each 90° from the equator.

The **poles of the ecliptic** are each 90° from the ecliptic.

The **equinoxes** are the points where the celestial equator and ecliptic intersect; that which the sun crosses when coming north being called the vernal equinox, and that which it crosses when going south the autumnal equinox.

The **declination** of a heavenly body is its distance, measured in degrees, north or south of the celestial equator.

The **right ascension** of a heavenly body is the distance, measured in degrees eastward on the celestial equator, from the vernal equinox to the great circle passing through the poles of the equator and this body.

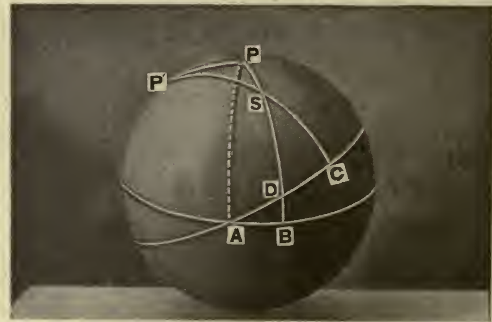
The **celestial latitude** of a heavenly body is the distance from the ecliptic measured in degrees on the great circle passing through the pole of the ecliptic and the body.

The **celestial longitude** of a heavenly body is the distance, measured in degrees eastward on the ecliptic, from

the vernal equinox to the great circle passing through the pole of the ecliptic and the body.

EXERCISES

(1.) The right ascension of a given star is $25^\circ\ 35'$, and its declination is $+$(north) $63^\circ\ 26'$. Assuming the angle between the celestial equator and the ecliptic to be $23^\circ\ 27'$, find the celestial latitude and celestial longitude.



In this figure AB is the celestial equator, AC the ecliptic, P the pole of the equator, P' the pole of the ecliptic. S is the position of the star, and the lines SB and SC are drawn through P and P' perpendicular to AB and AC. AB is the right ascension and BS the declination of the star, while AC is the longitude and SC the latitude of the star.

In the spherical triangle $P'PS$, it will be seen that $P'S$ is the complement of the celestial latitude, PS the complement of the declination, and $P'PS$ is 90° plus the right ascension. It is to be noted that A is the vernal equinox.

(2.) The declination of the sun on December 21st is $-$(south) $23^\circ\ 27'$. At what time will the sun rise as seen from a place whose latitude is $41^\circ\ 18'$ north?

The arc ZS which is the distance from the zenith to the centre of the sun when the sun's upper rim is on the horizon is $90^\circ\ 50'$. The $50'$ is made up of the sun's semi-diameter of $16'$, plus the correction for refraction of $34'$.

(3.) The declination of the sun on December 21st is — (south) 23° 27′. At what time would the sun set as seen from a place in latitude 50° 35′ north?

SUNRISE

SUNSET

In these figures P is the pole of the equator, Z the zenith, EQ the celestial equator. AS is the declination of the sun, $ZS=90° \; 50'$, $PS=90°+$declination, $PZ=90°-$latitude. The problem is to find the angle SPZ. An angle of 15° at the pole corresponds to 1 hour of time.

GEOGRAPHICAL PROBLEMS

102. The **meridian** of a place is the great circle passing through the place and the poles of the earth.

The **latitude** of a place is the arc of the meridian of the place extending from the equator to the place.

Latitude is measured north and south of the equator from 0° to 90°.

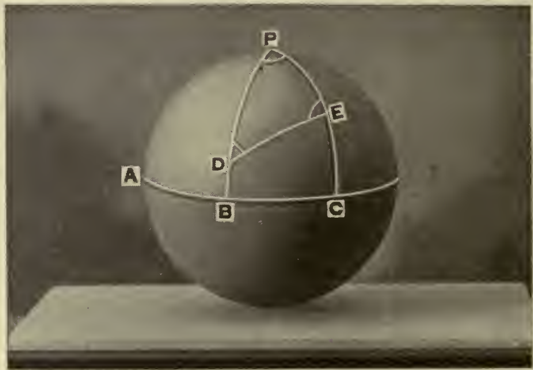
The **longitude** of a place is the arc of the equator extending from the zero meridian to the meridian of the place. The meridian of the Greenwich Observatory is usually taken as the zero meridian.

Longitude is measured east or west from 0° to 180°.

The longitude of a place is also the angle between the zero meridian and the meridian of the place.

In the following problems one minute is taken equal to one geographical mile.

(1.) Required the distance in geographical miles between two places, D and E, on the earth's surface. The longitude of D is 60° 15′ E., and the latitude 20° 10′ N. The longitude of E is 115° 20′ E., and the latitude 37° 20′ N.



In this figure AC represents the equator of the earth, P the north pole, and A the intersection of the meridian of Greenwich with the equator. PB and PC represent meridians drawn through D and E respectively. Then AB is the longitude and BD the latitude of D; AC the longitude and CE the latitude of E.

(2.) Required the distance from New York, latitude 40° 43′ N., longitude 74° 0′ W., to San Francisco, latitude 37° 48′ N., longitude 122° 28′ W., on the shortest route.

(3.) Required the distance from Sandy Hook, latitude 40° 28′ N., longitude 74° 1′ W., to Madeira, in latitude 32° 28′ N., longitude 16° 55, W., on the shortest route.

(4.) Required the distance from San Francisco, latitude 37° 48′ N., longitude 122° 28′ W., to Batavia in Java, latitude 6° 9′ S., longitude 106° 53′ E., on the shortest route.

(5.) Required the distance from San Francisco, latitude 37° 48′ N., longitude 122° 28′ W., to Valparaiso, latitude 33° 2′ S., longitude 71° 41′ W., on the shortest route.

CHAPTER XI

GRAPHICAL SOLUTION OF A SPHERICAL TRIANGLE

103. The given parts of a spherical triangle may be laid off, and then the required parts may be measured, by making use of a globe fitted to a hemispherical cup.

The sides of the spherical triangle are arcs of great circles, and may be drawn on the globe with a pencil, using the rim of the cup, which is a great circle, as a ruler. The rim of the cup is graduated from 0° to 180° in both directions.

The angle of a spherical triangle may be measured on a great circle drawn on the sphere at a distance of 90° from the vertex of the angle.*

CASE I. *Given the sides a, b, and c of a spherical triangle, to determine the angles A, B, and C.*

Place the globe in the cup, and draw upon it a line equal to the number of degrees in the side c, using the rim of the cup as a ruler. Mark the extremities of this line A and B. With A and B as centres, and b and a respectively as radii, draw with the dividers two arcs intersecting at C (Fig. 1). Then, placing the globe in the cup so that the points A and C shall rest on the rim, draw the line $AC=b$, and in the same way draw $BC=a$.

To measure the angle A place the arc AB in coincidence

* Slated globes, three inches in diameter, made of papier-maché, and held in metal hemispherical cups, are manufactured for the use of students of spherical trigonometry at a small cost.

with the rim of the cup, and make AE equal to 90°. Also make AF in AC produced equal to 90°. Then place the globe in the cup so that E and F shall be in the rim, and note the measure of the arc EF. This is the measure of the angle A. In the same way the angles B and C can be determined.

FIG. 1

FIG. 2

CASE II. *Given the angles A, B, and C, to find the sides* a, b, and c.

Subtract A, B, and C each from 180°, to obtain the sides a', b', and c' of the polar triangle. Construct this polar triangle according to the method employed in Case I. Mark its vertices A', B', and C'. With each of these vertices as a centre, and a radius equal to 90°, describe arcs with the dividers. The points of intersection of these arcs will be the vertices A, B, and C of the given triangle. The sides of this triangle a, b, and c can then be measured on the rim of the cup.

CASE III. *Given two sides, b and c, and the included angle A, to find B, C, and a.*

Lay off (Fig. 3) the line AB equal to c, and mark the point D in AB produced, so that AD equals 90°. With the dividers mark another point, F, at a distance of 90° from A. Turn the globe in the cup till D and F are both in the rim, and make DE equal to the number of degrees in the angle A. With A and E in the rim of the cup, draw the line AC equal to the number of degrees in the side b. Join C and B. The required parts of the triangle can then be measured.

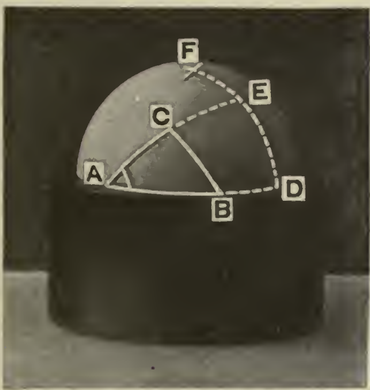


FIG. 3

FIG. 4

CASE IV. *Given the angles A and B and the included side c, to find a, b, and C.*

Lay off the line AB equal to c. Then construct the given angles at A and B, as in Case III., and extend their sides to intersect at C.

CASE V. *Given the sides b, a, and the angle A opposite one of these sides, to find c, B, and C.* (Ambiguous case.)

Lay off (Fig. 4) AC equal to b, and construct the angle A as in Case III. Take c in the dividers as a radius, and with C as a centre describe arcs cutting the other side of the triangle in B and B', and measure the remaining parts of the two triangles.

If the arc described with C as a centre does not cut the other side of the triangle, there is no solution. If tangent, there is one solution.

CASE VI. *Given the angles A, B, and the side a opposite one of the angles.*

Construct the polar triangle of the given triangle by Case V.; then construct the original triangle as in Case II., and measure the parts required.

The constructions given above include all cases of right and quadrantal triangles.

CHAPTER XII

RECAPITULATION OF FORMULAS

ELEMENTARY RELATIONS (§ 10)

$$\tan x = \frac{\sin x}{\cos x}, \qquad \cot x = \frac{\cos x}{\sin x},$$

$$\sec x = \frac{1}{\cos x}, \qquad \csc x = \frac{1}{\sin x}.$$

$$\tan x \cot x = 1,$$
$$\sin^2 x + \cos^2 x = 1,$$
$$1 + \tan^2 x = \sec^2 x,$$
$$1 + \cot^2 x = \csc^2 x.$$

RIGHT TRIANGLES (§§ 14 AND 27)

$$\sin A = \frac{a}{c}, \qquad \sin B = \frac{b}{c},$$

$$\cos A = \frac{b}{c}, \qquad \cos B = \frac{a}{c},$$

$$\tan A = \frac{a}{b}, \qquad \tan B = \frac{b}{a},$$

$$\cot A = \frac{b}{a}, \qquad \cot B = \frac{a}{b},$$

$$a^2 + b^2 = c^2,$$

where c = hypotenuse, a and b sides about the right angle; A and B the acute angles opposite a and b.

FUNCTIONS OF TWO ANGLES (§§ 30–34)

$$\sin(x+y) = \sin x \cos y + \cos x \sin y,$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y,$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y,$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y.$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y},$$

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y},$$

$$\cot(x+y)=\frac{\cot x\cot y-1}{\cot y+\cot x},$$

$$\cot(x-y)=\frac{\cot x\cot y+1}{\cot y-\cot x}.$$

FUNCTIONS OF TWICE AN ANGLE (§ 36)

$$\sin 2x=2\sin x\cos x,$$

$$\begin{aligned}\cos 2x&=\cos^2 x-\sin^2 x,\\&=1-2\sin^2 x,\\&=2\cos^2 x-1,\end{aligned}$$

$$\tan 2x=\frac{2\tan x}{1-\tan^2 x},$$

$$\cot 2x=\frac{\cot^2 x-1}{2\cot x}.$$

FUNCTIONS OF HALF AN ANGLE (§ 37)

$$\sin\tfrac{1}{2}x=\pm\sqrt{\frac{1-\cos x}{2}},$$

$$\cos\tfrac{1}{2}x=\pm\sqrt{\frac{1+\cos x}{2}},$$

$$\tan\tfrac{1}{2}x=\pm\sqrt{\frac{1-\cos x}{1+\cos x}},$$

$$\cot\tfrac{1}{2}x=\sqrt{\frac{1+\cos x}{1-\cos x}}.$$

SUMS AND DIFFERENCES OF FUNCTIONS (§ 38)

$$\sin u+\sin v=2\sin\tfrac{1}{2}(u+v)\cos\tfrac{1}{2}(u-v),$$

$$\sin u-\sin v=2\cos\tfrac{1}{2}(u+v)\sin\tfrac{1}{2}(u-v),$$

$$\cos u+\cos v=2\cos\tfrac{1}{2}(u+v)\cos\tfrac{1}{2}(u-v),$$

$$\cos u-\cos v=-2\sin\tfrac{1}{2}(u+v)\sin\tfrac{1}{2}(u-v).$$

$$\frac{\sin u+\sin v}{\sin u-\sin v}=\frac{\tan\frac{1}{2}(u+v)}{\tan\frac{1}{2}(u-v)}.$$

OBLIQUE TRIANGLES (§§ 42–45)

$$\frac{a}{b}=\frac{\sin A}{\sin B};\qquad \frac{a}{c}=\frac{\sin A}{\sin C};\qquad \frac{b}{c}=\frac{\sin B}{\sin C}.$$

$$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)},$$

$$\frac{a-c}{a+c}=\frac{\tan\frac{1}{2}(A-C)}{\tan\frac{1}{2}(A+C)},$$

$$\frac{b-c}{b+c}=\frac{\tan\frac{1}{2}(B-C)}{\tan\frac{1}{2}(B+C)}.$$

$$a^2=b^2+c^2-2bc\cos A,$$

$$b^2=c^2+a^2-2ca\cos B,$$

$$c^2=a^2+b^2-2ab\cos C.$$

$$\tan\tfrac{1}{2}A=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

$$\tan\tfrac{1}{2}B=\sqrt{\frac{(s-c)(s-a)}{s(s-b)}},$$

$$\tan\tfrac{1}{2}C=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}},$$

where $s=\frac{a+b+c}{2}$.

$$\tan\tfrac{1}{2}A=\frac{K}{s-a},\qquad \tan\tfrac{1}{2}B=\frac{K}{s-b},\qquad \tan\tfrac{1}{2}C=\frac{K}{s-c},$$

where $K=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$.

AREA OF A TRIANGLE (§ 46)

$$S=\tfrac{1}{2}ac\sin B.\quad S=\tfrac{1}{2}ba\sin C.\quad S=\tfrac{1}{2}cb\sin A.$$

$$S=\sqrt{s(s-a)(s-b)(s-c)}.$$

LOGARITHMIC, COSINE, SINE, AND EXPONENTIAL SERIES (§ 58)

$$\log_e(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+,\text{ etc.}$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+,\text{ etc.}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} +, \text{ etc.}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} +, \text{ etc.}$$

DE MOIVRE'S THEOREM (§ 71)

$$(\cos x + \sqrt{-1}\, \sin x)^n = \cos nx + \sqrt{-1}\, \sin nx.$$

$$\sin nx = n \cos^{n-1} x \sin x - \frac{n(n-1)(n-2)}{3!} \cos^{n-3} x \sin^3 x +, \text{ etc.}$$

$$\cos nx = n \cos^n x - \frac{n(n-1)}{2!} \cos^{n-2} x \sin^2 x +, \text{ etc.}$$

HYPERBOLIC FUNCTIONS (§ 75)

$$\sinh x = \frac{e^x - e^{-x}}{2},$$

$$\cosh x = \frac{e^x + e^{-x}}{2},$$

$$e^{ix} = \cos x + i \sin x.$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i},$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}.$$

$$\sin ix = \frac{i(e^x - e^{-x})}{2} = i \sinh x,$$

$$\cos ix = \frac{e^x + e^{-x}}{2} = \cosh x.$$

SPHERICAL TRIANGLES

RIGHT AND QUADRANTAL TRIANGLES (§§ 83, 87)

Use Napier's rules.

OBLIQUE TRIANGLES (§§ 89–93)

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a.$$

$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}}.$$

$$\tan \tfrac{1}{2} a = \sqrt{\frac{-\cos S \cos (S-A)}{\cos (S-B) \cos (S-C)}}.$$

$$\frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2} c}{\tan \frac{1}{2}(a-b)}.$$

$$\frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2} c}{\tan \frac{1}{2}(a+b)}.$$

$$\frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A-B)}.$$

$$\frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A+B)}.$$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}.$$

AREA OF SPHERICAL TRIANGLES (§ 101)

$$\text{area} = \pi R^2 \left(\frac{A+B+C-180^\circ}{180^\circ} \right)$$

$$\tan \left(\frac{A+B+C-180^\circ}{4} \right) = \sqrt{\tan \tfrac{1}{2} s \, \tan \tfrac{1}{2}(s-a) \, \tan \tfrac{1}{2}(s-b) \, \tan \tfrac{1}{2}(s-c)}.$$

11 formulae

APPENDIX

RELATIONS OF THE PLANE, SPHERICAL, AND PSEUDO-SPHERICAL TRIGONOMETRIES

We have up to the present considered the trigonometries which deal with figures on a plane or spherical surface. A characteristic feature of these two surfaces is that the curvature of the plane is zero, while that of the sphere is a positive constant ρ. If the radius of the sphere is increased indefinitely, its surface approaches the plane as a limit while its curvature ρ approaches 0.

In works on absolute geometry it is shown that there exists a surface which has a constant negative curvature: it is called a pseudo-sphere, and the trigonometry upon it pseudo-spherical trigonometry.

We observe that as ρ passes continuously from positive to negative values, we pass from the sphere through the plane to the pseudo-sphere. Thus the formulas of plane trigonometry are the limiting cases of those of either of the two other trigonometries.

In the treatment of spherical trigonometry the radius of the sphere has been taken as unity. If, however, the radius of the sphere is r, and a, b, and c denote the *lengths* of the sides of the spherical triangle, the formulas are changed, in that a is replaced by $\frac{a}{r}$, b by $\frac{b}{r}$, and c by $\frac{c}{r}$; thus,

$$\sin C = \frac{\sin c}{\sin a}$$

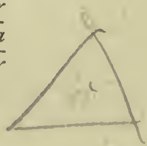
becomes
$$\sin C = \frac{\sin \dfrac{c}{r}}{\sin \dfrac{a}{r}}.$$

The formulas for pseudo-spherical trigonometry are the same as the formulas of spherical trigonometry, except that the hyperbolic functions of $\frac{a}{r}$, $\frac{b}{r}$, and $\frac{c}{r}$ are substituted for the trigonometric.

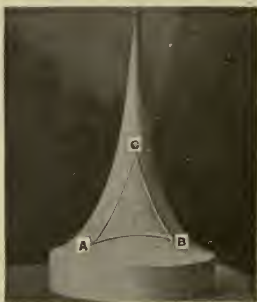
Thus, corresponding to the above formula of spherical trigonometry, is the formula

$$\sin C = \frac{\sinh \dfrac{c}{r}}{\sinh \dfrac{a}{r}}$$

of pseudo-spherical trigonometry.



SPHERE



PSEUDO-SPHERE

The pseudo-sphere is generated by revolving the curve whose equation is

$$y = r \log \frac{r + \sqrt{r^2 - x^2}}{x} - \sqrt{r^2 - x^2}$$

about its y axis. The radius of the base of the pseudo-sphere is r.

Hence the formulas of plane trigonometry can be derived from the formulas of either spherical or pseudo-spherical trigonometry by expressing the functions in series and allowing r to increase without limit.

Example.—Show that if r is increased indefinitely the following corresponding formulas for the spherical and pseudo-spherical right triangle

$$\cos\frac{a}{r}=\cos\frac{b}{r}\cos\frac{c}{r}, \tag{1}$$

$$\cosh\frac{a}{r}=\cosh\frac{b}{r}\cosh\frac{c}{r}, \tag{2}$$

reduce to the corresponding formula for a plane right triangle; that is, to

$$a^2=b^2+c^2. \tag{3}$$

Substituting the series $\cos\frac{a}{r}$, etc., in equation (1), we obtain

$$\left(1-\frac{1}{2\,!}\left(\frac{a}{r}\right)^2+\ldots\right)=\left(1-\frac{1}{2\,!}\left(\frac{b}{r}\right)^2+\ldots\right)\left(1-\frac{1}{2\,!}\left(\frac{c}{r}\right)^2+\ldots\right),$$

or
$$1-\frac{1}{2\,!}\frac{a^2}{r^2}+\frac{1}{4\,!}\frac{a^4}{r^4}+\ldots=1-\frac{1}{2\,!}\frac{b^2}{r^2}-\frac{1}{2\,!}\frac{c^2}{r^2}+\frac{1}{4\,!}\frac{b^4}{r^4}+\ldots \tag{4}$$

Substituting in equation (2) the series for $\cosh\frac{a}{r}$, etc., which we obtain from $\cosh x=\frac{e^x+e^{-x}}{2}$, we have

$$1+\frac{1}{2\,!}\left(\frac{a}{r}\right)^2+\ldots=\left(1+\frac{1}{2\,!}\left(\frac{b}{r}\right)^2+\ldots\right)\left(1+\frac{1}{2\,!}\left(\frac{c}{r}\right)^2+\ldots\right),$$

or
$$1+\frac{1}{2\,!}\frac{a^2}{r^2}+\frac{1}{4\,!}\frac{a^4}{r^4}+\ldots=1+\frac{1}{2\,!}\frac{b^2}{r^2}+\frac{1}{2\,!}\frac{c^2}{r^2}+\frac{1}{4\,!}\frac{b^4}{r^4}+\ldots \tag{5}$$

Cancelling 1 in equations (4) and (5), multiplying by r^2, and, finally, allowing r to increase without limit, we get from either equation

$$a^2=b^2+c^2.$$

EXERCISES

Derive each of the following formulas of plane trigonometry from the corresponding formula of spherical trigonometry, and also from the corresponding formula of pseudo-spherical trigonometry.

Right triangles; A = right angle.

(1.) Plane, $\sin C = \frac{c}{a}$.

Spherical, $\sin C = \frac{\sin c}{\sin a}$.

Pseudo-spherical, $\sin C = \frac{\sinh c}{\sinh a}$.

Oblique Triangles.

(2.) Plane, $a^2 = b^2 + c^2 - 2\,bc \cos A$.

Spherical, $\cos a = \cos b \cos c + \sin b \sin c \cos A$.

Pseudo-spherical, $\cosh a = \cosh b \cosh c + \sinh b \sinh c \cos A$.

(3.) Plane, $S = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$.

Spherical,

$$\tan \frac{(A+B+C-180^\circ)}{4} = \sqrt{\tan \tfrac{1}{2} \frac{s}{r} \tan \tfrac{1}{2} \frac{(s-a)}{r} \tan \tfrac{1}{2} \frac{(s-b)}{r} \tan \tfrac{1}{2} \frac{(s-c)}{r}}.$$

Pseudo-spherical,

$$\tan \frac{(180^\circ - A+B+C)}{4} = \sqrt{\tanh \tfrac{1}{2} \frac{s}{r} \tanh \tfrac{1}{2} \frac{(s-a)}{r} \tanh \tfrac{1}{2} \frac{(s-b)}{r} \tanh \tfrac{1}{2} \frac{(s-c)}{r}}.$$

ANSWERS TO EXERCISES

§ 4 (page 3).

(1.) $192° \; 51' \; 25\frac{5}{7}''$.
Quadrant III.

(2.) $25°$.

(3.) $287°$, $647°$.

(4.) Quadrant III.

§ 9 (page 9).

tan $1000°$ is negative.
cos $810°$ is 0.
sin $760°$ is positive.
cot $-70°$ is negative.
cos $-550°$ is negative.
tan $-560°$ is negative.
sec $300°$ is positive.
cot $1560°$ is negative.
sin $130°$ is positive.
cos $260°$ is negative.
tan $310°$ is negative.

§ 13 (page 11).

(3.) $\cos -30° = \frac{1}{2}\sqrt{3}$.
$\tan -30° = -\frac{1}{3}\sqrt{3}$,
$\cot -30° = -\sqrt{3}$,
$\sec -30° = \frac{2}{3}\sqrt{3}$,
$\csc -30° = -2$.

(4.) $\cos x = -\frac{2}{3}\sqrt{2}$,
$\tan x = \frac{1}{4}\sqrt{2}$,
$\cot x = 2\sqrt{2}$,
$\sec x = -\frac{3}{4}\sqrt{2}$,
$\csc x = -3$.

(5.) $\cos y = \frac{4}{5}$, $\tan y = -\frac{3}{4}$,
$\cot y = -\frac{4}{3}$, $\sec y = \frac{5}{4}$,
$\csc y = -\frac{5}{3}$.

(6.) $\sin 60° = \frac{1}{2}\sqrt{3}$,
$\tan 60° = \sqrt{3}$,
$\cot 60° = \frac{1}{3}\sqrt{3}$,
$\sec 60° = 2$,
$\csc 60° = \frac{2}{3}\sqrt{3}$.

(7.) $\cos 0° = 1$, $\tan 0° = 0$.

(8.) $\sin z = \frac{4}{5}$, $\cos z = \frac{3}{5}$,
$\cot z = \frac{3}{4}$, $\sec z = \frac{5}{3}$,
$\csc z = \frac{5}{4}$.

(9.) $\sin 45° = \cos 45° = \frac{1}{2}\sqrt{2}$,
$\tan 45° = 1$,
$\sec 45° = \csc 45° = \sqrt{2}$.

(10.) $\sin y = -\frac{1}{3}\sqrt{5}$, $\cos y = -\frac{2}{3}$,
$\cot y = \frac{2}{5}\sqrt{5}$, $\sec y = -\frac{3}{2}$,
$\csc y = -\frac{3}{5}\sqrt{5}$.

(11.) $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{1}{2}\sqrt{3}$,
$\tan 30° = \frac{1}{3}\sqrt{3}$,
$\sec 30° = \frac{2}{3}\sqrt{3}$,
$\csc 30° = 2$.

(12.) $\sin x = \frac{4}{5}$, $\cos x = -\frac{3}{5}$.

(13.) $\sqrt{\frac{1}{2} \pm \frac{1}{6}\sqrt{5}}$.

§ 17 (page 14).

(1.) $\sin 70° = \cos 20°$,
$\cos 60° = \sin 30°$,
$\cos 89° \; 31' = \sin 29'$,
$\cot 47° = \tan 43°$,

$\tan 63^\circ = \cot 27^\circ$,
$\sin 72^\circ 39' = \cos 17^\circ 21'$.

(2.) $x = 30^\circ$.

(3.) $x = 22^\circ 30'$.

(4.) $x = 18^\circ$.

(5.) $x = 15^\circ$.

§ 25 (page 21).

(1.) 225° and 315°,
60° and 240°.

(2.) 60°, 120°, 420°, 480°.

(3.) $\sin -30^\circ = -\frac{1}{2}$,
$\cos -30^\circ = \frac{1}{2}\sqrt{3}$,
$\sin 765^\circ = \cos 765 = \frac{1}{2}\sqrt{2}$,
$\sin 120^\circ = \frac{1}{2}\sqrt{3}$,
$\cos 120^\circ = -\frac{1}{2}$,
$\sin 210^\circ = -\frac{1}{2}$,
$\cos 210^\circ = -\frac{1}{2}\sqrt{3}$.

(4.) The functions of 405° are equal to the functions of 45°.
$\sin 600^\circ = -\frac{1}{2}\sqrt{3}$,
$\cos 600^\circ = -\frac{1}{2}$,
$\tan 600^\circ = \sqrt{3}$,
$\cot 600^\circ = \frac{1}{3}\sqrt{3}$.
$\sec 600^\circ = -2$,
$\csc 600^\circ = -\frac{2}{3}\sqrt{3}$.

The functions of 1125° are equal to the functions of 45°.
$\sin -45^\circ = -\frac{1}{2}\sqrt{2}$,
$\cos -45^\circ = \frac{1}{2}\sqrt{2}$,
$\tan -45^\circ = \cot -45^\circ = -1$,
$\sec -45^\circ = \sqrt{2}$,
$\csc -45^\circ = -\sqrt{2}$.
$\sin 225^\circ = \cos 225^\circ = -\frac{1}{2}\sqrt{2}$,
$\tan 225^\circ = \cot 225^\circ = 1$,
$\sec 225^\circ = \csc 225^\circ = -\sqrt{2}$.

(5.) The functions of -120° are the same as those of 600° given in (4).
$\sin -225^\circ = \frac{1}{2}\sqrt{2}$,
$\cos -225^\circ = -\frac{1}{2}\sqrt{2}$,
$\tan -225^\circ = \cot -225^\circ = -1$,
$\sec -225^\circ = -\sqrt{2}$,
$\csc -225^\circ = \sqrt{2}$,
$\sin -420^\circ = -\frac{1}{2}\sqrt{3}$,
$\cos -420^\circ = \frac{1}{2}$,
$\tan -420^\circ = -\sqrt{3}$,
$\cot -420^\circ = -\frac{1}{3}\sqrt{3}$,
$\sec -420^\circ = 2$,
$\csc -420^\circ = -\frac{2}{3}\sqrt{3}$.

The functions of 3270° are equal to the functions of 30°.

(6.) $\sin 233^\circ = -\cos 37^\circ$,
$\cos 233^\circ = -\sin 37^\circ$,
$\tan 233^\circ = \cot 37^\circ$,
$\cot 233^\circ = \tan 37^\circ$,
$\sec 233^\circ = -\csc 37^\circ$,
$\csc 233^\circ = -\sec 37^\circ$.
$\sin -197^\circ = \sin 17^\circ$,
$\cos -197^\circ = -\cos 17^\circ$,
$\tan -197^\circ = -\tan 17^\circ$,
$\cot -197^\circ = -\cot 17^\circ$,
$\sec -197^\circ = -\sec 17^\circ$,
$\csc -197^\circ = \csc 17^\circ$.
$\sin 894^\circ = \sin 6^\circ$,
$\cos 894^\circ = -\cos 6^\circ$,
$\tan 894^\circ = -\tan 6^\circ$,
$\cot 894^\circ = -\cot 6^\circ$,
$\sec 894^\circ = -\sec 6^\circ$,
$\csc 894^\circ = \csc 6^\circ$.

(7.) $\sin 267^\circ = -\sin 87^\circ$,
$\tan -254^\circ = -\tan 74^\circ$,
$\cos 950^\circ = -\cos 50^\circ$.

(8.) -0.28.

(9.) $2 \sin^2 x$.

(10.) $1 + \sec^2 x$.

(11.) $\sin(x - 90°) = -\cos x$,
$\cos(x - 90°) = \sin x$,
$\tan(x - 90°) = -\cot x$,
$\cot(x - 90°) = -\tan x$,
$\sec(x - 90°) = \csc x$,
$\csc(x - 90°) = -\sec x$.

§ 28 (page 24).

(1.) $a = 62.324$,
$A = 32° \, 52' \, 40''$.

(2.) $b = 21.874$,
$A = 39° \, 45' \, 28''$,
$B = 50° \, 14' \, 32''$.

(3.) $a = 300.95$,
$b = 683.96$,
$B = 66° \, 15'$.

(4.) $b = 26.608$,
$c = 45.763$,
$B = 35° \, 33'$.
area $= 495.34$.

(5.) $b = 3.9973$,
$c = 4.1537$,
$A = 15° \, 46' \, 33''$,
area $= 2.257$.

(6.) $b = 0.01729$.

(7.) $a = 298.5$.

(8.) $A = 39° \, 42' \, 24''$.

(9.) $c = 2346.7$.

(10.) $B = 28° \, 57' \, 8''$.

(11.) 444.16 ft.

(12.) 186.32 ft.

(13.) $34° \, 33' \, 44''$.

(14.) 303.99 ft.

(15.) 238.33 ft.

(16.) 15 miles (about).

(17.) 79,079 ft.

(18.) 165.68 ft.

(19.) $53° \, 33'$.

(20.) 115.136 ft.

(21.) 76.355 ft.

(22.) $B = 80° \, 32''$,
$A = C = 49° \, 59' \, 44''$.

(23.) $B = 53° \, 16' \, 36''$,
$b = 12.0518$ in.,
area $= 72.392$ sq. in.

(24.) $b = 130.52$ in.,
area $= 24246$ sq. in.

(25.) 23.263 ft.

(26.) $17° \, 48''$.

(27.) 5.3546 in.

(28.) 1084950 sq. ft.

(29.) 17 ft., 885 sq. ft.

(30.) radius $= 24.882$ in.,
apothem $= 20.13$ in.,
area $= 1472$ sq. in.

(31.) 12.861.

(32.) 1782.3 sq. ft.

(33.) 38168 ft.

(34.) 20.21 ft.

(35.) 2518.2 ft.

§ 29 (page 28).

(1.) $A = 22° \, 58'$,
$b = 7.07$,
$c = 9.0046$.

(2.) $b = 79.435$,
$A = 45° \, 27' \, 14''$,
$C = 95° \, 24' \, 46''$.

(3.) $AB = 7.6745$,
$AB' = 2.6435$,
$B = 46° \, 43' \, 50''$,
$B' = 133° \, 16' \, 10''$,
$ACB = 105° \, 53' \, 10''$,
$ACB' = 19° \, 20' \, 50''$.

(4.) $A = 37° \, 53'$,
$B = 43° \, 52' \, 25''$,

$C = 98^\circ\ 14'\ 35''$.

(5.) 902.94.

(6.) 1253.2 ft.

(7.) 357.224 ft.

(8.) $A = 44^\circ\ 2'\ 9''$,
$B = 51^\circ\ 28'\ 11''$,
$C = 84^\circ\ 29'\ 40''$,
area $= 126100$ sq. ft.

(9.) 407.89 ft.

(10.) $B = 121^\circ\ 7'\ 16''$,
$C = 92^\circ\ 20'\ 38''$,
$D = 71^\circ\ 11'\ 6''$.

(11.) $BC = 6.6885$,
$DC = 1.9915$.

§ 34 (page 34).

(2.) $\sin(45^\circ + x) = \frac{1}{2}\sqrt{2}\,(\cos x + \sin x)$,
$\cos(45^\circ + x) = \frac{1}{2}\sqrt{2}\,(\cos x - \sin x)$,
$\sin(30^\circ - x) = \frac{1}{2}(\cos x - \sqrt{3}\sin x)$,
$\cos(30^\circ - x) = \frac{1}{2}(\sqrt{3}\cos x + \sin x)$,
$\sin(60^\circ + x) = \frac{1}{2}(\sqrt{3}\cos x + \sin x)$,
$\cos(60^\circ + x) = \frac{1}{2}(\cos x - \sqrt{3}\sin x)$.

(3.) $\sin(x + y) = \frac{56}{65}$,
$\sin(x - y) = \frac{16}{65}$.

(4.) $\sin 75^\circ = \dfrac{\sqrt{6} + \sqrt{2}}{4}$,
$\cos 75^\circ = \dfrac{\sqrt{6} - \sqrt{2}}{4}$.

(5.) $\sin 15^\circ = \dfrac{\sqrt{6} - \sqrt{2}}{4}$,
$\cos 15^\circ = \dfrac{\sqrt{6} + \sqrt{2}}{4}$.

(6.) $\sin(x + y) = -\dfrac{\sqrt{15} + \sqrt{3}}{8}$,
$\cos(x - y) = \dfrac{3\sqrt{5} + 1}{8}$.

§ 39 (page 37).

(5.) $\sin(45^\circ - x) = \frac{1}{2}\sqrt{2}\,(\cos x - \sin x)$,
$\cos(45^\circ - x) = \frac{1}{2}\sqrt{2}\,(\cos x + \sin x)$,
$\sin(45^\circ + x) = \frac{1}{2}\sqrt{2}\,(\cos x + \sin x)$,
$\cos(45^\circ + x) = \frac{1}{2}\sqrt{2}\,(\cos x - \sin x)$:

(6.) $\tan 75^\circ = 2 + \sqrt{3}$,
$\tan 15^\circ = 2 - \sqrt{3}$.

(14.) $\sin \frac{1}{2}y = \sqrt{\dfrac{3 - \sqrt{5}}{6}}$,
$\cos \frac{1}{2}y = \sqrt{\dfrac{3 + \sqrt{5}}{6}}$,
$\tan \frac{1}{2}y = \dfrac{3 - \sqrt{5}}{2}$.

(15.) $\sin 2x = -\frac{24}{25}$,
$\cos 2x = -\frac{7}{25}$.

(16.) $\sin 22\frac{1}{2}^\circ = \frac{1}{2}\sqrt{2 - \sqrt{2}}$,
$\cos 22\frac{1}{2}^\circ = \frac{1}{2}\sqrt{2 + \sqrt{2}}$,
$\tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$,
$\cot 22\frac{1}{2}^\circ = \sqrt{2} + 1$,
$\sec 22\frac{1}{2}^\circ = \sqrt{4 - 2\sqrt{2}}$,
$\csc 22\frac{1}{2}^\circ = \sqrt{4 + 2\sqrt{2}}$.

(17.) $\dfrac{\sqrt{5} - 1}{2}$.

(18.) $\sin 15^\circ = \frac{1}{2}\sqrt{2 - \sqrt{3}}$,
$\cos 15^\circ = \frac{1}{2}\sqrt{2 + \sqrt{3}}$.

$\tan 15° = 2 - \sqrt{3}$,

$\cot 15° = 2 + \sqrt{3}$,

$\sec 15° = 2\sqrt{2-\sqrt{3}}$,

$\csc 15° = 2\sqrt{2+\sqrt{3}}$.

(20.) $\sin 5x =$
$5 \sin x - 20 \sin^3 x + 16 \sin^5 x$.

(21.) $\cos 5x =$
$5 \cos x - 20 \cos^3 x + 16 \cos^5 x$.

(23.) The values of $x < 360°$ are, $0°, 30°, 150°, 180°, 210°, 330°$.

(36.) $\tan x \tan y$.

§ 41 (page 40).

(1.) $\sin^{-1} \frac{1}{2}\sqrt{2} = 45°, 135°, 45° + 360°$, etc.,
$\cos^{-1} \frac{1}{2} = 60°, 300°$, etc.,
$\tan^{-1}(-1) = 135°, 315°$, etc.,
$\cos^{-1} 1 = 0°, 360°$, etc.,
$\sin^{-1}(-\frac{1}{2}) = 210°, 330°$, etc.

(2.) $\tan x = 3$.

(3.) $\cos x = \pm \frac{4}{5}$, $\tan x = \pm \frac{3}{4}$.

(4.) $\sin(\tan^{-1} \frac{1}{3}\sqrt{3}) = \pm \frac{1}{2}$.

(5.) $\sin(\cos^{-1} \frac{4}{5}) = \pm \frac{3}{5}$.

(6.) $\cot(\tan^{-1} \frac{1}{17}) = 17$.

(7.) $a = \frac{1}{2}\sqrt{3}$.

(8.) $45°, 225°$.

(9.) $x = 45°, y = 180°$.

(10.) $\sin^{-1} a = 225°$.

§ 48 (page 46).

(1.) $C = 121° \, 33'$,
$b = 2133.5$,
$c = 2477.8$.

(2.) $C = 55° \, 41'$,
$b = 534.05$,
$c = 653.52$.

(3.) $C = 45° \, 34'$,
$a = 1548.1$,
$b = 1293.7$.

(4.) $A = 105° \, 59'$,
$a = 54.018$,
$c = 47.738$.

(5.) $B = 68° \, 58'$,
$b = 5274.9$,
$c = 3730$.

(6.) $B = 54° \, 58'$,
$a = 923.4$,
$c = 1187.7$.

§ 49 (page 47).

(1.) (1.) Two solutions.
(2.) One solution, a right triangle.
(3.) One solution.
(4.) Two solutions.

(2.) $B = 16° \, 57' \, 21''$,
$C = 15° \, 50' \, 39''$,
$c = 0.32122$.

(3.) $c = 2.5719$,
$B = 13° \, 15' \, 1''$,
$C = 142° \, 13' \, 59''$.

(4.) $c = 93.59$, $c' = 54.069$,
$B = 26° \, 52' \, 7''$, $B' = 133° \, 7' \, 53''$,
$C = 131° \, 46' \, 53''$, $C' = 25° \, 31' \, 7''$.

(5.) No solution.

(6.) $b = 1.0916$, $b' = 0.36276$,
$A = 39° \, 37' \, 16''$, $A' = 140° \, 22' \, 44''$,
$B = 117° \, 50' \, 44''$, $B' = 17° \, 5' \, 16''$.

§ 50 (page 48).

(1.) $a = 0.0971$,
$B = 90° \, 35' \, 36''$,
$C = 48° \, 9' \, \cancel{34''}$, 24
$S = 0.0053261$.

(2.) $c = 14.211$,
$A = 76° 20' 5''$,
$B = 44° 52' 55''$
$S = 80.962$.

(3.) $b = 85.892$,
$A = 67° 21' 42''$,
$C = 62° 48' 18''$,
$S = 3962.8$.

(4.) $a = 0.6767$,
$B = 15° 9' 21''$,
$C = 131° 19' 39''$,
$S = 0.08141$.

(5.) $c = 72.87$,
$A = 40° 50' 32''$.
$B = 11° 2' 28''$.
$S = 422.65$.

§ 51 (page 49).

(1.) $A = 55° 20' 42''$,
$B = 106° 35' 36''$,
$C = 18° 3' 42''$,
$S = 267.92$.

(2.) $A = 34° 24' 26''$,
$B = 73° 14' 56''$,
$C = 72° 20' 36''$,
$S = 3.6143$.

(3.) $A = 52° 20' 24''$,
$B = 107° 19' 14''$,
$C = 20° 20' 24''$,
$S = 1437.5$.

(4.) $A = 97° 48'$,
$B = 18° 21' 48''$,
$C = 63° 50' 12''$,
$S = 193.13$.

(5.) $A = 54° 20' 16''$,
$B = 70° 27' 46''$,
$C = 54° 72'$,
$S = 6090$.

(6.) $A = 35° 59' 30''$,
$B = 48° 44' 32''$,
$C = 95° 15' 56''$,
$S = 0.60709$.

§ 52 (page 50).

(1.) 1116.6 ft.
(2.) 3081.8 yards.
(3.) 638.34 ft.,
14653 sq. ft.
(4.) 4.1 and 8.1.
(5.) 13.27 miles.
(6.) 6667 ft. One solution.
(7.) 121.97.
(8.) $44° 2' 56''$.
(9.) 32.151 sq. miles.
(11.) $54° 29' 12''$.
(12.) $a = 12296$ ft.,
$c = 13055$ ft.
(13.) 294.77 ft.
(14.) 222.1 ft.
(16.) 4202.1 ft. 4211.8
(17.) 72.613 miles.
(18.) 50.977 ft.
(19.) 0.85872 miles.
(20.) 2.98 miles.
(21.) 1393.9 ft.
(22.) 8.2 miles.
(23.) 187.39 ft.
(24.) 0.6011.
(25.) 4.8112 miles.
(26.) $60° 51' 8''$.
(27.) 37.365 ft.
(28.) 3.2103 miles.
(29.) 10.532 miles.
(30.) 851.22 yards.
(31.) 9.5722 miles.
(32.) 6.1271 miles.
(33.) 280.47 ft.
(34.) 123.33 ft.

(35.) 4.8112 miles.
(36.) 2666.1 ft.

§ 53 (page 56).

(1.) $30^\circ = 0.5236$,
$45^\circ = 0.7854$,
$60^\circ = 1.0472$,
$120^\circ = 2.0944$,
$135^\circ = 2.3562$,
$720^\circ = 12.5664$,
$990^\circ = 17.2788$.

(2.) $\frac{\pi}{8} = 22^\circ\ 30'$,
$\frac{\pi}{10} = 18^\circ$,
$\frac{1}{2} = 28^\circ\ 38'\ 53''$,
$\frac{7}{4} = 100^\circ\ 16'\ 4''$.

(3.) 1.35, 0.54.

§ 74 (page 73).

(1.) $\sin 4x = 4\cos^3 x \sin x - 4\cos x \sin^3 x$,
$\cos 4x = \cos^4 x - 6\cos^2 x \sin^2 x + \sin^4 x$.

(2.) $\sin 6x = 6\cos^5 x \sin x - 20\cos^3 x \sin^3 x + 6\cos x \sin^5 x$,
$\cos 6x = \cos^6 x - 15\cos^4 x \sin^2 x + 15\cos^2 x \sin^4 x - \sin^6 x$.

(3.) $x_0 = 1$, $x_1 = \frac{1}{2} + i\frac{\sqrt{3}}{2}$,
$x_2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, $x_3 = -1$,
$x_4 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$,
$x_5 = \frac{1}{2} - i\frac{\sqrt{3}}{2}$.

(4.) $x_0 = 1$, $x_1 = 0.3090 + i\,0.9511$,
$x_2 = -0.8090 + i\,0.5878$,
$x_3 = -0.8090 - i\,0.5878$,
$x_4 = 0.3090 - i\,0.9511$.

§ 77 (page 78).

(23.) $x = 30^\circ$.
(24.) $y = 30^\circ$.
(25.) $x = 0^\circ$ or 45°.
(26.) $x = 60^\circ$.
(27.) $y = 45^\circ$.
(28.) $y = 45^\circ$.
(29.) $x = 45^\circ$.
(30.) $x = 30^\circ$.
(31.) $x = 60^\circ$.
(32.) $x = 30^\circ$.
(33.) No angle $< 90^\circ$.
(34.) $x = 30^\circ$.
(35.) $\sin 92^\circ = \cos 2^\circ$.
(36.) $\cos 127^\circ = -\sin 37^\circ$.
(37.) $\tan 320^\circ = -\tan 40^\circ$.
(38.) $\cot 350^\circ = -\cot 10^\circ$.
(39.) $\sin 265^\circ = -\cos 5^\circ$.
(40.) $\tan 171^\circ = -\tan 9^\circ$.

(41.) $\cos x = -\frac{1}{7}\sqrt{33}$,
$\tan x = -\frac{4}{33}\sqrt{33}$,
$\cot x = -\frac{1}{4}\sqrt{33}$,
$\sec x = -\frac{7}{33}\sqrt{33}$,
$\csc x = \frac{7}{4}$.

(42.) $\sin x = -\frac{1}{8}\sqrt{55}$,
$\tan x = \frac{1}{3}\sqrt{55}$,
$\cot x = \frac{3}{55}\sqrt{55}$,
$\sec x = -\frac{8}{3}$,
$\csc x = -\frac{8}{55}\sqrt{55}$.

(43.) $\sin x = -\frac{3}{13}\sqrt{13}$,
$\cos x = -\frac{2}{13}\sqrt{13}$,
$\cot x = \frac{2}{3}$, $\sec x = -\frac{1}{2}\sqrt{13}$.

$\csc x = -\frac{1}{8}\sqrt{13}$.

(44.) $\sin x = -\frac{5}{74}\sqrt{74}$,

$\cos x = \frac{7}{74}\sqrt{74}$,

$\tan x = -\frac{5}{7}$, $\sec x = \frac{1}{7}\sqrt{74}$,

$\csc x = -\frac{1}{5}\sqrt{74}$.

(45.) Quadrant II or IV.

(46.) Quadrant I or II.

(47.) Quadrant III or IV.

(48.) Quadrant I or II.

(49.) $x = 0°, 120°, 180°, 240°$.

(50.) $x = 30°, 135°, 150°, 315°$.

(51.) $x = 0°, 90°, 120°, 180°, 240°, 270°$.

(57.) 0.

(58.) a.

(59.) $2(a-b)$.

(60.) $\frac{1}{2}(a^2 - b^2)$.

§ 78 (page 80).

(1.) 306.32 ft.

(2.) 831.06 ft.

(3.) 53° 28′ 14″.

(4.) 49.39 ft.

(5.) 0.43498 mile.

(6.) 209.53 ft.

(7.) 7.3188 ft.

(8.) 37° 36′ 30″.

(9.) 109.28 ft.

(10.) 502.46 ft.

(11.) 6799.8 ft.

(12.) 219.05 ft.

(13.) 491.76 ft.

(14.) 50° 32′ 44″.

(15.) 49° 44′ 38″.

(16.) 34.063 ft.

(17.) 32.326 ft., 29° 6′ 35″.

(18.) 5.6569 miles an hour.

(19.) 56.295 ft.

(20.) 103.09 ft.

(21.) 71° 33′ 54″.

(22.) 858,160 miles.

(23.) 238,850 miles.

(24.) 2163.4 miles.

(25.) 90,824,000 miles.

(26.) 432.08 ft.

(27.) 60.191 ft.

(28.) 0.32149 mile.

(29.) 193.77 ft.

§ 79 (page 83).

(1.) 3.416 ft.

(2.) 3.7865 ft.

(3.) 20.45 ft.

(4.) 36.024 ft.

(5.) 8.6058 sq. ft.

(6.) 181.23 in.

(7.) 2.9943 ft.

(8.) 5.1311 in.

(9.) 25.92 ft.

(10.) 92° 1′ 24″.

(11.) 1.2491.

(12.) 33° 12′ 4″.

(13.) 11248 ft.

(14.) 0.60965 miles.

(15.) 1.3764.

(16.) 1.9755.

(17.) 19.882.

(18.) 0.9397.

(19.) 6.4984.

(20.) 3.4641.

(21.) 6.1981.

(22.) 6.9978.

(23.) 15.25.

§ 80 (page 84).

(78.) $x = 90°, 120°, 240°, 270°$.

(79.) $x = 0°, 20°, 45°, 90°, 100°, 135°, 140°, 180°, 220°, 225°, 260°, 270°, 315°, 340°$.

(80.) $x = 0°, 30°, 90°, 150°, 180°, 270°$.

(81.) $x = 0°, 45°, 120°, 240°, 225°, 270°$.

(82.) $x = 0°, 90°, 180°, 270°$.

(83.) $x = 0°, 90°, 210°, 330°$.

(84.) $x = 240°, 300°$.

(85.) $x = 210°, 330°$.

(86.) $x = 0°, 90°$.

(87.) $x = 0°, 180°$.

(88.) $x = 0°, 180°$.

(89.) $x = 0°, 90°, 120°, 180°, 240°, 270°$.

(90.) $x = 45°, 135°, 225°, 315°$.

(91.) $x = 30°, 150°, 210°, 330°$.

§ 81 (page 88).

(1.) 2145.1 ft.
(2.) 12.458 miles.
(3.) 1.1033 miles.
(4.) 1508.4 ft.
(5.) 1719.3 yards.
(6.) 1.2564 miles.
(7.) 1346.3 ft.
(8.) 387.1 yards.
(9.) 5.1083 miles.
(10.) 3791.8 ft.
(11.) 4.4152 ft.
(12.) 28° 57′ 20″.
(13.) 115.27.
(14.) 44.358 ft.
(15.) 92.258 ft.
(16.) 101° 32′ 16″.
(17.) 0.83732 mile.
(18.) 539.1 ft.
(19.) 1.239.
(20.) 152.31 and 238.3.
(21.) 68.673 ft.
(22.) 32.071 ft.
(23.) 137.78 ft.
(24.) 55.74 ft.
(25.) 247.52 ft.
(26.) 556.34 ft.
(27.) 465.72 ft.
(28.) 109.22 ft.
(29.) 2639.4 ft.
(30.) 396.54 ft.
(31.) 287.75 ft.
(32.) 2280.6 ft.
(33.) 64.62 ft.
(34.) 127.98 ft.
(35.) 45.183 ft.
(36.) 4365.2 ft.
(37.) 140.17 ft.
(38.) 610.45 ft.
(39.) 156.66 ft.
(40.) 41° 48′ 39″ and 125° 25′ 57″.
(41.) 51,288,000.
(42.) 366680.
(43.) 11586.
(44.) 947460.
(45.) 0.89782.
(46.) 9929.3.
(47.) 751.62 sq. ft.
(48.) 3145.9.
(49.) 855.1.
(50.) 876.34.

§ 88 (page 98).

(1.) $c = 54° 59' 47''$,
$B = 45° 41' 28''$,
$C = 65° 45' 58''$,

(2.) $C = 71° 36' 47''$,
$b = 95° 22'$,
$c = 71° 32' 14''$,

(3.) $C = 64° 14' 30''$,
$C' = 115° 45' 30''$,
$a = 48° 22' 55''$,
$a' = 131° 37' 5''$,
$c = 42° 19' 17''$.

$c = 137° 40' 43''$.

(4.) $C = 65° 49' 54''$,
$a = 63° 10' 6''$,
$b = 38° 59' 12''$.

(5.) $a = 75° 13' 1''$,
$B = 58° 25' 46''$,
$C = 67° 27' 1''$.

(6.) $a = 76° 30' 37''$,
$b = 65° 28' 58,''$
$c = 55° 47' 44''$.

(7.) $\cancel{B} = 54° 44' 23''$, = b'
B' $\cancel{b} = 64° 36' 39''$,
$c = 47° 57' 45''$.

(8.) $B = 96° 13' 23''$,
$a = 73° 17' 29''$,
$c = 70° 8' 38''$.

(9.) $B = 66° 58'$,
$a = 11° 35' 49''$,
$c = 4° 35' 26''$.

(10.) $a = 61° 4' 55''$,
$b = 40° 30' 22''$,
$c = 50° 30' 32''$.

§ 99 (page 107).

(1.) $c = 155° 35' 22''$, 155-33-36
$B = 10° 19' 34''$, 10-19-10
$C = 171° 48' 22''$. 171-49-00

(2.) $a = 131° 36' 36''$,
$b = 116° 36' 38''$,
$c = 29° 11' 42''$.

(3.) $a = 107° 7' 45''$,
$B = 48° 57' 29''$,
$C = 62° 31' 40''$.

(4.) $B = 62° 54' 43''$,
$a = 114° 30' 26''$,
$c = 56° 39' 10''$.

(5.) $A = 130° 35' 56''$,
$B = 30° 25' 34''$,
$C = 31° 26' 32''$.

(6.) $a = 98° 21' 22''$,
$b = 109° 50' 8''$,
$c = 115° 13' 4''$.

(7.) $B = 32° 26' 9''$,
$a = 84° 14' 32''$,
$c = 51° 6' 12''$.

(8.) $a = 80° 5' 8''$,
$b = 70° 10' 36''$,
$c = 145° 5' 2''$.

(9.) $A = 70° 39' 4''$,
$B = 48° 36' 2''$,
$C = 119° 15' 2''$.

(10.) $a = 40° 0' 12''$,
$B = 42° 15' 11''$,
$C = 121° 36' 19''$.

§ 100 (page 109).

(1.) 80.895 sq. in.
(2.) 26.869 sq. in.
(3.) 158.41 sq. in.
(4.) 39990 sq. miles.

§ 101 (page 112).

(1.) $SC = 48° 2' 43''$,
$AC = 52° 53' 9''$.
(2.) 7 : 24 A.M.
(3.) 4 P.M.

§ 102 (page 114).

(1.) $3029\frac{1}{12}$ miles.
(2.) 2229.8 miles.
(3.) 2748.5 miles.
(4.) 7516.3 miles.
(5.) 5108.9 miles.

THE END

LOGARITHMIC

AND

TRIGONOMETRIC TABLES

FIVE-PLACE AND FOUR-PLACE

PHILLIPS-LOOMIS MATHEMATICAL SERIES

LOGARITHMIC

AND

TRIGONOMETRIC TABLES

FIVE-PLACE AND FOUR-PLACE

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CONTENTS

INTRODUCTION TO THE TABLES

COMMON LOGARITHMS.

1. The **common logarithm** of a number is the index of the power to which 10 must be raised to give the number.

Thus,

$$\begin{aligned} \log 100 &= 2, &\quad \text{because } 100 &= 10^2 \\ \log 1 &= 0, &\quad \text{“} \quad 1 &= 10^0 \\ \log .1 &= -1, &\quad \text{“} \quad .1 &= 10^{-1} \\ \log 3 &= .47712, &\quad \text{“} \quad 3 &= 10^{.47712} \end{aligned}$$

In general, $\log m = x$ if $m = 10^x$.

2. *To multiply two numbers, add their logarithms. The result is the logarithm of the product.*

Proof.— If $m = 10^x$ so that $\log m = x$,

and $n = 10^y$ " " $\log n = y$,

then $mn = 10^{x+y}$ " " $\log mn = x+y$.

Hence $\log mn = \log m + \log n$.

3. *To divide one number by another, subtract the logarithm of the divisor from the logarithm of the dividend. The result is the logarithm of the quotient.*

Proof.—
$$\frac{m}{n} = \frac{10^x}{10^y} = 10^{x-y};$$

Hence
$$\log \frac{m}{n} = x - y = \log m - \log n.$$

4. *To raise a number to a power, multiply the logarithm of the number by the index of the power. The result is the logarithm of the power.*

Proof.— $m^a = (10^x)^a = 10^{ax}$;

Hence $\log m^a = ax = a \log m$.

5. *To extract a root of a number, divide the logarithm of the number by the index of the root. The result is the logarithm of the root.*

Proof.— $\sqrt[b]{m} = \sqrt[b]{10^x} = 10^{\frac{x}{b}}$.

Hence $\log \sqrt[b]{m} = \frac{x}{b} = \frac{\log m}{b}$.

6. Restatement of laws:

$$\log mn = \log m + \log n;$$

$$\log \frac{m}{n} = \log m - \log n;$$

$$\log m^a = a \log m;$$

$$\log \sqrt[b]{m} = \frac{\log m}{b}.$$

7. Most numbers are not integral powers of 10; hence most logarithms are of decimal form.

Thus, $\log 2.2 = .34242$, $\log 4 = .60206$.

8. If a logarithm is negative, it is expressed for convenience as a *negative* integer plus a *positive* decimal.

The logarithm of a number less than 1 is negative.

The negative integer is usually expressed in the form $9-10$, $8-10$, etc.

Thus, $\log .21544 = -1 + .33333$, written $9.33333 - 10$;
$\log .021544 = -2 + .33333$, " $8.33333 - 10$;
$\log .0021544 = -3 + .33333$, " $7.33333 - 10$.

Remark.—In some books the negative integer is written $\bar{1}$, $\bar{2}$, etc., instead of $9-10$, $8-10$, etc.

The integral part of a logarithm is the **characteristic**; the decimal part is the **mantissa**.

Thus, $\log 215.44 = 2.33333$; the characteristic is $+2$; the mantissa

is $+.33333$: $\log .021544 = 8.33333 - 10$; the characteristic is $8 - 10 = -2$; the mantissa is $+.33333$.

9. It is evident that the larger a number the larger its logarithm. Hence the logarithm of any number

between	1	and	10	is	$0 +$ a mantissa,
"	10	"	100	"	$1 +$ " "
"	.1	"	1	"	$-1 +$ " "
"	.01	"	.1	"	$-2 +$ " " etc.

We have, then, the following rule for obtaining the characteristic:

10. *Count the number of places the first left-hand digit of the number is removed from the unit's place.*

If this digit is to the left of the unit's place, the result is the required characteristic.

If this digit is to the right of the unit's place, the result taken with a minus sign is the required characteristic.

If this digit is in the unit's place, the characteristic is zero.

Thus the characteristic of the logarithm of	21550	is	4
" " " " " " "	21.55	"	1
" " " " " " "	2.155	"	0
" " " " " " "	.2155	"	-1
" " " " " " "	.02155	"	-2

11. The logarithms of numbers which differ only in the position of the decimal point have the same mantissa.

For to change the position of the decimal point is to multiply or divide by an integral power of 10; that is, an integer is added to or subtracted from the logarithm, and consequently only the characteristic is changed.

Thus,

$$\log 2154.4 = 3.33333$$
$$\log 2.1544 = 0.33333$$
$$\log .21544 = 9.33333 - 10$$
$$\log .021544 = 8.33333 - 10$$

Therefore, in finding the mantissa of the logarithm of a number the decimal point may be disregarded. The mantissa is found from the tables of logarithms.

USE OF THE TABLE OF LOGARITHMS OF NUMBERS.
(TABLE I.)

12. *To find the logarithm of a number.*

Look in the column at the head of which is "N" for the first three figures of the number, and in the line with "N," for the fourth figure. In the line opposite the first three figures and in the column under the fourth is the desired mantissa.

Only the last three figures of the mantissa are found thus; the first two must be taken from the first column; they are found either in the same line or in the first line above which gives the whole mantissa, except when a * occurs. If a * precedes the last three figures of the mantissa the first two are found in the following line:

The characteristic is obtained by § 10.

Example.—To find the logarithm of 105400.

The characteristic $= 5$. § 10

The mantissa $= .02284$ (opposite 105 and under 4 in the tables);

Hence $\log 105400 = 5.02284$.

13. If there are five or more figures in a number the figures beyond the fourth are treated as a decimal. The corresponding mantissa is between two successive mantissas of the tables.

Example.—To find the logarithm of 10543.

The characteristic $= 4$. § 10

The mantissa is not in the tables, but is between the mantissa of

$$1055 = .02325$$

and the mantissa of $1054 = \underline{.02284}$

Their difference $= 41$

Hence an increase of one in the fourth figure of the number produces an increase of 41 in the mantissa. Then an increase of .3 must produce an increase of $41 \times .3$ in the mantissa.

$$41 \times .3 = 12.3 = 12 \text{ nearly.}$$

Hence the mantissa of $10543 = .02284 + 12 = .02296$.

Therefore $\log 10543 = 4.02296$.

An easy method of multiplying 41 by .3 is to use the table of proportional parts at the bottom of the page in the tables.

Under 41 and opposite 3 is $12.3 (= 41 \times .3)$.

14. Figures beyond the fifth are usually omitted in the use of a five-place table, as their retention does not add much to the accuracy of the result. For the fifth figure, however, we choose the one which gives most nearly the true value of the number.

Thus, if the number is 157.032, we use 157.03;
" " " " 157.036, " " 157.04;
" " " " 157.035, " " 157.04.

15. *To find a number from its logarithm.*

The process is the reverse of finding the logarithm from the number; it is illustrated by the following examples:

Find the number of which $9.12872 - 10$ is the logarithm.

Since the characteristic $= -1$, the decimal point will be before the first figure of the number.

.12872 is opposite 134 and under 5 in the tables.

Hence $.12872 =$ the mantissa of 1345,

and $9.12872 - 10 = \log .1345$.

Find the number of which $9.12895 - 10$ is the logarithm.

The mantissa .12895 is not in the tables, but is

between $.12905 =$ mantissa of 1346

and $.12872 =$ " " 1345.

$.00033 =$ the difference.

$.12895 =$ mantissa given,

$.12872 =$ mantissa of 1345, the smaller number,

$23 =$ the difference.

Change $\frac{23}{33}$ into a decimal. The first figure of this decimal will be the figure in the fifth place of the number.

$\frac{23}{33} = .7$ nearly.

Hence $9.12895 - 10 = \log .13457$.

An easy method of changing $\frac{23}{33}$ into a decimal is to use the table of proportional parts.

Under 33 is found 23.1 (= 23 nearly), which is opposite 7.

Hence $\frac{23}{33} = .7$ nearly.

The process we have employed in finding the logarithm of a number of more than four figures, or the number corresponding to a mantissa not given in the table, is called interpolation.

EXAMPLES FOR THE USE OF LOGARITHMS.

16. Multiply 5789.2 by .018315.

$$\begin{aligned} \log 5789.2 &= 3.76262 \\ \log .018315 &= \underline{8.26281} - 10 \\ & 2.02543 = \log 106.03 \end{aligned}$$

Multiply 9.8764 by .10013.

$$\begin{aligned} \log 9.8764 &= 0.99460 \\ \log .10013 &= \underline{9.00056} - 10 \\ & 9.99516 - 10 = \log .98892 \end{aligned}$$

Find the value of $3.1416 \times 7638.6 \times .017829$.

$$\begin{aligned} \log 3.1416 &= 0.49715 \\ \log 7638.6 &= 3.88302 \\ \log .017829 &= \underline{8.25113} - 10 \\ & 2.63130 = \log 427.86 \end{aligned}$$

Divide 81.321 by 3.1416.

$$\begin{aligned} \log 81.312 &= 1.91021 \\ \log 3.1416 &= \underline{0.49715} \\ & 1.41306 = \log 25.886 \end{aligned}$$

Find the value of $(2.1345)^5$.

$$\begin{aligned} \log 2.1345 &= 0.32930 \\ & \underline{5} \\ & 1.64650 = \log 44.310 \end{aligned}$$

Find the value of $\sqrt[3]{.01021}$.

$$\begin{aligned} \log .01021 &= 8.00903 - 10 \\ &= 28.00903 - 30 \end{aligned}$$

$$\frac{28.00903 - 30}{3} = 9.33634 - 10 = \log .21694$$

17. The logarithm of $\frac{1}{m}$ is called the **cologarithm** of m, and is obtained by subtracting $\log m$ from zero.

Thus, if $\log m = 9.76423 - 10$, $\text{colog}\, m = 0.23577$.

It is frequently shorter to add $\text{colog}\, m$ than to subtract $\log m$ when we wish to divide by a number m.

The following example illustrates this:

Find the value of $\frac{57.98 \times 42.24}{644.32}$.

$$\begin{aligned} \log 57.98 &= 1.76328 \\ \log 42.24 &= 1.62572 \\ \text{colog}\, 644.32 &= \underline{7.19090} - 10 \\ & 0.57990 = \log 3.801 \end{aligned}$$

USE OF THE TABLE OF LOGARITHMS OF TRIGONOMETRIC FUNCTIONS. (TABLE II.)

18. For an angle less than 45°, the degrees are at the *head* of the page, the minutes in the column at the *left*, and "L. Sin.," "L. Tang.," etc., at the head of the corresponding columns. For angles between 45° and 90°, the degrees are at the *foot* of the page, the minutes in the column at the *right*, and "L. Sin.," "L. Tang.," etc., at the foot of the corresponding columns.

The characteristic is printed 10 too large where it would otherwise be negative. Hence, in using this table, −10 is to be supplied, except for the cotangent of angles less than 45° and the tangent of angles from 45° to 90°.

EXAMPLES.

$\log \sin 15° 25' = 9.42461 - 10.$

$\log \tan 28° 17' = 9.73084 - 10.$

$\log \cos 62° 14' = 9.66827 - 10.$

$\log \cot 25° 34' = 0.32020.$

19. If the given angle contains seconds, we may reduce the seconds to a decimal of a minute and proceed as in finding the logarithms of numbers. It must be remembered, however, that log cos and log cot decrease as the angle increases.

In practice we remember that $6''$ is one-tenth of a minute, and divide the number of seconds by $6''$, then use the table of proportional parts at the bottom of the page.

EXAMPLES.

Find log sin $28^\circ\ 14'\ 36''$ ($=$log sin $28^\circ\ 14.6'$).

$$\log \sin 28^\circ\ 15' - \log \sin 28^\circ\ 14' = 23 \text{ (found in column "d.")}$$

$$\begin{aligned} \log \sin 28^\circ\ 14' &= 9.67492 - 10 \\ 23 \times .6 = 13.8 &= \underline{\quad 14 \text{ nearly}} \\ \log \sin 28^\circ\ 14'\ 36'' &= 9.67506 - 10 \end{aligned}$$

Find log cos $39^\circ\ 17'\ 22''$ ($=$log cos $39^\circ\ 17.3\frac{2}{3}'$).

$$\begin{aligned} \log \cos 39^\circ\ 17' &= 9.88875 - 10 \\ 10 \times .3\tfrac{2}{3} &= \underline{\qquad 4 \qquad} \\ \log \cos 39^\circ\ 17'\ 22'' &= 9.88871 - 10 \end{aligned}$$

Find log tan $51^\circ\ 27'\ 44''$ ($=$log tan $51^\circ\ 27.7\frac{1}{3}'$).

$$\begin{aligned} \log \tan 51^\circ\ 27' &= .09862 \\ 26 \times .7\tfrac{1}{3} &= \underline{\quad 19} \\ \log \tan 51^\circ\ 27'\ 44'' &= .09881 \end{aligned}$$

Find log cot $67^\circ\ 18'\ 46''$.

$$\begin{aligned} \log \cot 67^\circ\ 18' &= 9.62150 - 10 \\ 36 \times .7\tfrac{2}{3} &= \underline{\qquad 28 \qquad} \end{aligned}$$

Hence $\log \cot 67^\circ\ 18'\ 46'' = 9.62122 - 10$

20. The process of finding an angle, if its logarithmic sine or tangent, etc., is given, is the reverse of the preceding.

EXAMPLES.

Given $\log \sin x = 9.67433 - 10$; find x.

$$\log \sin 28^\circ\ 11' = \underline{9.67421 - 10}$$

$$\log \sin x - \log \sin 28^\circ\ 11' = \quad 12$$

and $\log \sin 28^\circ\ 12' - \log \sin 28^\circ\ 11' = \quad 24$

Hence $x = 28^\circ\ 11'\ 30''$ ($\frac{12}{24}$ of $1'$ being $30''$).

Find the angle whose $\log \cos = 9.88231 - 10$.

$$\log \cos 40^\circ\ 18' = 9.88234 - 10.$$

$$60'' \times \tfrac{3}{11} = 16''.$$

Hence $\log \cos 40^\circ\ 18'\ 16'' = 9.88231 - 10$.

Find the angle whose $\log \tan = 0.17844$.

$$\log \tan 56^\circ\ 27' = 0.17839.$$

$$60'' \times \tfrac{5}{28} = 11''.$$

Hence $\log \tan 56^\circ\ 27'\ 11'' = 0.17844$.

Find the angle whose $\log \cot = 9.87432 - 10$.

$$\log \cot 53^\circ\ 10' = 9.87448 - 10.$$

$$60'' \times \tfrac{16}{26} = 37''.$$

Hence $\log \cot 53^\circ\ 10'\ 37'' = 9.87432 - 10$.

EXPLANATION OF THE TABLES.

21. A dash above the terminal 5 of a mantissa, as $\bar{5}$, denotes that the true value is less than 5.

Thus, $\log 389 = 2.5899496$ to seven places, but to five places $\log 389 = 2.5899\bar{5}$.

Tables I and II have already been explained.

TABLE III.

22. The logarithmic sine and tangent cannot be obtained very accurately from Table II if the angle contains seconds and is less than 2°.

Table III is to be used when greater accuracy in the sine or tangent of a small angle is desired than can be obtained

by the use of Table II. It is to be noted that the first page of Table III gives the sine and tangent to every second for angles less than $8'$.

TABLE IV.

23. Naperian or "natural" logarithms are logarithms to the base e $(=2.71828+)$. The whole logarithm is given, since the integral part cannot be supplied by inspection, as with common logarithms.

TABLES V AND VI.

24. Four-place logarithms and logarithmic functions are used instead of five-place if the results are sufficiently accurate for the purpose in view.

In Table VI both the degrees and minutes are in the columns at the sides of the page, otherwise this table does not differ in form from Table II.

TABLE VII.

25. This table is identical with Table VI in form, but gives the trigonometric functions themselves, instead of their logarithms.

TABLES VIII, IX, X.

26. These tables require no explanation.

TABLE I

FIVE-PLACE LOGARITHMS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

PP	44	43	42		41	40	39		38	37	36
1	4.4	4.3	4.2	1	4.1	4.0	3.9	1	3.8	3.7	3.6
2	8.8	8.6	8.4	2	8.2	8.0	7.8	2	7.6	7.4	7.2
3	13.2	12.9	12.6	3	12.3	12.0	11.7	3	11.4	11.1	10.8
4	17.6	17.2	16.8	4	16.4	16.0	15.6	4	15.2	14.8	14.4
5	22.0	21.5	21.0	5	20.5	20.0	19.5	5	19.0	18.5	18.0
6	26.4	25.8	25.2	6	24.6	24.0	23.4	6	22.8	22.2	21.6
7	30.8	30.1	29.4	7	28.7	28.0	27.3	7	26.6	25.9	25.2
8	35.2	34.4	33.6	8	32.8	32.0	31.2	8	30.4	29.6	28.8
9	39.6	38.7	37.8	9	36.9	36.0	35.1	9	34.2	33.3	32.4

N	0	1	2	3	4	5	6	7	8	9
130	11 394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905̄	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	45̄0	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925̄	956
138	988	*019	*051	*082	*114	*145̄	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675̄	706	737	768	799	829	860	891
141	922	953	983	*014	*045̄	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625̄	655̄	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495̄	524	554	584	613	643	673	702
147	732	761	791	820	85̄0	879	909	938	967	997
148	17 026	056	085̄	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725̄	754	782	811	840	869
151	898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865̄	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285̄
156	312	340	368	396	424	451	479	507	535̄	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385̄
160	412	439	466	493	520	548	575̄	602	629	656
N	0	1	2	3	4	5	6	7	8	9

PP	35	34	33		32	31	30		29	28	27
1	3.5	3.4	3.3	1	3.2	3.1	3.0	1	2.9	2.8	2.7
2	7.0	6.8	6.6	2	6.4	6.2	6.0	2	5.8	5.6	5.4
3	10.5	10.2	9.9	3	9.6	9.3	9.0	3	8.7	8.4	8.1
4	14.0	13.6	13.2	4	12.8	12.4	12.0	4	11.6	11.2	10.8
5	17.5	17.0	16.5	5	16.0	15.5	15.0	5	14.5	14.0	13.5
6	21.0	20.4	19.8	6	19.2	18.6	18.0	6	17.4	16.8	16.2
7	24.5	23.8	23.1	7	22.4	21.7	21.0	7	20.3	19.6	18.9
8	28.0	27.2	26.4	8	25.6	24.8	24.0	8	23.2	22.4	21.6
9	31.5	30.6	29.7	9	28.8	27.9	27.0	9	26.1	25.2	24.3

N	0	1	2	3	4	5	6	7	8	9
160	20 412	439	466	493	520	548	575	602	629	656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	304	329	353	378	403	428	452	477	502	527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	717	741	764	788	811	834	858	881	905	928
186	951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
N	0	1	2	3	4	5	6	7	8	9

PP	27	26	25		24	23	22		21	20	19
1	2.7	2.6	2.5	1	2.4	2.3	2.2	1	2.1	2.0	1.9
2	5.4	5.2	5.0	2	4.8	4.6	4.4	2	4.2	4.0	3.8
3	8.1	7.8	7.5	3	7.2	6.9	6.6	3	6.3	6.0	5.7
4	10.8	10.4	10.0	4	9.6	9.2	8.8	4	8.4	8.0	7.6
5	13.5	13.0	12.5	5	12.0	11.5	11.0	5	10.5	10.0	9.5
6	16.2	15.6	15.0	6	14.4	13.8	13.2	6	12.6	12.0	11.4
7	18.9	18.2	17.5	7	16.8	16.1	15.4	7	14.7	14.0	13.3
8	21.6	20.8	20.0	8	19.2	18.4	17.6	8	16.8	16.0	15.2
9	24.3	23.4	22.5	9	21.6	20.7	19.8	9	18.9	18.0	17.1

N	0	1	2	3	4	5	6	7	8	9
190	27 875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40 140	157	175	192	209	226	243	261	278	295
253	312	329	346	364	381	398	415	432	449	466
254	483	500	518	535	552	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	875	892	909	926	943	960	976
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41 162	179	196	212	229	246	263	280	296	313
259	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
N	0	1	2	3	4	5	6	7	8	9

PP	19	18	17		16	15	14
1	1.9	1.8	1.7	1	1.6	1.5	1.4
2	3.8	3.6	3.4	2	3.2	3.0	2.8
3	5.7	5.4	5.1	3	4.8	4.5	4.2
4	7.6	7.2	6.8	4	6.4	6.0	5.6
5	9.5	9.0	8.5	5	8.0	7.5	7.0
6	11.4	10.8	10.2	6	9.6	9.0	8.4
7	13.3	12.6	11.9	7	11.2	10.5	9.8
8	15.2	14.4	13.6	8	12.8	12.0	11.2
9	17.1	16.2	15.3	9	14.4	13.5	12.6

N	0	1	2	3	4	5	6	7	8	9
260	41 497	514	531	547	564	581	597	614	631	647
261	664	681	697	714	731	747	764	780	797	814
262	830	847	863	880	896	913	929	946	963	979
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
264	42 160	177	193	210	226	243	259	275	292	308
265	325	341	357	374	390	406	423	439	455	472
266	488	504	521	537	553	570	586	602	619	635
267	651	667	684	700	716	732	749	765	781	797
268	813	830	846	862	878	894	911	927	943	959
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	933	949	965	981	996	*012	*028	*044	*059	*075
276	44 091	107	122	138	154	170	185	201	217	232
277	248	264	279	295	311	326	342	358	373	389
278	404	420	436	451	467	483	498	514	529	545
279	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
281	871	886	902	917	932	948	963	979	994	*010
282	45 025	040	056	071	086	102	117	133	148	163
283	179	194	209	225	240	255	271	286	301	317
284	332	347	362	378	393	408	423	439	454	469
285	484	500	515	530	545	561	576	591	606	621
286	637	652	667	682	697	712	728	743	758	773
287	788	803	818	834	849	864	879	894	909	924
288	939	954	969	984	*000	*015	*030	*045	*060	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
291	389	404	419	434	449	464	479	494	509	523
292	538	553	568	583	598	613	627	642	657	672
293	687	702	716	731	746	761	776	790	805	820
294	835	850	864	879	894	909	923	938	953	967
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47 129	144	159	173	188	202	217	232	246	261
297	276	290	305	319	334	349	363	378	392	407
298	422	436	451	465	480	494	509	524	538	553
299	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
301	857	871	885	900	914	929	943	958	972	986
302	48 001	015	029	044	058	073	087	101	116	130
303	144	159	173	187	202	216	230	244	259	273
304	287	302	316	330	344	359	373	387	401	416
305	430	444	458	473	487	501	515	530	544	558
306	572	586	601	615	629	643	657	671	686	700
307	714	728	742	756	770	785	799	813	827	841
308	855	869	883	897	911	926	940	954	968	982
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
311	276	290	304	318	332	346	360	374	388	402
312	415	429	443	457	471	485	499	513	527	541
313	554	568	582	596	610	624	638	651	665	679
314	693	707	721	734	748	762	776	790	803	817
315	831	845	859	872	886	900	914	927	941	955
316	969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50 106	120	133	147	161	174	188	202	215	229
318	243	256	270	284	297	311	325	338	352	365
319	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
321	651	664	678	691	705	718	732	745	759	772
322	786	799	813	826	840	853	866	880	893	907
323	920	934	947	961	974	987	*001	*014	*028	*041
324	51 055	068	081	095	108	121	135	148	162	175
325	188	202	215	228	242	255	268	282	295	308
326	322	335	348	362	375	388	402	415	428	441
327	455	468	481	495	508	521	534	548	561	574
328	587	601	614	627	640	654	667	680	693	706
329	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
N	0	1	2	3	4	5	6	7	8	9

PP	15	14	13		12	11
1	1.5	1.4	1.3	1	1.2	1.1
2	3.0	2.8	2.6	2	2.4	2.2
3	4.5	4.2	3.9	3	3.6	3.3
4	6.0	5.6	5.2	4	4.8	4.4
5	7.5	7.0	6.5	5	6.0	5.5
6	9.0	8.4	7.8	6	7.2	6.6
7	10.5	9.8	9.1	7	8.4	7.7
8	12.0	11.2	10.4	8	9.6	8.8
9	13.5	12.6	11.7	9	10.8	9.9

N	0	1	2	3	4	5	6	7	8	9
330	51 851	865̄	878	891	904	917	930	943	957	970
331	983	996	*009	*022	*035	*048	*061	*075̄	*088	*101
332	52 114	127	140	153	166	179	192	205	218	231
333	244	257	270	284	297	310	323	336	349	362
334	375̄	388	401	414	427	440	453	466	479	492
335	504	517	530	543	556	569	582	595	608	621
336	634	647	660	673	686	699	711	724	737	750
337	763	776	789	802	815̄	827	840	853	866	879
338	892	905̄	917	930	943	956	969	982	994	*007
339	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	25̄0	263
341	275	288	301	314	326	339	352	364	377	390
342	403	415	428	441	453	466	479	491	504	517
343	529	542	555̄	567	580	593	605	618	631	643
344	656	668	681	694	706	719	732	744	757	769
345	782	794	807	820	832	845̄	857	870	882	895
346	908	920	933	945	958	970	983	995	*008	*020
347	54 033	045	058	070	083	095	108	120	133	145
348	158	170	183	195	208	220	233	245	258	270
349	283	295̄	307	320	332	345̄	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518
351	531	543	555	568	580	593	605̄	617	630	642
352	654	667	679	691	704	716	728	741	753	765
353	777	790	802	814	827	839	851	864	876	888
354	900	913	925̄	937	949	962	974	986	998	*011
355	55 023	035	047	060	072	084	096	108	121	133
356	145̄	157	169	182	194	206	218	230	242	255̄
357	267	279	291	303	315	328	340	352	364	376
358	388	400	413	425̄	437	449	461	473	485	497
359	509	522	534	546	558	570	582	594	606	618
360	630	642	654	666	678	691	703	715̄	727	739
361	751	763	775̄	787	799	811	823	835̄	847	859
362	871	883	895̄	907	919	931	943	955̄	967	979
363	991	*003	*015̄	*027	*038	*050	*062	*074	*086	*098
364	56 110	122	134	146	158	170	182	194	205	217
365	229	241	253	265̄	277	289	301	312	324	336
366	348	360	372	384	396	407	419	431	443	455̄
367	467	478	490	502	514	526	538	549	561	573
368	585̄	597	608	620	632	644	656	667	679	691
369	703	714	726	738	750	761	773	785̄	797	808
370	820	832	844	855	867	879	891	902	914	926
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9
370	56 820	832	844	855	867	879	891	902	914	926
371	937	949	961	972	984	996	*008	*019	*031	*043
372	57 054	066	078	089	101	113	124	136	148	159
373	171	183	194	206	217	229	241	252	264	276
374	287	299	310	322	334	345	357	368	380	392
375	403	41$\bar{5}$	426	438	449	461	473	484	496	507
376	519	530	542	553	56$\bar{5}$	576	588	600	611	623
377	634	646	657	669	680	692	703	71$\bar{5}$	726	738
378	749	761	772	784	795	807	818	830	841	852
379	864	875	887	898	910	921	933	944	955	967
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58 092	104	115	127	138	149	161	172	184	19$\bar{5}$
382	206	218	229	240	252	263	274	286	297	309
383	320	331	343	354	365	377	388	399	410	422
384	433	444	456	467	478	490	501	512	524	53$\bar{5}$
385	546	557	569	580	591	602	614	62$\bar{5}$	636	647
386	659	670	681	692	704	71$\bar{5}$	726	737	749	760
387	771	782	794	80$\bar{5}$	816	827	838	8$\bar{5}$0	861	872
388	883	894	906	917	928	939	950	961	973	984
389	99$\bar{5}$	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
391	218	229	240	251	262	273	284	295	306	318
392	329	340	351	362	373	384	395	406	417	428
393	439	450	461	472	483	494	506	517	528	539
394	5$\bar{5}$0	561	572	583	594	60$\bar{5}$	616	627	638	649
395	660	671	682	693	704	71$\bar{5}$	726	737	748	759
396	770	780	791	802	813	824	835	846	857	868
397	879	890	901	912	923	934	94$\bar{5}$	956	966	977
398	988	999	*010	*021	*032	*043	*054	*06$\bar{5}$	*076	*086
399	60 097	108	119	130	141	152	163	173	184	195
400	206	217	228	239	249	260	271	282	293	304
N	0	1	2	3	4	5	6	7	8	9

PP	12	11		10	9
1	1.2	1.1	1	1.0	0.9
2	2.4	2.2	2	2.0	1.8
3	3.6	3.3	3	3.0	2.7
4	4.8	4.4	4	4.0	3.6
5	6.0	5.5	5	5.0	4.5
6	7.2	6.6	6	6.0	5.4
7	8.4	7.7	7	7.0	6.3
8	9.6	8.8	8	8.0	7.2
9	10.8	9.9	9	9.0	8.1

N	0	1	2	3	4	5	6	7	8	9
400	60 206	217	228	239	249	260	271	282	293	304
401	314	325	336	347	358	369	379	390	401	412
402	423	433	444	455	466	477	487	498	509	520
403	531	541	552	563	574	584	595	606	617	627
404	638	649	660	670	681	692	703	713	724	735
405	746	756	767	778	788	799	810	821	831	842
406	853	863	874	885	895	906	917	927	938	949
407	959	970	981	991	*002	*013	*023	*034	*045	*055
408	61 066	077	087	098	109	119	130	140	151	162
409	172	183	194	204	215	225	236	247	257	268
410	278	289	300	310	321	331	342	352	363	374
411	384	395	405	416	426	437	448	458	469	479
412	490	500	511	521	532	542	553	563	574	584
413	595	606	616	627	637	648	658	669	679	690
414	700	711	721	731	742	752	763	773	784	794
415	805	815	826	836	847	857	868	878	888	899
416	909	920	930	941	951	962	972	982	993	*003
417	62 014	024	034	045	055	066	076	086	097	107
418	118	128	138	149	159	170	180	190	201	211
419	221	232	242	252	263	273	284	294	304	315
420	325	335	346	356	366	377	387	397	408	418
421	428	439	449	459	469	480	490	500	511	521
422	531	542	552	562	572	583	593	603	613	624
423	634	644	655	665	675	685	696	706	716	726
424	737	747	757	767	778	788	798	808	818	829
425	839	849	859	870	880	890	900	910	921	931
426	941	951	961	972	982	992	*002	*012	*022	*033
427	63 043	053	063	073	083	094	104	114	124	134
428	144	155	165	175	185	195	205	215	225	236
429	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
431	448	458	468	478	488	498	508	518	528	538
432	548	558	568	579	589	599	609	619	629	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	849	859	869	879	889	899	909	919	929	939
436	949	959	969	979	988	998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	108	118	128	137
438	147	157	167	177	187	197	207	217	227	237
439	246	256	266	276	286	296	306	316	326	335
440	345	355	365	375	385	395	404	414	424	434
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441	444	454	464	473	483	493	503	513	523	532
442	542	552	562	572	582	591	601	611	621	631
443	640	650	660	670	680	689	699	709	719	729
444	738	748	758	768	777	787	797	807	816	826
445	836	846	856	865	875	885̄	895̄	904	914	924
446	933	943	953	963	972	982	992	*002	*011	*021
447	65 031	040	050	060	070	079	089	099	108	118
448	128	137	147	157	167	176	186	196	205	215̄
449	225̄	234	244	254	263	273	283	292	302	312
450	321	331	341	350	360	369	379	389	398	408
451	418	427	437	447	456	466	475	485	495̄	504
452	514	523	533	543	552	562	571	581	591	600
453	610	619	629	639	648	658	667	677	686	696
454	706	715	725̄	734	744	753	763	772	782	792
455	801	811	820	830	839	849	858	868	877	887
456	896	906	916	925	935̄	944	954	963	973	982
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077
458	66 087	096	106	115̄	124	134	143	153	162	172
459	181	191	200	210	219	229	238	247	257	266
460	276	285	295̄	304	314	323	332	342	351	361
461	370	380	389	398	408	417	427	436	445	455̄
462	464	474	483	492	502	511	521	530	539	549
463	558	567	577	586	596	605̄	614	624	633	642
464	652	661	671	680	689	699	708	717	727	736
465	745	755̄	764	773	783	792	801	811	820	829
466	839	848	857	867	876	885	894	904	913	922
467	932	941	950	960	969	978	987	997	*006	*015
468	67 025̄	034	043	052	062	071	080	089	099	108
469	117	127	136	145	154	164	173	182	191	201
470	210	219	228	237	247	256	265	274	284	293
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471	302	311	321	330	339	348	357	367	376	385
472	394	403	413	422	431	440	449	459	468	477
473	486	495	504	514	523	532	541	550	560	569
474	578	587	596	605	614	624	633	642	651	660
475	669	679	688	697	706	715	724	733	742	752
476	761	770	779	788	797	806	815	825	834	843
477	852	861	870	879	888	897	906	916	925	934
478	943	952	961	970	979	988	997	*006	*015	*024
479	68 034	043	052	061	070	079	088	097	106	115
480	124	133	142	151	160	169	178	187	196	205
481	215	224	233	242	251	260	269	278	287	296
482	305	314	323	332	341	350	359	368	377	386
483	395	404	413	422	431	440	449	458	467	476
484	485	494	502	511	520	529	538	547	556	565
485	574	583	592	601	610	619	628	637	646	655
486	664	673	681	690	699	708	717	726	735	744
487	753	762	771	780	789	797	806	815	824	833
488	842	851	860	869	878	886	895	904	913	922
489	931	940	949	958	966	975	984	993	*002	*011
490	69 020	028	037	046	055	064	073	082	090	099
491	108	117	126	135	144	152	161	170	179	188
492	197	205	214	223	232	241	249	258	267	276
493	285	294	302	311	320	329	338	346	355	364
494	373	381	390	399	408	417	425	434	443	452
495	461	469	478	487	496	504	513	522	531	539
496	548	557	566	574	583	592	601	609	618	627
497	636	644	653	662	671	679	688	697	705	714
498	723	732	740	749	758	767	775	784	793	801
499	810	819	827	836	845	854	862	871	880	888
500	897	906	914	923	932	940	949	958	966	975
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062
502	70 070	079	088	096	105	114	122	131	140	148
503	157	165	174	183	191	200	209	217	226	234
504	243	252	260	269	278	286	295	303	312	321
505	329	338	346	355	364	372	381	389	398	406
506	415	424	432	441	449	458	467	475	484	492
507	501	509	518	526	535	544	552	561	569	578
508	586	595	603	612	621	629	638	646	655	663
509	672	680	689	697	706	714	723	731	740	749
510	757	766	774	783	791	800	808	817	825	834
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510	70 757	766	774	783	791	800	808	817	825	834
511	842	851	859	868	876	885	893	902	910	919
512	927	935	944	952	961	969	978	986	995	*003
513	71 012	020	029	037	046	054	063	071	079	088
514	096	105	113	122	130	139	147	155	164	172
515	181	189	198	206	214	223	231	240	248	257
516	265	273	282	290	299	307	315	324	332	341
517	349	357	366	374	383	391	399	408	416	425
518	433	441	450	458	466	475	483	492	500	508
519	517	525	533	542	550	559	567	575	584	592
520	600	609	617	625	634	642	650	659	667	675
521	684	692	700	709	717	725	734	742	750	759
522	767	775	784	792	800	809	817	825	834	842
523	850	858	867	875	883	892	900	908	917	925
524	933	941	950	958	966	975	983	991	999	*008
525	72 016	024	032	041	049	057	066	074	082	090
526	099	107	115	123	132	140	148	156	165	173
527	181	189	198	206	214	222	230	239	247	255
528	263	272	280	288	296	304	313	321	329	337
529	346	354	362	370	378	387	395	403	411	419
530	428	436	444	452	460	469	477	485	493	501
531	509	518	526	534	542	550	558	567	575	583
532	591	599	607	616	624	632	640	648	656	665
533	673	681	689	697	705	713	722	730	738	746
534	754	762	770	779	787	795	803	811	819	827
535	835	843	852	860	868	876	884	892	900	908
536	916	925	933	941	949	957	965	973	981	989
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070
538	73 078	086	094	102	111	119	127	135	143	151
539	159	167	175	183	191	199	207	215	223	231
540	239	247	255	263	272	280	288	296	304	312
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541	320	328	336	344	352	360	368	376	384	392
542	400	408	416	424	432	440	448	456	464	472
543	480	488	496	504	512	520	528	536	544	552
544	560	568	576	584	592	600	608	616	624	632
545	640	648	656	664	672	679	687	695	703	711
546	719	727	735	743	751	759	767	775	783	791
547	799	807	815	823	830	838	846	854	862	870
548	878	886	894	902	910	918	926	933	941	949
549	957	965	973	981	989	997	*005	*013	*020	*028
550	74 036	044	052	060	068	076	084	092	099	107
551	115	123	131	139	147	155	162	170	178	186
552	194	202	210	218	225	233	241	249	257	265
553	273	280	288	296	304	312	320	327	335	343
554	351	359	367	374	382	390	398	406	414	421
555	429	437	445	453	461	468	476	484	492	500
556	507	515	523	531	539	547	554	562	570	578
557	586	593	601	609	617	624	632	640	648	656
558	663	671	679	687	695	702	710	718	726	733
559	741	749	757	764	772	780	788	796	803	811
560	819	827	834	842	850	858	865	873	881	889
561	896	904	912	920	927	935	943	950	958	966
562	974	981	989	997	*005	*012	*020	*028	*035	*043
563	75 051	059	066	074	082	089	097	105	113	120
564	128	136	143	151	159	166	174	182	189	197
565	205	213	220	228	236	243	251	259	266	274
566	282	289	297	305	312	320	328	335	343	351
567	358	366	374	381	389	397	404	412	420	427
568	435	442	450	458	465	473	481	488	496	504
569	511	519	526	534	542	549	557	565	572	580
570	587	595	603	610	618	626	633	641	648	656
571	664	671	679	686	694	702	709	717	724	732
572	740	747	755	762	770	778	785	793	800	808
573	815	823	831	838	846	853	861	868	876	884
574	891	899	906	914	921	929	937	944	952	959
575	967	974	982	989	997	*005	*012	*020	*027	*035
576	76 042	050	057	065	072	080	087	095	103	110
577	118	125	133	140	148	155	163	170	178	185
578	193	200	208	215	223	230	238	245	253	260
579	268	275	283	290	298	305	313	320	328	335
580	343	350	358	365	373	380	388	395	403	410
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581	418	425	433	440	448	45$\bar{5}$	462	470	477	48$\bar{5}$
582	492	$\bar{5}$00	507	51$\bar{5}$	522	530	537	54$\bar{5}$	552	559
583	567	574	582	589	597	604	612	619	626	634
584	641	649	656	664	671	678	686	693	701	708
585	716	723	730	738	745	753	760	768	77$\bar{5}$	782
586	790	797	80$\bar{5}$	812	819	827	834	842	849	856
587	864	871	879	886	893	901	908	916	923	930
588	938	945	953	960	967	97$\bar{5}$	982	989	997	*004
589	77 012	019	026	034	041	048	056	063	070	078
590	085	093	100	107	11$\bar{5}$	122	129	137	144	151
591	159	166	173	181	188	195	203	210	217	22$\bar{5}$
592	232	240	247	254	262	269	276	283	291	298
593	305	313	320	327	33$\bar{5}$	342	349	357	364	371
594	379	386	393	401	408	415	422	430	437	444
595	452	459	466	474	481	488	495	503	510	517
596	52$\bar{5}$	532	539	546	554	561	568	576	583	590
597	597	60$\bar{5}$	612	619	627	634	641	648	656	663
598	670	677	68$\bar{5}$	692	699	706	714	721	728	735
599	743	7$\bar{5}$0	757	764	772	779	786	793	801	808
600	815	822	830	837	844	851	859	866	873	880
601	887	89$\bar{5}$	902	909	916	924	931	938	945	952
602	960	967	974	981	988	996	*003	*010	*017	*02$\bar{5}$
603	78 032	039	046	053	061	068	07$\bar{5}$	082	089	097
604	104	111	118	125	132	140	147	154	161	168
605	176	183	190	197	204	211	219	226	233	240
606	247	254	262	269	276	283	290	297	30$\bar{5}$	312
607	319	326	333	340	347	35$\bar{5}$	362	369	376	383
608	390	398	40$\bar{5}$	412	419	426	433	440	447	45$\bar{5}$
609	462	469	476	483	490	497	504	512	519	526
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610	78 533	540	547	554	561	569	576	583	590	597
611	604	611	618	625	633	640	647	654	661	668
612	675	682	689	696	704	711	718	725̄	732	739
613	746	753	760	767	774	781	789	796	803	810
614	817	824	831	838	845	852	859	866	873	880
615	888	895̄	902	909	916	923	930	937	944	951
616	958	965	972	979	986	993	*000	*007	*014	*021
617	79 029	036	043	050	057	064	071	078	085̄	092
618	099	106	113	120	127	134	141	148	155	162
619	169	176	183	190	197	204	211	218	225	232
620	239	246	253	260	267	274	281	288	295	302
621	309	316	323	330	337	344	351	358	365	372
622	379	386	393	400	407	414	421	428	435̄	442
623	449	456	463	470	477	484	491	498	505̄	511
624	518	525	532	539	546	553	560	567	574	581
625	588	595̄	602	609	616	623	630	637	644	650
626	657	664	671	678	685	692	699	706	713	720
627	727	734	741	748	754	761	768	775	782	789
628	796	803	810	817	824	831	837	844	851	858
629	865	872	879	886	893	900	906	913	920	927
630	934	941	948	955̄	962	969	975	982	989	996
631	80 003	010	017	024	030	037	044	051	058	065̄
632	072	079	085	092	099	106	113	120	127	134
633	140	147	154	161	168	175̄	182	188	195	202
634	209	216	223	229	236	243	250	257	264	271
635	277	284	291	298	305̄	312	318	325	332	339
636	346	353	359	366	373	380	387	393	400	407
637	414	421	428	434	441	448	455̄	462	468	475
638	482	489	496	502	509	516	523	530	536	543
639	550	557	564	570	577	584	591	598	604	611
640	618	625̄	632	638	645	652	659	665	672	679
641	686	693	699	706	713	720	726	733	740	747
642	754	760	767	774	781	787	794	801	808	814
643	821	828	835̄	841	848	855̄	862	868	875	882
644	889	895	902	909	916	922	929	936	943	949
645	956	963	969	976	983	990	996	*003	*010	*017
646	81 023	030	037	043	050	057	064	070	077	084
647	090	097	104	111	117	124	131	137	144	151
648	158	164	171	178	184	191	198	204	211	218
649	224	231	238	245̄	251	258	265̄	271	278	285̄
650	291	298	305̄	311	318	325̄	331	338	345̄	351
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651	358	365	371	378	385	391	398	405	411	418
652	425	431	438	445	451	458	465	471	478	485
653	491	498	505	511	518	525	531	538	544	551
654	558	564	571	578	584	591	598	604	611	617
655	624	631	637	644	651	657	664	671	677	684
656	690	697	704	710	717	723	730	737	743	750
657	757	763	770	776	783	790	796	803	809	816
658	823	829	836	842	849	856	862	869	875	882
659	889	895	902	908	915	921	928	935	941	948
660	954	961	968	974	981	987	994	*000	*007	*014
661	82 020	027	033	040	046	053	060	066	073	079
662	086	092	099	105	112	119	125	132	138	145
663	151	158	164	171	178	184	191	197	204	210
664	217	223	230	236	243	249	256	263	269	276
665	282	289	295	302	308	315	321	328	334	341
666	347	354	360	367	373	380	387	393	400	406
667	413	419	426	432	439	445	452	458	465	471
668	478	484	491	497	504	510	517	523	530	536
669	543	549	556	562	569	575	582	588	595	601
670	607	614	620	627	633	640	646	653	659	666
671	672	679	685	692	698	705	711	718	724	730
672	737	743	750	756	763	769	776	782	789	795
673	802	808	814	821	827	834	840	847	853	860
674	866	872	879	885	892	898	905	911	918	924
675	930	937	943	950	956	963	969	975	982	988
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052
677	83 059	065	072	078	085	091	097	104	110	117
678	123	129	136	142	149	155	161	168	174	181
679	187	193	200	206	213	219	225	232	238	245
680	251	257	264	270	276	283	289	296	302	308
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682	378	385̄	391	398	404	410	417	423	429	436
683	442	448	455̄	461	467	474	480	487	493	499
684	506	512	518	525̄	531	537	544	550	556	563
685	569	575	582	588	594	601	607	613	620	626
686	632	639	645	651	658	664	670	677	683	689
687	696	702	708	715̄	721	727	734	740	746	753
688	759	765	771	778	784	790	797	803	809	816
689	822	828	835̄	841	847	853	860	866	872	879
690	885̄	891	897	904	910	916	923	929	935	942
691	948	954	960	967	973	979	985	992	998	*004
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698	386	392	398	404	410	417	423	429	435	442
699	448	454	460	466	473	479	485̄	491	497	504
700	510	516	522	528	535̄	541	547	553	559	566
701	572	578	584	590	597	603	609	615	621	628
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706	880	887	893	899	905	911	917	924	930	936
707	942	948	954	960	967	973	979	985̄	991	997
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712	248	254	260	266	272	278	285̄	291	297	303
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714	370	376	382	388	394	400	406	412	418	425̄
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971	722	726	731	735	740	744	749	753	758	762
972	767	771	776	780	784	789	793	798	802	807
973	811	816	820	825	829	834	838	843	847	851
974	856	860	865	869	874	878	883	887	892	896
975	900	905	909	914	918	923	927	932	936	941
976	945	949	954	958	963	967	972	976	981	985
977	989	994	998	*003	*007	*012	*016	*021	*025	*029
978	99 034	038	043	047	052	056	061	065	069	074
979	078	083	087	092	096	100	105	109	114	118
980	123	127	131	136	140	145	149	154	158	162
981	167	171	176	180	185	189	193	198	202	207
982	211	216	220	224	229	233	238	242	247	251
983	255	260	264	269	273	277	282	286	291	295
984	300	304	308	313	317	322	326	330	335	339
985	344	348	352	357	361	366	370	374	379	383
986	388	392	396	401	405	410	414	419	423	427
987	432	436	441	445	449	454	458	463	467	471
988	476	480	484	489	493	498	502	506	511	515
989	520	524	528	533	537	542	546	550	555	559
990	564	568	572	577	581	585	590	594	599	603
991	607	612	616	621	625	629	634	638	642	647
992	651	656	660	664	669	673	677	682	686	691
993	695	699	704	708	712	717	721	726	730	734
994	739	743	747	752	756	760	765	769	774	778
995	782	787	791	795	800	804	808	813	817	822
996	826	830	835	839	843	848	852	856	861	865
997	870	874	878	883	887	891	896	900	904	909
998	913	917	922	926	930	935	939	944	948	952
999	957	961	965	970	974	978	983	987	991	996
1000	00 000	004	009	013	017	022	026	030	035	039
N	0	1	2	3	4	5	6	7	8	9

TABLE II

FIVE-PLACE LOGARITHMS
OF THE
TRIGONOMETRIC FUNCTIONS

TO EVERY MINUTE

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
0	—		—		—	0.00 000	**60**
1	6.46 373	30103	6.46 373	30103	3.53 627	0.00 000	59
2	6.76 476	17609	6.76 476	17609	3.23 524	0.00 000	58
3	6.94 085	12494	6.94 085	12494	3.05 915	0.00 000	57
4	7.06 579	9691	7.06 579	9691	2.93 421	0.00 000	56
5	7.16 270	7918	7.16 270	7918	2.83 730	0.00 000	55
6	7.24 188	6694	7.24 188	6694	2.75 812	0.00 000	54
7	7.30 882	5800	7.30 882	5800	2.69 118	0.00 000	53
8	7.36 682	5115	7.36 682	5115	2.63 318	0.00 000	52
9	7.41 797	4576	7.41 797	4576	2.58 203	0.00 000	51
10	7.46 373	4139	7.46 373	4139	2.53 627	0.00 000	**50**
11	7.50 512	3779	7.50 512	3779	2.49 488	0.00 000	49
12	7.54 291	3476	7.54 291	3476	2.45 709	0.00 000	48
13	7.57 767	3218	7.57 767	3219	2.42 233	0.00 000	47
14	7.60 985	2997	7.60 986	2996	2.39 014	0.00 000	46
15	7.63 982	2802	7.63 982	2803	2.36 018	0.00 000	45
16	7.66 784	2633	7.66 785	2633	2.33 215	0.00 000	44
17	7.69 417	2483	7.69 418	2482	2.30 582	9.99 999	43
18	7.71 900	2348	7.71 900	2348	2.28 100	9.99 999	42
19	7.74 248	2227	7.74 248	2228	2.25 752	9.99 999	41
20	7.76 475	2119	7.76 476	2119	2.23 524	9.99 999	**40**
21	7.78 594	2021	7.78 595	2020	2.21 405	9.99 999	39
22	7.80 615	1930	7.80 615	1931	2.19 385	9.99 999	38
23	7.82 545	1848	7.82 546	1848	2.17 454	9.99 999	37
24	7.84 393	1773	7.84 394	1773	2.15 606	9.99 999	36
25	7.86 166	1704	7.86 167	1704	2.13 833	9.99 999	35
26	7.87 870	1639	7.87 871	1639	2.12 129	9.99 999	34
27	7.89 509	1579	7.89 510	1579	2.10 490	9.99 999	33
28	7.91 088	1524	7.91 089	1524	2.08 911	9.99 999	32
29	7.92 612	1472	7.92 613	1473	2.07 387	9.99 998	31
30	7.94 084		7.94 086		2.05 914	9.99 998	**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

89° 30′.

PP	9691	4576	2997		2483	2119	1848		1704	1579	1472
.1	969	458	300	.1	248	212	185	.1	170	158	147
.2	1938	915	599	.2	497	424	370	.2	341	316	294
.3	2907	1372	899	.3	745	636	554	.3	511	474	442
.4	3876	1830	1199	.4	993	848	739	.4	682	632	589
.5	4846	2288	1498	.5	1242	1060	924	.5	852	789	736
.6	5815	2646	1798	.6	1490	1271	1109	.6	1022	947	883
.7	6784	3203	2098	.7	1738	1483	1294	.7	1193	1105	1030
.8	7753	3661	2398	.8	1986	1695	1478	.8	1363	1263	1178
.9	8722	4118	2697	.9	2235	1907	1663	.9	1534	1421	1325

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
30	7.94 084		7.94 086		2.05 914	9.99 998	**30**
31	7.95 508	1424	7.95 510	1424	2.04 490	9.99 998	29
32	7.96 887	1379	7.96 889	1379	2.03 111	9.99 998	28
33	7.98 223	1336	7.98 225	1336	2.01 77$\bar{5}$	9.99 998	27
34	7.99 520	1297	7.99 522	1297	2.00 478	9.99 998	26
35	8.00 779	1259	8.00 781	1259	1.99 219	9.99 998	25
36	8.02 002	1223	8.02 004	1223	1.97 996	9.99 998	24
37	8.03 192	1190	8.03 194	1190	1.96 806	9.99 997	23
38	8.04 350	1158	8.04 353	1159	1.95 647	9.99 997	22
39	8.05 478	1128	8.05 481	1128	1.94 519	9.99 997	21
40	8.06 578	1100	8.06 581	1100	1.93 419	9.99 997	**20**
41	8.07 650	1072	8.07 653	1072	1.92 347	9.99 997	19
42	8.08 696	1046	8.08 700	1047	1.91 300	9.99 997	18
43	8.09 718	1022	8.09 722	1022	1.90 278	9.99 997	17
44	8.10 717	999	8.10 720	998	1.89 280	9.99 996	16
45	8.11 693	976	8.11 696	976	1.88 304	9.99 996	15
46	8.12 647	954	8.12 651	955	1.87 349	9.99 996	14
47	8.13 581	934	8.13 585	934	1.86 41$\bar{5}$	9.99 996	13
48	8.14 495	914	8.14 $\bar{5}$00	915	1.85 500	9.99 996	12
49	8.15 391	896	8.15 395	895	1.84 60$\bar{5}$	9.99 996	11
50	8.16 268	877	8.16 273	878	1.83 727	9.99 995	**10**
51	8.17 128	860	8.17 133	860	1.82 867	9.99 995	9
52	8.17 971	843	8.17 976	843	1.82 024	9.99 995	8
53	8.18 798	827	8.18 804	828	1.81 196	9.99 99$\bar{5}$	7
54	8.19 610	812	8.19 616	812	1.80 384	9.99 99$\bar{5}$	6
55	8.20 407	797	8.20 413	797	1.79 587	9.99 994	5
56	8.21 189	782	8.21 195	782	1.78 80$\bar{5}$	9.99 994	4
57	8.21 958	769	8.21 964	769	1.78 036	9.99 994	3
58	8.22 713	755	8.22 720	756	1.77 280	9.99 994	2
59	8.23 456	743	8.23 462	742	1.76 538	9.99 994	1
60	8.24 186	730	8.24 192	730	1.75 808	9.99 993	**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

89°.

PP	1379	1223	1100		999	914	860		812	769	730
.1	138	122	110	.1	100	91	86	.1	81	77	73
.2	276	245	220	.2	200	183	172	.2	162	154	146
.3	414	367	330	.3	300	274	258	.3	244	231	219
.4	552	489	440	.4	400	366	344	.4	325	308	292
.5	690	612	550	.5	500	457	430	.5	406	385	365
.6	827	734	660	.6	599	548	516	.6	487	461	438
.7	965	856	770	.7	699	640	602	.7	568	538	511
.8	1103	978	880	.8	799	731	688	.8	650	615	584
.9	1241	1101	990	.9	899	823	774	.9	731	692	657

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
0	8.24 186	717	8.24 192	718	1.75 808	9.99 993	60
1	8.24 903	706	8.24 910	706	1.75 090	9.99 993	59
2	8.25 609	695	8.25 616	696	1.74 384	9.99 993	58
3	8.26 304	684	8.26 312	684	1.73 688	9.99 993	57
4	8.26 988	673	8.26 996	673	1.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	1.72 331	9.99 992	55
6	8.28 324	653	8.28 332	654	1.71 668	9.99 992	54
7	8.28 977	644	8.28 986	643	1.71 014	9.99 992	53
8	8.29 621	634	8.29 629	634	1.70 371	9.99 992	52
9	8.30 255	624	8.30 263	625	1.69 737	9.99 991	51
10	8.30 879	616	8.30 888	617	1.69 112	9.99 991	50
11	8.31 495	608	8.31 505	607	1.68 495	9.99 991	49
12	8.32 103	599	8.32 112	599	1.67 888	9.99 990	48
13	8.32 702	590	8.32 711	591	1.67 289	9.99 990	47
14	8.33 292	583	8.33 302	584	1.66 698	9.99 990	46
15	8.33 875	575	8.33 886	575	1.66 114	9.99 990	45
16	8.34 450	568	8.34 461	568	1.65 539	9.99 989	44
17	8.35 018	560	8.35 029	561	1.64 971	9.99 989	43
18	8.35 578	553	8.35 590	553	1.64 410	9.99 989	42
19	8.36 131	547	8.36 143	546	1.63 857	9.99 989	41
20	8.36 678	539	8.36 689	540	1.63 311	9.99 988	40
21	8.37 217	533	8.37 229	533	1.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	1.62 238	9.99 988	38
23	8.38 276	520	8.38 289	520	1.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	1.61 191	9.99 987	36
25	8.39 310	508	8.39 323	509	1.60 677	9.99 987	35
26	8.39 818	502	8.39 832	502	1.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	1.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	1.59 170	9.99 986	32
29	8.41 307	485	8.41 321	486	1.58 679	9.99 985	31
30	8.41 792		8.41 807		1.58 193	9.99 985	30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

88° 30′.

PP	706	663	634		599	575	553		533	514	496
.1	70.6	66.3	63.4	.1	59.9	57.5	55.3	.1	53.3	51.4	49.6
.2	141.2	132.6	126.8	.2	119.8	115.0	110.6	.2	106.6	102.8	99.2
.3	211.8	198.9	190.2	.3	179.7	172.5	165.9	.3	159.9	154.2	148.8
.4	282.4	265.2	253.6	.4	239.6	230.0	221.2	.4	213.2	205.6	198.4
.5	353.0	331.5	317.0	.5	299.5	287.5	276.5	.5	266.5	257.0	248.0
.6	423.6	397.8	380.4	6	359.4	345.0	331.8	.6	319.8	308.4	297.6
.7	494.2	464.1	443.8	.7	419.3	402.5	387.1	.7	373.1	359.8	347.2
.8	564.8	530.4	507.2	.8	479.2	460.0	442.4	.8	426.4	411.2	396.8
.9	635.4	596.7	570.6	.9	539.1	517.5	497.7	.9	479.7	462.6	446.4

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
30	8.41 792		8.41 807		1.58 193	9.99 985	30
31	8.42 272	480	8.42 287	480	1.57 713	9.99 985̄	29
32	8.42 746	474	8.42 762	475	1.57 238	9.99 984	28
33	8.43 216	470	8.43 232	470	1.56 768	9.99 984	27
34	8.43 680	464	8.43 696	464	1.56 304	9.99 984	26
35	8.44 139	459	8.44 156	460	1.55 844	9.99 983	25
36	8.44 594	455	8.44 611	455	1.55 389	9.99 983	24
37	8.45 044	450	8.45 061	450	1.54 939	9.99 983	23
38	8.45 489	445	8.45 507	446	1.54 493	9.99 982	22
39	8.45 930	441	8.45 948	441	1.54 052	9.99 982	21
40	8.46 366	436	8.46 385̄	437	1.53 615	9.99 982	20
41	8.46 799	433	8.46 817	432	1.53 183	9.99 981	19
42	8.47 226	427	8.47 245	428	1.52 755̄	9.99 981	18
43	8.47 650̄	424	8.47 669	424	1.52 331	9.99 981	17
44	8.48 069	419	8.48 089	420	1.51 911	9.99 980	16
45	8.48 485̄	416	8.48 505	416	1.51 495̄	9.99 980	15
46	8.48 896	411	8.48 917	412	1.51 083	9.99 979	14
47	8.49 304	408	8.49 325	408	1.50 675̄	9.99 979	13
48	8.49 708	404	8.49 729	404	1.50 271	9.99 979	12
49	8.50 108	400	8.50 130	401	1.49 870	9.99 978	11
50	8.50 504	396	8.50 527	397	1.49 473	9.99 978	10
51	8.50 897	393	8.50 920	393	1.49 080	9.99 977	9
52	8.51 287	390	8.51 310	390	1.48 690	9.99 977	8
53	8.51 673	386	8.51 696	386	1.48 304	9.99 977	7
54	8.52 055	382	8.52 079	383	1.47 921	9.99 976	6
55	8.52 434	379	8.52 459	380	1.47 541	9.99 976	5
56	8.52 810	376	8.52 835̄	376	1.47 165	9.99 975	4
57	8.53 183	373	8.53 208	373	1.46 792	9.99 975̄	3
58	8.53 552	369	8.53 578	370	1.46 422	9.99 974	2
59	8.53 919	367	8.53 945̄	367	1.46 055	9.99 974	1
60	8.54 282	363	8.54 308	363	1.45 692	9.99 974	0
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

88°.

PP	470	455	441		424	408	396		386	376	367
.1	47.0	45.5	44.1	.1	42.4	40.8	39.6	.1	38.6	37.6	36.7
.2	94.0	91.0	88.2	.2	84.8	81.6	79.2	.2	77.2	75.2	73.4
.3	141.0	136.5	132.3	.3	127.2	122.4	118.8	.3	115.8	112.8	110.1
.4	188.0	182.0	176.4	.4	169.6	163.2	158.4	.4	154.4	150.4	146.8
.5	235.0	227.5	220.5	.5	212.0	204.0	198.0	.5	193.0	188.0	183.5
.6	282.0	273.0	264.6	.6	254.4	244.8	237.6	.6	231.6	225.6	220.2
.7	329.0	318.5	308.7	.7	296.8	285.6	277.2	.7	270.2	263.2	256.9
.8	376.0	364.0	352.8	.8	339.2	326.4	316.8	.8	308.8	300.8	293.6
.9	423.0	409.5	396.9	.9	381.6	367.2	356.4	.9	347.4	338.4	330.3

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
0	8.54 282	360	8.54 308	361	1.45 692	9.99 974	**60**
1	8.54 642	357	8.54 669	358	1.45 331	9.99 973	59
2	8.54 999	355	8.55 027	355	1.44 973	9.99 973	58
3	8.55 354	351	8.55 382	352	1.44 618	9.99 972	57
4	8.55 705	349	8.55 734	349	1.44 266	9.99 972	56
5	8.56 054	346	8.56 083	346	1.43 917	9.99 971	55
6	8.56 400	343	8.56 429	344	1.43 571	9.99 971	54
7	8.56 743	341	8.56 773	341	1.43 227	9.99 970	53
8	8.57 084	337	8.57 114	338	1.42 886	9.99 970	52
9	8.57 421	336	8.57 452	336	1.42 548	9.99 969	51
10	8.57 757	332	8.57 788	333	1.42 212	9.99 969	**50**
11	8.58 089	330	8.58 121	330	1.41 879	9.99 968	49
12	8.58 419	328	8.58 451	328	1.41 549	9.99 968	48
13	8.58 747	325	8.58 779	326	1.41 221	9.99 967	47
14	8.59 072	323	8.59 105	323	1.40 895̄	9.99 967	46
15	8.59 395̄	320	8.59 428	321	1.40 572	9.99 967	45
16	8.59 715	318	8.59 749	319	1.40 251	9.99 966	44
17	8.60 033	316	8.60 068	316	1.39 932	9.99 966	43
18	8.60 349	313	8.60 384	314	1.39 616	9.99 965̄	42
19	8.60 662	311	8.60 698	311	1.39 302	9.99 964	41
20	8.60 973	309	8.61 009	310	1.38 991	9.99 964	**40**
21	8.61 282	307	8.61 319	307	1.38 681	9.99 963	39
22	8.61 589	305	8.61 626	305	1.38 374	9.99 963	38
23	8.61 894	302	8.61 931	303	1.38 069	9.99 962	37
24	8.62 196	301	8.62 234	301	1.37 766	9.99 962	36
25	8.62 497	298	8.62 535	299	1.37 465̄	9.99 961	35
26	8.62 795̄	296	8.62 834	297	1.37 166	9.99 961	34
27	8.63 091	294	8.63 131	295	1.36 869	9.99 960	33
28	8.63 385	293	8.63 426	292	1.36 574	9.99 960	32
29	8.63 678	290	8.63 718	291	1.36 282	9.99 959	31
30	8.63 968		8.64 009		1.35 991	9.99 959	**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

87° 30′.

PP	360	350	340		330	320	310		300	290	285
.1	36	35	34	.1	33	32	31	.1	30	29	28.5
.2	72	70	68	.2	66	64	62	.2	60	58	57.0
.3	108	105	102	.3	99	96	93	.3	90	87	85.5
.4	144	140	136	.4	132	128	124	.4	120	116	114.0
.5	180	175	170	.5	165	160	155	.5	150	145	142.5
.6	216	210	204	.6	198	192	186	.6	180	174	171.0
.7	252	245	238	.7	231	224	217	.7	210	203	199.5
.8	288	280	272	.8	264	256	248	.8	240	232	228.0
.9	324	315	306	.9	297	288	279	.9	270	161	256.5

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
30	8.63 968		8.64 009		1.35 991	9.99 959	**30**
31	8.64 256	288	8.64 298	289	1.35 702	9.99 958	29
32	8.64 543	287	8.64 585	287	1.35 41$\bar{5}$	9.99 958	28
33	8.64 827	284	8.64 870	285	1.35 130	9.99 957	27
34	8.65 110	283	8.65 154	284	1.34 846	9.99 956	26
35	8.65 391	281	8.65 435	281	1.34 56$\bar{5}$	9.99 956	25
36	8.65 670	279	8.65 71$\bar{5}$	280	1.34 285	9.99 955	24
37	8.65 947	277	8.65 993	278	1.34 007	9.99 95$\bar{5}$	23
38	8.66 223	276	8.66 269	276	1.33 731	9.99 954	22
39	8.66 497	274	8.66 543	274	1.33 457	9.99 954	21
40	8.66 769	272	8.66 816	273	1.33 184	9.99 953	**20**
41	8.67 039	270	8.67 087	271	1.32 913	9.99 952	19
42	8.67 308	269	8.67 356	269	1.32 644	9.99 952	18
43	8.67 575	267	8.67 624	268	1.32 376	9.99 951	17
44	8.67 841	266	8.67 890	266	1.32 110	9.99 951	16
45	8.68 104	263	8.68 154	264	1.31 846	9.99 95$\bar{0}$	15
46	8.68 367	263	8.68 417	263	1.31 583	9.99 949	14
47	8.68 627	260	8.68 678	261	1.31 322	9.99 949	13
48	8.68 886	259	8.68 938	260	1.31 062	9.99 948	12
49	8.69 144	258	8.69 196	258	1.30 804	9.99 948	11
50	8.69 400	256	8.69 453	257	1.30 547	9.99 947	**10**
51	8.69 654	254	8.69 708	255	1.30 292	9.99 946	9
52	8.69 907	253	8.69 962	254	1.30 038	9.99 946	8
53	8.70 159	252	8.70 214	252	1.29 786	9.99 94$\bar{5}$	7
54	8.70 409	250	8.70 46$\bar{5}$	251	1.29 535	9.99 944	6
55	8.70 658	249	8.70 714	249	1.29 286	9.99 944	5
56	8.70 90$\bar{5}$	247	8.70 962	248	1.29 038	9.99 943	4
57	8.71 151	246	8.71 208	246	1.28 792	9.99 942	3
58	8.71 395	244	8.71 453	245	1.28 547	9.99 942	2
59	8.71 638	243	8.71 697	244	1.28 303	9.99 941	1
60	8.71 880	242	8.71 940	243	1.28 060	9.99 940	**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

87°.

PP	280	275	270		265	260	255		250	245	240
.1	28.0	27.5	27.0	.1	26.5	26.0	25.5	.1	25.0	24.5	24.0
.2	56.0	55.0	54.0	.2	53.0	52.0	51.0	.2	50.0	49.0	48.0
.3	84.0	82.5	81.0	.3	79.5	78.0	76.5	.3	75.0	73.5	72.0
.4	112.0	110.0	108.0	.4	106.0	104.0	102.0	.4	100.0	98.0	96.0
.5	140.0	137.5	135.0	.5	132.5	130.0	127.5	.5	125.0	122.5	120.0
.6	168.0	165.0	162.0	.6	159.0	156.0	153.0	.6	150.0	147.0	144.0
.7	196.0	192.5	189.0	.7	185.5	182.0	178.5	.7	175.0	171.5	168.0
.8	224.0	220.0	216.0	.8	212.0	208.0	204.0	.8	200.0	196.0	192.0
.9	252.0	247.5	243.0	.9	238.5	234.0	229.5	.9	225.0	220.5	216.0

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
0	8.71 880	240	8.71 940	241	1.28 060	9.99 940	**60**
1	8.72 120	239	8.72 181	239	1.27 819	9.99 940	59
2	8.72 359	238	8.72 420	239	1.27 580	9.99 939	58
3	8.72 597	237	8.72 659	237	1.27 341	9.99 938	57
4	8.72 834	235	8.72 896	236	1.27 104	9.99 938	56
5	8.73 069	234	8.73 132	234	1.26 868	9.99 937	55
6	8.73 303	232	8.73 366	234	1.26 634	9.99 936	54
7	8.73 535	232	8.73 600	232	1.26 400	9.99 936	53
8	8.73 767	230	8.73 832	231	1.26 168	9.99 935	52
9	8.73 997	229	8.74 063	229	1.25 937	9.99 934	51
10	8.74 226	228	8.74 292	229	1.25 708	9.99 934	**50**
11	8.74 454	226	8.74 521	227	1.25 479	9.99 933	49
12	8.74 680	226	8.74 748	226	1.25 252	9.99 932	48
13	8.74 906	224	8.74 974	225	1.25 026	9.99 932	47
14	8.75 130	223	8.75 199	224	1.24 801	9.99 931	46
15	8.75 353	222	8.75 423	222	1.24 577	9.99 930	45
16	8.75 57$\bar{5}$	220	8.75 645	222	1.24 35$\bar{5}$	9.99 929	44
17	8.75 795	220	8.75 867	220	1.24 133	9.99 929	43
18	8.76 015	219	8.76 087	219	1.23 913	9.99 928	42
19	8.76 234	217	8.76 306	219	1.23 694	9.99 927	41
20	8.76 451	216	8.76 52$\bar{5}$	217	1.23 475	9.99 926	**40**
21	8.76 667	216	8.76 742	216	1.23 258	9.99 926	39
22	8.76 883	214	8.76 958	215	1.23 042	9.99 92$\bar{5}$	38
23	8.77 097	213	8.77 173	214	1.22 827	9.99 924	37
24	8.77 310	212	8.77 387	213	1.22 613	9.99 923	36
25	8.77 522	211	8.77 600	211	1.22 400	9.99 923	35
26	8.77 733	210	8.77 811	211	1.22 189	9.99 922	34
27	8.77 943	209	8.78 022	210	1.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	1.21 768	9.99 920	32
29	8.78 360	208	8.78 441	208	1.21 559	9.99 920	31
30	8.78 568		8.78 649		1.21 351	9.99 919	**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

86° 30′.

PP	238	234	229		225	220	216		212	208	204
.1	23.8	23.4	22.9	.1	22.5	22.0	21.6	.1	21.2	20.8	20.4
.2	47.6	46.8	45.8	.2	45.0	44.0	43.2	.2	42.4	41.6	40.8
.3	71.4	70.2	68.7	.3	67.5	66.0	64.8	.3	63.6	62.4	61.2
.4	95.2	93.6	91.6	.4	90.0	88.0	86.4	.4	84.8	83.2	81.6
.5	119.0	117.0	114.5	.5	112.5	110.0	108.0	.5	106.0	104.0	102.0
.6	142.8	140.4	137.4	.6	135.0	132.0	129.6	.6	127.2	124.8	122.4
.7	166.6	163.8	160.3	.7	157.5	154.0	151.2	.7	148.4	145.6	142.8
.8	190.4	187.2	183.2	.8	180.0	176.0	172.8	.8	169.6	166.4	163.2
.9	214.2	210.6	206.1	.9	202.5	198.0	194.4	.9	190.8	187.2	183.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
30	8.78 568	206	8.78 649	206	1.21 351	9.99 919	**30**
31	8.78 774	205	8.78 855	206	1.21 14$\bar{5}$	9.99 918	29
32	8.78 979	204	8.79 061	205	1.20 939	9.99 917	28
33	8.79 183	203	8.79 266	204	1.20 734	9.99 917	27
34	8.79 386	202	8.79 470	203	1.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	1.20 327	9.99 915	25
36	8.79 789	201	8.79 875	201	1.20 12$\bar{5}$	9.99 914	24
37	8.79 990	199	8.80 076	201	1.19 924	9.99 913	23
38	8.80 189	199	8.80 277	199	1.19 723	9.99 913	22
39	8.80 388	197	8.80 476	198	1.19 524	9.99 912	21
40	8.80 585	197	8.80 674	198	1.19 326	9.99 911	**20**
41	8.80 782	196	8.80 872	196	1.19 128	9.99 910	19
42	8.80 978	195	8.81 068	196	1.18 932	9.99 909	18
43	8.81 173	194	8.81 264	195	1.18 736	9.99 909	17
44	8.81 367	193	8.81 459	194	1.18 541	9.99 908	16
45	8.81 560	192	8.81 653	193	1.18 347	9.99 907	15
46	8.81 752	192	8.81 846	192	1.18 154	9.99 906	14
47	8.81 944	190	8.82 038	192	1.17 962	9.99 905	13
48	8.82 134	190	8.82 230	190	1.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	1.17 580	9.99 904	11
50	8.82 513	188	8.82 610	189	1.17 390	9.99 903	**10**
51	8.82 701	187	8.82 799	188	1.17 201	9.99 902	9
52	8.82 888	187	8.82 987	188	1.17 013	9.99 901	8
53	8.83 07$\bar{5}$	186	8.83 17$\bar{5}$	186	1.16 825	9.99 900	7
54	8.83 261	185	8.83 361	186	1.16 639	9.99 899	6
55	8.83 446	184	8.83 547	185	1.16 453	9.99 898	5
56	8.83 630	183	8.83 732	184	1.16 268	9.99 898	4
57	8.83 813	183	8.83 916	184	1.16 084	9.99 897	3
58	8.83 996	181	8.84 100	182	1.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	1.15 718	9.99 89$\bar{5}$	1
60	8.84 358		8.84 464		1.15 536	9.99 894	**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

86°.

PP	201	198	195		192	189	187		185	183	181
.1	20.1	19.8	19.5	.1	19.2	18.9	18.7	.1	18.5	18.3	18.1
.2	40.2	39.6	39.0	.2	38.4	37.8	37.4	.2	37.0	36.6	36.2
.3	60.3	59.4	58.5	.3	57.6	56.7	56.1	.3	55.5	54.9	54.3
.4	80.4	79.2	78.0	.4	76.8	75.6	74.8	.4	74.0	73.2	72.4
.5	100.5	99.0	97.5	.5	96.0	94.5	93.5	.5	92.5	91.5	90.5
.6	120.6	118.8	117.0	.6	115.2	113.4	112.2	.6	111.0	109.8	108.6
.7	140.7	138.6	136.5	.7	134.4	132.3	130.9	.7	129.5	128.1	126.7
.8	160.8	158.4	156.0	.8	153.6	151.2	149.6	.8	148.0	146.4	144.8
.9	180.9	178.2	175.5	.9	172.8	170.1	168.3	.9	166.5	164.7	162.9

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
0	8.84 358	181	8.84 464	182	1.15 536	9.99 894	60
1	8.84 539	179	8.84 646	180	1.15 354	9.99 893	59
2	8.84 718	179	8.84 826	180	1.15 174	9.99 892	58
3	8.84 897	178	8.85 006	179	1.14 994	9.99 891	57
4	8.85 075	177	8.85 185	178	1.14 815	9.99 891	56
5	8.85 252	177	8.85 363	177	1.14 637	9.99 890	55
6	8.85 429	176	8.85 540	177	1.14 460	9.99 889	54
7	8.85 605	175	8.85 717	176	1.14 283	9.99 888	53
8	8.85 780	175	8.85 893	176	1.14 107	9.99 887	52
9	8.85 955	173	8.86 069	174	1.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	1.13 757	9.99 885	50
11	8.86 301	173	8.86 417	174	1.13 583	9.99 884	49
12	8.86 474	171	8.86 591	172	1.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47
14	8.86 816	171	8.86 935	171	1.13 065	9.99 881	46
15	8.86 987	169	8.87 106	171	1.12 894	9.99 880	45
16	8.87 156	169	8.87 277	170	1.12 723	9.99 879	44
17	8.87 325	169	8.87 447	169	1.12 553	9.99 879	43
18	8.87 494	167	8.87 616	169	1.12 384	9.99 878	42
19	8.87 661	168	8.87 785	168	1.12 215	9.99 877	41
20	8.87 829	166	8.87 953	167	1.12 047	9.99 876	40
21	8.87 995	166	8.88 120	167	1.11 880	9.99 875	39
22	8.88 161	165	8.88 287	166	1.11 713	9.99 874	38
23	8.88 326	164	8.88 453	165	1.11 547	9.99 873	37
24	8.88 490	164	8.88 618	165	1.11 382	9.99 872	36
25	8.88 654	163	8.88 783	165	1.11 217	9.99 871	35
26	8.88 817	163	8.88 948	163	1.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	1.10 889	9.99 869	33
28	8.89 142	162	8.89 274	163	1.10 726	9.99 868	32
29	8.89 304	160	8.89 437	161	1.10 563	9.99 867	31
30	8.89 464		8.89 598		1.10 402	9.99 866	30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

85° 30′.

PP	181	179	177		175	173	171		168	166	164
.1	18.1	17.9	17.7	.1	17.5	17.3	17.1	.1	16.8	16.6	16.4
.2	36.2	35.8	35.4	.2	35.0	34.6	34.2	.2	33.6	33.2	32.8
.3	54.3	53.7	53.1	.3	52.5	51.9	51.3	.3	50.4	49.8	49.2
.4	72.4	71.6	70.8	.4	70.0	69.2	68.4	.4	67.2	66.4	65.6
.5	90.5	89.5	88.5	.5	87.5	86.5	85.5	.5	84.0	83.0	82.0
.6	108.6	107.4	106.2	.6	105.0	103.8	102.6	.6	100.8	99.6	98.4
.7	126.7	125.3	123.9	.7	122.5	121.1	119.7	.7	117.6	116.2	114.8
.8	144.8	143.2	141.6	.8	140.0	138.4	136.8	.8	134.4	132.8	131.2
.9	162.9	161.1	159.3	.9	157.5	155.7	153.9	.9	151.2	149.4	147.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
30	8.89 464	161	8.89 598	162	1.10 402	9.99 866	**30**
31	8.89 625̄	159	8.89 760	160	1.10 240	9.99 865̄	29
32	8.89 784	159	8.89 920	160	1.10 080	9.99 864	28
33	8.89 943	159	8.90 080	160	1.09 920	9.99 863	27
34	8.90 102	158	8.90 240	159	1.09 760	9.99 862	26
35	8.90 260	157	8.90 399	158	1.09 601	9.99 861	25
36	8.90 417	157	8.90 557	158	1.09 443	9.99 860	24
37	8.90 574	156	8.90 715̄	157	1.09 285	9.99 859	23
38	8.90 730	155	8.90 872	157	1.09 128	9.99 858	22
39	8.90 885	155	8.91 029	156	1.08 971	9.99 857	21
40	8.91 040	155	8.91 185̄	155	1.08 815	9.99 856	**20**
41	8.91 195̄	154	8.91 340	155	1.08 660	9.99 855̄	19
42	8.91 349	153	8.91 495	155	1.08 505̄	9.99 854	18
43	8.91 502	153	8.91 650̄	153	1.08 350	9.99 853	17
44	8.91 655	152	8.91 803	154	1.08 197	9.99 852	16
45	8.91 807	152	8.91 957	153	1.08 043	9.99 851	15
46	8.91 959	151	8.92 110	152	1.07 890	9.99 850̄	14
47	8.92 110	151	8.92 262	152	1.07 738	9.99 848	13
48	8.92 261	150	8.92 414	151	1.07 586	9.99 847	12
49	8.92 411	150	8.92 565̄	151	1.07 435	9.99 846	11
50	8.92 561	149	8.92 716	150	1.07 284	9.99 845	**10**
51	8.92 710	149	8.92 866	150	1.07 134	9.99 844	9
52	8.92 859	148	8.93 016	149	1.06 984	9.99 843	8
53	8.93 007	147	8.93 165̄	148	1.06 835	9.99 842	7
54	8.93 154	147	8.93 313	149	1.06 687	9.99 841	6
55	8.93 301	147	8.93 462	147	1.06 538	9.99 840	5
56	8.93 448	146	8.93 609	147	1.06 391	9.99 839	4
57	8.93 594	146	8.93 756	147	1.06 244	9.99 838	3
58	8.93 740	145	8.93 903	146	1.06 097	9.99 837	2
59	8.93 885̄	145	8.94 049	146	1.05 951	9.99 836	1
60	8.94 030		8.94 195		1.05 805̄	9.99 834	**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

85°.

PP	162	160	159		157	155	153		151	149	147
.1	16.2	16.0	15.9	.1	15.7	15.5	15.3	.1	15.1	14.9	14.7
.2	32.4	32.0	31.8	.2	31.4	31.0	30.6	.2	30.2	29.8	29.4
.3	48.6	48.0	47.7	.3	47.1	46.5	45.9	.3	45.3	44.7	44.1
.4	64.8	64.0	63.6	.4	62.8	62.0	61.2	.4	60.4	59.6	58.8
.5	81.0	80.0	79.5	.5	78.5	77.5	76.5	.5	75.5	74.5	73.5
.6	97.2	96.0	95.4	.6	94.2	93.0	91.8	.6	90.6	89.4	88.2
.7	113.4	112.0	111.3	.7	109.9	108.5	107.1	.7	105.7	104.3	102.9
.8	129.6	128.0	127.2	.8	125.6	124.0	122.4	.8	120.8	119.2	117.6
.9	145.8	144.0	143.1	.9	141.3	139.5	137.7	.9	135.9	134.1	132.3

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
0	8.94 030		8.94 195		1.05 805	9.99 834	60
1	8.94 174	144	8.94 340	145	1.05 660	9.99 833	59
2	8.94 317	143	8.94 485	145	1.05 515	9.99 832	58
3	8.94 461	144	8.94 630	145	1.05 370	9.99 831	57
4	8.94 603	142	8.94 773	143	1.05 227	9.99 830	56
5	8.94 746	143	8.94 917	144	1.05 083	9.99 829	55
6	8.94 887	141	8.95 060	143	1.04 940	9.99 828	54
7	8.95 029	142	8.95 202	142	1.04 798	9.99 827	53
8	8.95 170	141	8.95 344	142	1.04 656	9.99 825	52
9	8.95 310	140	8.95 486	142	1.04 514	9.99 824	51
10	8.95 450	140	8.95 627	141	1.04 373	9.99 823	50
11	8.95 589	139	8.95 767	140	1.04 233	9.99 822	49
12	8.95 728	139	8.95 908	141	1.04 092	9.99 821	48
13	8.95 867	139	8.96 047	139	1.03 953	9.99 820	47
14	8.96 005	138	8.96 187	140	1.03 813	9.99 819	46
15	8.96 143	138	8.96 325	138	1.03 675	9.99 817	45
16	8.96 280	137	8.96 464	139	1.03 536	9.99 816	44
17	8.96 417	137	8.96 602	138	1.03 398	9.99 815	43
18	8.96 553	136	8.96 739	137	1.03 261	9.99 814	42
19	8.96 689	136	8.96 877	138	1.03 123	9.99 813	41
20	8.96 825	136	8.97 013	136	1.02 987	9.99 812	40
21	8.96 960	135	8.97 150	137	1.02 850	9.99 810	39
22	8.97 095	135	8.97 285	135	1.02 715	9.99 809	38
23	8.97 229	134	8.97 421	136	1.02 579	9.99 808	37
24	8.97 363	134	8.97 556	135	1.02 444	9.99 807	36
25	8.97 496	133	8.97 691	135	1.02 309	9.99 806	35
26	8.97 629	133	8.97 825	134	1.02 175	9.99 804	34
27	8.97 762	133	8.97 959	134	1.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32
29	8.98 026	132	8.98 225	133	1.01 775	9.99 801	31
30	8.98 157	131	8.98 358	133	1.01 642	9.99 800	30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

84° 30′.

PP	145	143	141		139	138	136		135	133	131
.1	14.5	14.3	14.1	.1	13.9	13.8	13.6	.1	13.5	13.3	13.1
.2	29.0	28.6	28.2	.2	27.8	27.6	27.2	.2	27.0	26.6	26.2
.3	43.5	42.9	42.3	.3	41.7	41.4	40.8	.3	40.5	39.9	39.3
.4	58.0	57.2	56.4	.4	55.6	55.2	54.4	.4	54.0	53.2	52.4
.5	72.5	71.5	70.5	.5	69.5	69.0	68.0	.5	67.5	66.5	65.5
.6	87.0	85.8	84.6	.6	83.4	82.8	81.6	.6	81.0	79.8	78.6
.7	101.5	100.1	98.7	.7	97.3	96.6	95.2	.7	94.5	93.1	91.7
.8	116.0	114.4	112.8	.8	111.2	110.4	108.8	.8	108.0	106.4	104.8
.9	130.5	128.7	126.9	.9	125.1	124.2	122.4	.9	121.5	119.7	117.9

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
30	8.98 157	131	8.98 358	132	1.01 642	9.99 800	30
31	8.98 288	131	8.98 490	132	1.01 510	9.99 798	29
32	8.98 419	130	8.98 622	131	1.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27
34	8.98 679	129	8.98 884	131	1.01 116	9.99 795̄	26
35	8.98 808	129	8.99 015̄	130	1.00 985	9.99 793	25
36	8.98 937	129	8.99 145	130	1.00 855̄	9.99 792	24
37	8.99 066	128	8.99 275	130	1.00 725̄	9.99 791	23
38	8.99 194	128	8.99 405̄	129	1.00 595	9.99 790	22
39	8.99 322	128	8.99 534	128	1.00 466	9.99 788	21
40	8.99 45̄0	127	8.99 662	129	1.00 338	9.99 787	20
41	8.99 577	127	8.99 791	128	1.00 209	9.99 786	19
42	8.99 704	126	8.99 919	127	1.00 081	9.99 785̄	18
43	8.99 830	126	9.00 046	128	0.99 954	9.99 783	17
44	8.99 956	126	9.00 174	127	0.99 826	9.99 782	16
45	9.00 082	125	9.00 301	126	0.99 699	9.99 781	15
46	9.00 207	125	9.00 427	126	0.99 573	9.99 780	14
47	9.00 332	124	9.00 553	126	0.99 447	9.99 778	13
48	9.00 456	125	9.00 679	126	0.99 321	9.99 777	12
49	9.00 581	123	9.00 805̄	125	0.99 195	9.99 776	11
50	9.00 704	124	9.00 930	125	0.99 070	9.99 775̄	10
51	9.00 828	123	9.01 055̄	124	0.98 945	9.99 773	9
52	9.00 951	123	9.01 179	124	0.98 821	9.99 772	8
53	9.01 074	122	9.01 303	124	0.98 697	9.99 771	7
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	0.98 45̄0	9.99 768	5
56	9.01 440	121	9.01 673	123	0.98 327	9.99 767	4
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3
58	9.01 682	121	9.01 918	122	0.98 082	9.99 764	2
59	9.01 803	120	9.02 040	122	0.97 960	9.99 763	1
60	9.01 923		9.02 162		0.97 838	9.99 761	0
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

84°.

PP	130	129	128		126	125	123		122	121	120
.1	13.0	12.9	12.8	.1	12.6	12.5	12.3	.1	12.2	12.1	12.0
.2	26.0	25.8	25.6	.2	25.2	25.0	24.6	.2	24.4	24.2	24.0
.3	39.0	38.7	38.4	.3	37.8	37.5	36.9	.3	36.6	36.3	36.0
.4	52.0	51.6	51.2	.4	50.4	50.0	49.2	.4	48.8	48.4	48.0
.5	65.0	64.5	64.0	.5	63.0	62.5	61.5	.5	61.0	60.5	60.0
.6	78.0	77.4	76.8	.6	75.6	75.0	73.8	.6	73.2	72.6	72.0
.7	91.0	90.3	89.6	.7	88.2	87.5	86.1	.7	85.4	84.7	84.0
.8	104.0	103.2	102.4	.8	100.8	100.0	98.4	.8	97.6	96.8	96.0
.9	117.0	116.1	115.2	.9	113.4	112.5	110.7	.9	109.8	108.9	108.0

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
0	9.01 923	120	9.02 162	121	0.97 838	9.99 761	**60**
1	9.02 043	120	9.02 283	121	0.97 717	9.99 760	59
2	9.02 163	120	9.02 404	121	0.97 596	9.99 759	58
3	9.02 283	119	9.02 525	120	0.97 475̄	9.99 757	57
4	9.02 402	118	9.02 645	121	0.97 355̄	9.99 756	56
5	9.02 520	119	9.02 766	119	0.97 234	9.99 755̄	55
6	9.02 639	118	9.02 885	120	0.97 115̄	9.99 753	54
7	9.02 757	117	9.03 005̄	119	0.96 995	9.99 752	53
8	9.02 874	118	9.03 124	118	0.96 876	9.99 751	52
9	9.02 992	117	9.03 242	119	0.96 758	9.99 749	51
10	9.03 109	117	9.03 361	118	0.96 639	9.99 748	**50**
11	9.03 226	116	9.03 479	118	0.96 521	9.99 747	49
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48
13	9.03 458	116	9.03 714	118	0.96 286	9.99 744	47
14	9.03 574	116	9.03 832	116	0.96 168	9.99 742	46
15	9.03 690	115	9.03 948	117	0.96 052	9.99 741	45
16	9.03 805̄	115	9.04 065	116	0.95 935̄	9.99 740	44
17	9.03 920	114	9.04 181	116	0.95 819	9.99 738	43
18	9.04 034	115	9.04 297	116	0.95 703	9.99 737	42
19	9.04 149	113	9.04 413	115	0.95 587	9.99 736	41
20	9.04 262	114	9.04 528	115	0.95 472	9.99 734	**40**
21	9.04 376	114	9.04 643	115	0.95 357	9.99 733	39
22	9.04 490	113	9.04 758	115	0.95 242	9.99 731	38
23	9.04 603	112	9.04 873	114	0.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	0.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	0.94 899	9.99 727	35
26	9.04 940	112	9.05 214	114	0.94 786	9.99 726	34
27	9.05 052	112	9.05 328	113	0.94 672	9.99 724	33
28	9.05 164	111	9.05 441	112	0.94 559	9.99 723	32
29	9.05 275̄	111	9.05 553	113	0.94 447	9.99 721	31
30	9.05 386		9.05 666		0.94 334	9.99 720	**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

83° 30′.

PP	121	120	119		118	117	116		115	114	113
.1	12.1	12.0	11.9	.1	11.8	11.7	11.6	.1	11.5	11.4	11.3
.2	24.2	24.0	23.8	.2	23.6	23.4	23.2	.2	23.0	22.8	22.6
.3	36.3	36.0	35.7	.3	35.4	35.1	34.8	.3	34.5	34.2	33.9
.4	48.4	48.0	47.6	.4	47.2	46.8	46.4	.4	46.0	45.6	45.2
.5	60.5	60.0	59.5	.5	59.0	58.5	58.0	.5	57.5	57.0	56.5
.6	72.6	72.0	71.4	.6	70.8	70.2	69.6	.6	69.0	68.4	67.8
.7	84.7	84.0	83.3	.7	82.6	81.9	81.2	.7	80.5	79.8	79.1
.8	96.8	96.0	95.2	.8	94.4	93.6	92.8	.8	92.0	91.2	90.4
.9	108.9	108.0	107.1	.9	106.2	105.3	104.4	.9	103.5	102.6	101.7

6° 30′.

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
30	9.05 386		9.05 666		0.94 334	9.99 720	**30**
31	9.05 497	111	9.05 778	112	0.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	0.94 110	9.99 717	28
33	9.05 717	110	9.06 002	112	0.93 998	9.99 716	27
34	9.05 827	110	9.06 113	111	0.93 887	9.99 714	26
35	9.05 937	110	9.06 224	111	0.93 776	9.99 713	25
36	9.06 046	109	9.06 33$\bar{5}$	111	0.93 665	9.99 711	24
37	9.06 155	109	9.06 445	110	0.93 55$\bar{5}$	9.99 710	23
38	9.06 264	109	9.06 556	111	0.93 444	9.99 708	22
39	9.06 372	108	9.06 666	110	0.93 334	9.99 707	21
40	9.06 481	109	9.06 775	109	0.93 22$\bar{5}$	9.99 705	**20**
41	9.06 589	108	9.06 88$\bar{5}$	110	0.93 115	9.99 704	19
42	9.06 696	107	9.06 994	109	0.93 006	9.99 702	18
43	9.06 804	108	9.07 103	109	0.92 897	9.99 701	17
44	9.06 911	107	9.07 211	108	0.92 789	9.99 699	16
45	9.07 018	107	9.07 320	109	0.92 680	9.99 698	15
46	9.07 124	106	9.07 428	108	0.92 572	9.99 696	14
47	9.07 231	107	9.07 536	108	0.92 464	9.99 69$\bar{5}$	13
48	9.07 337	106	9.07 643	107	0.92 357	9.99 693	12
49	9.07 442	105	9.07 751	108	0.92 249	9.99 692	11
50	9.07 548	106	9.07 858	107	0.92 142	9.99 690	**10**
51	9.07 653	105	9.07 964	106	0.92 036	9.99 689	9
52	9.07 758	105	9.08 071	107	0.91 929	9.99 687	8
53	9.07 863	105	9.08 177	106	0.91 823	9.99 686	7
54	9.07 968	105	9.08 283	106	0.91 717	9.99 684	6
55	9.08 072	104	9.08 389	106	0.91 611	9.99 683	5
56	9.08 176	104	9.08 49$\bar{5}$	106	0.91 505	9.99 681	4
57	9.08 280	104	9.08 600	105	0.91 400	9.99 680	3
58	9.08 383	103	9.08 705	105	0.91 29$\bar{5}$	9.99 678	2
59	9.08 486	103	9.08 810	105	0.91 190	9.99 677	1
60	9.08 589	103	9.08 914	104	0.91 086	9.99 675	**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

83°.

PP	112	111	110		109	108	107		106	105	104
.1	11.2	11.1	11.0	.1	10.9	10.8	10.7	.1	10.6	10.5	10.4
.2	22.4	22.2	22.0	.2	21.8	21.6	21.4	.2	21.2	21.0	20.8
.3	33.6	33.3	33.0	.3	32.7	32.4	32.1	.3	31.8	31.5	31.2
.4	44.8	44.4	44.0	.4	43.6	43.2	42.8	.4	42.4	42.0	41.6
.5	56.0	55.5	55.0	.5	54.5	54.0	53.5	.5	53.0	52.5	52.0
.6	67.2	66.6	66.0	.6	65.4	64.8	64.2	.6	63.6	63.0	62.4
.7	78.4	77.7	77.0	.7	76.3	75.6	74.9	.7	74.2	73.5	72.8
.8	89.6	88.8	88.0	.8	87.2	86.4	85.6	.8	84.8	84.0	83.2
.9	100.8	99.9	99.0	.9	98.1	97.2	96.3	.9	95.4	94.5	93.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
0	9.08 589	103	9.08 914	105	0.91 086	9.99 675	**60**
1	9.08 692	103	9.09 019	104	0.90 981	9.99 674	59
2	9.08 795̄	102	9.09 123	104	0.90 877	9.99 672	58
3	9.08 897	102	9.09 227	103	0.90 773	9.99 670	57
4	9.08 999	102	9.09 330	104	0.90 670	9.99 669	56
5	9.09 101	101	9.09 434	103	0.90 566	9.99 667	55
6	9.09 202	102	9.09 537	103	0.90 463	9.99 666	54
7	9.09 304	101	9.09 640	102	0.90 360	9.99 664	53
8	9.09 405̄	101	9.09 742	103	0.90 258	9.99 663	52
9	9.09 506	100	9.09 845̄	102	0.90 155	9.99 661	51
10	9.09 606	101	9.09 947	102	0.90 053	9.99 659	**50**
11	9.09 707	100	9.10 049	101	0.89 951	9.99 658	49
12	9.09 807	100	9.10 150	102	0.89 85̄0	9.99 656	48
13	9.09 907	99	9.10 252	101	0.89 748	9.99 655̄	47
14	9.10 006	100	9.10 353	101	0.89 647	9.99 653	46
15	9.10 106	99	9.10 454	101	0.89 546	9.99 651	45
16	9.10 205̄	99	9.10 555̄	101	0.89 445	9.99 65̄0	44
17	9.10 304	98	9.10 656	100	0.89 344	9.99 648	43
18	9.10 402	99	9.10 756	100	0.89 244	9.99 647	42
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645̄	41
20	9.10 599	98	9.10 956	100	0.89 044	9.99 643	**40**
21	9.10 697	98	9.11 056	99	0.88 944	9.99 642	39
22	9.10 795	98	9.11 155	99	0.88 845̄	9.99 640	38
23	9.10 893	97	9.11 254	99	0.88 746	9.99 638	37
24	9.10 990	97	9.11 353	99	0.88 647	9.99 637	36
25	9.11 087	97	9.11 452	99	0.88 548	9.99 635	35
26	9.11 184	97	9.11 551	98	0.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	0.88 351	9.99 632	33
28	9.11 377	97	9.11 747	98	0.88 253	9.99 630	32
29	9.11 474	96	9.11 845	98	0.88 155̄	9.99 629	31
30	9.11 570		9.11 943		0.88 057	9.99 627	**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

82° 30′.

PP	105	104	103		102	101	100		99	98	97
.1	10.5	10.4	10.3	.1	10.2	10.1	10.0	.1	9.9	9.8	9.7
.2	21.0	20.8	20.6	.2	20.4	20.2	20.0	.2	19.8	19.6	19.4
.3	31.5	31.2	30.9	.3	30.6	30.3	30.0	.3	29.7	29.4	29.1
.4	42.0	41.6	41.2	.4	40.8	40.4	40.0	.4	39.6	39.2	38.8
.5	52.5	52.0	51.5	.5	51.0	50.5	50.0	.5	49.5	49.0	48.5
.6	63.0	62.4	61.8	.6	61.2	60.6	60.0	.6	59.4	58.8	58.2
.7	73.5	72.8	72.1	.7	71.4	70.7	70.0	.7	69.3	68.6	67.9
.8	84.0	83.2	82.4	.8	81.6	80.8	80.0	.8	79.2	78.4	77.6
.9	94.5	93.6	92.7	.9	91.8	90.9	90.0	.9	89.1	88.2	87.3

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
30	9.11 570	96	9.11 943	97	0.88 057	9.99 627	**30**
31	9.11 666	95	9.12 040	98	0.87 960	9.99 625	29
32	9.11 761	96	9.12 138	97	0.87 862	9.99 624	28
33	9.11 857	95	9.12 235̄	97	0.87 765	9.99 622	27
34	9.11 952	95	9.12 332	96	0.87 668	9.99 620	26
35	9.12 047	95	9.12 428	97	0.87 572	9.99 618	25
36	9.12 142	94	9.12 525̄	96	0.87 475	9.99 617	24
37	9.12 236	95	9.12 621	96	0.87 379	9.99 615	23
38	9.12 331	94	9.12 717	96	0.87 283	9.99 613	22
39	9.12 425̄	94	9.12 813	96	0.87 187	9.99 612	21
40	9.12 519	93	9.12 909	95	0.87 091	9.99 610	**20**
41	9.12 612	94	9.13 004	95	0.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	0.86 901	9.99 607	18
43	9.12 799	93	9.13 194	95	0.86 806	9.99 605̄	17
44	9.12 892	93	9.13 289	95	0.86 711	9.99 603	16
45	9.12 985	93	9.13 384	94	0.86 616	9.99 601	15
46	9.13 078	93	9.13 478	95	0.86 522	9.99 600	14
47	9.13 171	92	9.13 573	94	0.86 427	9.99 598	13
48	9.13 263	92	9.13 667	94	0.86 333	9.99 596	12
49	9.13 355	92	9.13 761	93	0.86 239	9.99 595̄	11
50	9.13 447	92	9.13 854	94	0.86 146	9.99 593	**10**
51	9.13 539	91	9.13 948	93	0.86 052	9.99 591	9
52	9.13 630	92	9.14 041	93	0.85 959	9.99 589	8
53	9.13 722	91	9.14 134	93	0.85 866	9.99 588	7
54	9.13 813	91	9.14 227	93	0.85 773	9.99 586	6
55	9.13 904	90	9.14 320	92	0.85 680	9.99 584	5
56	9.13 994	91	9.14 412	92	0.85 588	9.99 582	4
57	9.14 085	90	9.14 504	93	0.85 496	9.99 581	3
58	9.14 175	91	9.14 597	91	0.85 403	9.99 579	2
59	9.14 266	90	9.14 688	92	0.85 312	9.99 577	1
60	9.14 356		9.14 780		0.85 220	9.99 575	**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

82°.

PP	97	96	95		94	93	92		91	90
.1	9.7	9.6	9.5	.1	9.4	9.3	9.2	.1	9.1	9.0
.2	19.4	19.2	19.0	.2	18.8	18.6	18.4	.2	18.2	18.0
.3	29.1	28.8	28.5	.3	28.2	27.9	27.6	.3	27.3	27.0
.4	38.8	38.4	38.0	.4	37.6	37.2	36.8	.4	36.4	36.0
.5	48.5	48.0	47.5	.5	47.0	46.5	46.0	.5	45.5	45.0
.6	58.2	57.6	57.0	.6	56.4	55.8	55.2	.6	54.6	54.0
.7	67.9	67.2	66.5	.7	65.8	65.1	64.4	.7	63.7	63.0
.8	77.6	76.8	76.0	8	75.2	74.4	73.6	.8	72.8	72.0
.9	87.3	86.4	85.5	.9	84.6	83.7	82.8	.9	81.9	81.0

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
0	9.14 356	89	9.14 780	92	0.85 220	9.99 575	60
1	9.14 445	90	9.14 872	91	0.85 128	9.99 574	59
2	9.14 535	89	9.14 963	91	0.85 037	9.99 572	58
3	9.14 624	90	9.15 054	91	0.84 946	9.99 570	57
4	9.14 714	89	9.15 145	91	0.84 855	9.99 568	56
5	9.14 803	88	9.15 236	91	0.84 764	9.99 566	55
6	9.14 891	89	9.15 327	90	0.84 673	9.99 565	54
7	9.14 980	89	9.15 417	91	0.84 583	9.99 563	53
8	9.15 069	88	9.15 508	90	0.84 492	9.99 561	52
9	9.15 157	88	9.15 598	90	0.84 402	9.99 559	51
10	9.15 245	88	9.15 688	89	0.84 312	9.99 557	50
11	9.15 333	88	9.15 777	90	0.84 223	9.99 556	49
12	9.15 421	87	9.15 867	89	0.84 133	9.99 554	48
13	9.15 508	88	9.15 956	90	0.84 044	9.99 552	47
14	9.15 596	87	9.16 046	89	0.83 954	9.99 550	46
15	9.15 683	87	9.16 135	89	0.83 865	9.99 548	45
16	9.15 770	87	9.16 224	88	0.83 776	9.99 546	44
17	9.15 857	87	9.16 312	89	0.83 688	9.99 545	43
18	9.15 944	86	9.16 401	88	0.83 599	9.99 543	42
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41
20	9.16 116	87	9.16 577	88	0.83 423	9.99 539	40
21	9.16 203	86	9.16 665	88	0.83 335	9.99 537	39
22	9.16 289	85	9.16 753	88	0.83 247	9.99 535	38
23	9.16 374	86	9.16 841	87	0.83 159	9.99 533	37
24	9.16 460	85	9.16 928	88	0.83 072	9.99 532	36
25	9.16 545	86	9.17 016	87	0.82 984	9.99 530	35
26	9.16 631	85	9.17 103	87	0.82 897	9.99 528	34
27	9.16 716	85	9.17 190	87	0.82 810	9.99 526	33
28	9.16 801	85	9.17 277	86	0.82 723	9.99 524	32
29	9.16 886	84	9.17 363	87	0.82 637	9.99 522	31
30	9.16 970		9.17 450		0.82 550	9.99 520	30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

81° 30′.

PP	92	91	90		89	88		87	86
.1	9.2	9.1	9.0	.1	8.9	8.8	.1	8.7	8.6
.2	18.4	18.2	18.0	.2	17.8	17.6	.2	17.4	17.2
.3	27.6	27.3	27.0	.3	26.7	26.4	.3	26.1	25.8
.4	36.8	36.4	36.0	.4	35.6	35.2	.4	34.8	34.4
.5	46.0	45.5	45.0	.5	44.5	44.0	.5	43.5	43.0
.6	55.2	54.6	54.0	.6	53.4	52.8	.6	52.2	51.6
.7	64.4	63.7	63.0	.7	62.3	61.6	.7	60.9	60.2
.8	73.6	72.8	72.0	.8	71.2	70.4	.8	69.6	68.8
.9	82.8	81.9	81.0	.9	80.1	79.2	.9	78.3	77.4

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
30	9.16 970		9.17 450		0.82 550	9.99 520	**30**
31	9.17 055	85	9.17 536	86	0.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	0.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	0.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	0.82 206	9.99 513	26
35	9.17 391	84	9.17 880	86	0.82 120	9.99 511	25
36	9.17 474	83	9.17 965	85	0.82 035	9.99 509	24
37	9.17 558	84	9.18 051	86	0.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21
40	9.17 807	83	9.18 306	85	0.81 694	9.99 501	**20**
41	9.17 890	83	9.18 391	85	0.81 609	9.99 499	19
42	9.17 973	83	9.18 475	84	0.81 525	9.99 497	18
43	9.18 055	82	9.18 560	85	0.81 440	9.99 495	17
44	9.18 137	82	9.18 644	84	0.81 356	9.99 494	16
45	9.18 220	83	9.18 728	84	0.81 272	9.99 492	15
46	9.18 302	82	9.18 812	84	0.81 188	9.99 490	14
47	9.18 383	81	9.18 896	84	0.81 104	9.99 488	13
48	9.18 465	82	9.18 979	83	0.81 021	9.99 486	12
49	9.18 547	82	9.19 063	84	0.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	**10**
51	9.18 709	81	9.19 229	83	0.80 771	9.99 480	9
52	9.18 790	81	9.19 312	83	0.80 688	9.99 478	8
53	9.18 871	81	9.19 395	83	0.80 605	9.99 476	7
54	9.18 952	81	9.19 478	83	0.80 522	9.99 474	6
55	9.19 033	81	9.19 561	83	0.80 439	9.99 472	5
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	0.80 275	9.99 468	3
58	9.19 273	80	9.19 807	82	0.80 193	9.99 466	2
59	9.19 353	80	9.19 889	82	0.80 111	9.99 464	1
60	9.19 433	80	9.19 971	82	0.80 029	9.99 462	**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

81°.

PP	86	85	84		83	82		81	80
.1	8.6	8.5	8.4	.1	8.3	8.2	.1	8.1	8.0
.2	17.2	17.0	16.8	.2	16.6	16.4	.2	16.2	16.0
.3	25.8	25.5	25.2	.3	24.9	24.6	.3	24.3	24.0
.4	34.4	34.0	33.6	.4	33.2	32.8	.4	32.4	32.0
.5	43.0	42.5	42.0	.5	41.5	41.0	.5	40.5	40.0
.6	51.6	51.0	50.4	.6	49.8	49.2	.6	48.6	48.0
.7	60.2	59.5	58.8	.7	58.1	57.4	.7	56.7	56.0
.8	68.8	68.0	67.2	.8	66.4	65.6	.8	64.8	64.0
.9	77.4	76.5	75.6	.9	74.7	73.8	.9	72.9	72.0

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
0	9.19 433	80	9.19 971	82	0.80 029	9.99 462	**60**
1	9.19 513	79	9.20 053	81	0.79 947	9.99 460	59
2	9.19 592	80	9.20 134	82	0.79 866	9.99 458	58
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	55
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54
7	9.19 988	79	9.20 540	81	0.79 460	9.99 448	53
8	9.20 067	78	9.20 621	80	0.79 379	9.99 446	52
9	9.20 145	78	9.20 701	81	0.79 299	9.99 444	51
10	9.20 223	79	9.20 782	80	0.79 218	9.99 442	**50**
11	9.20 302	78	9.20 862	80	0.79 138	9.99 440	49
12	9.20 380	78	9.20 942	80	0.79 058	9.99 438	48
13	9.20 458	77	9.21 022	80	0.78 978	9.99 436	47
14	9.20 535	78	9.21 102	80	0.78 898	9.99 434	46
15	9.20 613	78	9.21 182	79	0.78 818	9.99 432	45
16	9.20 691	77	9.21 261	80	0.78 739	9.99 429	44
17	9.20 768	77	9.21 341	79	0.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	**40**
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39
22	9.21 153	76	9.21 736	78	0.78 264	9.99 417	38
23	9.21 229	77	9.21 814	79	0.78 186	9.99 415	37
24	9.21 306	76	9.21 893	78	0.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	35
26	9.21 458	76	9.22 049	78	0.77 951	9.99 409	34
27	9.21 534	76	9.22 127	78	0.77 873	9.99 407	33
28	9.21 610	75	9.22 205	78	0.77 795	9.99 404	32
29	9.21 685	76	9.22 283	78	0.77 717	9.99 402	31
30	9.21 761		9.22 361		0.77 639	9.99 400	**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

80° 30′.

PP	82	81	80		79	78		77	76
.1	8.2	8.1	8.0	.1	7.9	7.8	.1	7.7	7.6
.2	16.4	16.2	16.0	.2	15.8	15.6	.2	15.4	15.2
.3	24.6	24.3	24.0	.3	23.7	23.4	.3	23.1	22.8
.4	32.8	32.4	32.0	.4	31.6	31.2	.4	30.8	30.4
.5	41.0	40.5	40.0	.5	39.5	39.0	.5	38.5	38.0
.6	49.2	48.6	48.0	.6	47.4	46.8	.6	46.2	45.6
.7	57.4	56.7	56.0	.7	55.3	54.6	.7	53.9	53.2
.8	65.6	64.8	64.0	.8	63.2	62.4	.8	61.6	60.8
.9	73.8	72.9	72.0	.9	71.1	70.2	.9	69.3	68.4

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	
30	9.21 761	75	9.22 361	77	0.77 639	9.99 400	**30**
31	9.21 836	76	9.22 438	78	0.77 562	9.99 398	29
32	9.21 912	75	9.22 516	77	0.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26
35	9.22 137	74	9.22 747	77	0.77 253	9.99 390	25
36	9.22 211	75	9.22 824	77	0.77 176	9.99 388	24
37	9.22 286	75	9.22 901	76	0.77 099	9.99 385	23
38	9.22 361	74	9.22 977	77	0.77 023	9.99 383	22
39	9.22 435	74	9.23 054	76	0.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	**20**
41	9.22 583	74	9.23 206	77	0.76 794	9.99 377	19
42	9.22 657	74	9.23 283	76	0.76 717	9.99 375	18
43	9.22 731	74	9.23 359	76	0.76 641	9.99 372	17
44	9.22 805	73	9.23 435	75	0.76 565	9.99 370	16
45	9.22 878	74	9.23 510	76	0.76 490	9.99 368	15
46	9.22 952	73	9.23 586	75	0.76 414	9.99 366	14
47	9.23 025	73	9.23 661	76	0.76 339	9.99 364	13
48	9.23 098	73	9.23 737	75	0.76 263	9.99 362	12
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11
50	9.23 244	73	9.23 887	75	0.76 113	9.99 357	**10**
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9
52	9.23 390	72	9.24 037	75	0.75 963	9.99 353	8
53	9.23 462	73	9.24 112	74	0.75 888	9.99 351	7
54	9.23 535	72	9.24 186	75	0.75 814	9.99 348	6
55	9.23 607	72	9.24 261	74	0.75 739	9.99 346	5
56	9.23 679	73	9.24 335	75	0.75 665	9.99 344	4
57	9.23 752	71	9.24 410	74	0.75 590	9.99 342	3
58	9.23 823	72	9.24 484	74	0.75 516	9.99 340	2
59	9.23 895	72	9.24 558	74	0.75 442	9.99 337	1
60	9.23 967		9.24 632		0.75 368	9.99 335	**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	′

80°.

PP	77	76	75		74	73		72	71
.1	7.7	7.6	7.5	.1	7.4	7.3	.1	7.2	7.1
.2	15.4	15.2	15.0	.2	14.8	14.6	.2	14.4	14.2
.3	23.1	22.8	22.5	.3	22.2	21.9	.3	21.6	21.3
.4	30.8	30.4	30.0	.4	29.6	29.2	.4	28.8	28.4
.5	38.5	38.0	37.5	.5	37.0	36.5	.5	36.0	35.5
.6	46.2	45.6	45.0	.6	44.4	43.8	.6	43.2	42.6
.7	53.9	53.2	52.5	.7	51.8	51.1	.7	50.4	49.7
.8	61.6	60.8	60.0	.8	59.2	58.4	.8	57.6	56.8
.9	69.3	68.4	67.5	.9	66.6	65.7	.9	64.8	63.9

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.23 967	72	9.24 632	74	0.75 368	9.99 335	2	60
1	9.24 039	71	9.24 706	73	0.75 294	9.99 333	2	59
2	9.24 110	71	9.24 779	74	0.75 221	9.99 331	3	58
3	9.24 181	72	9.24 853	73	0.75 147	9.99 328	2	57
4	9.24 253	71	9.24 926	74	0.75 074	9.99 326	2	56
5	9.24 324	71	9.25 000	73	0.75 000	9.99 324	2	55
6	9.24 395	71	9.25 073	73	0.74 927	9.99 322	3	54
7	9.24 466	70	9.25 146	73	0.74 854	9.99 319	2	53
8	9.24 536	71	9.25 219	73	0.74 781	9.99 317	2	52
9	9.24 607	70	9.25 292	73	0.74 708	9.99 315	2	51
10	9.24 677	71	9.25 365	72	0.74 635	9.99 313	3	50
11	9.24 748	70	9.25 437	73	0.74 563	9.99 310	2	49
12	9.24 818	70	9.25 510	72	0.74 490	9.99 308	2	48
13	9.24 888	70	9.25 582	73	0.74 418	9.99 306	2	47
14	9.24 958	70	9.25 655	72	0.74 345	9.99 304	3	46
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	2	45
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	2	44
17	9.25 168	69	9.25 871	72	0.74 129	9.99 297	3	43
18	9.25 237	70	9.25 943	72	0.74 057	9.99 294	2	42
19	9.25 307	69	9.26 015	71	0.73 985	9.99 292	2	41
20	9.25 376	69	9.26 086	72	0.73 914	9.99 290	2	40
21	9.25 445	69	9.26 158	71	0.73 842	9.99 288	3	39
22	9.25 514	69	9.26 229	72	0.73 771	9.99 285	2	38
23	9.25 583	69	9.26 301	71	0.73 699	9.99 283	2	37
24	9.25 652	69	9.26 372	71	0.73 628	9.99 281	3	36
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	2	35
26	9.25 790	68	9.26 514	71	0.73 486	9.99 276	2	34
27	9.25 858	69	9.26 585	70	0.73 415	9.99 274	3	33
28	9.25 927	68	9.26 655	71	0.73 345	9.99 271	2	32
29	9.25 995	68	9.26 726	71	0.73 274	9.99 269	2	31
30	9.26 063		9.26 797		0.73 203	9.99 267		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

79° 30′.

PP	74	73	72		71	70	69		68	3
.1	7.4	7.3	7.2	.1	7.1	7.0	6.9	.1	6.8	0.3
.2	14.8	14.6	14.4	.2	14.2	14.0	13.8	.2	13.6	0.6
.3	22.2	21.9	21.6	.3	21.3	21.0	20.7	.3	20.4	0.9
.4	29.6	29.2	28.8	.4	28.4	28.0	27.6	.4	27.2	1.2
.5	37.0	36.5	36.0	.5	35.5	35.0	34.5	.5	34.0	1.5
.6	44.4	43.8	43.2	.6	42.6	42.0	41.4	.6	40.8	1.8
.7	51.8	51.1	50.4	.7	49.7	49.0	48.3	.7	47.6	2.1
.8	59.2	58.4	57.6	.8	56.8	56.0	55.2	.8	54.4	2.4
.9	66.6	65.7	64.8	.9	63.9	63.0	62.1	.9	61.2	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.26 063	68	9.26 797	70	0.73 203	9.99 267	3	**30**
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	2	29
32	9.26 199	68	9.26 937	71	0.73 063	9.99 262	2	28
33	9.26 267	68	9.27 008	70	0.72 992	9.99 260	3	27
34	9.26 335	68	9.27 078	70	0.72 922	9.99 257	2	26
35	9.26 403	67	9.27 148	70	0.72 852	9.99 255̄	3	25
36	9.26 470	68	9.27 218	70	0.72 782	9.99 252	2	24
37	9.26 538	67	9.27 288	69	0.72 712	9.99 250	2	23
38	9.26 605	67	9.27 357	70	0.72 643	9.99 248	3	22
39	9.26 672	67	9.27 427	69	0.72 573	9.99 245	2	21
40	9.26 739	67	9.27 496	70	0.72 504	9.99 243	2	**20**
41	9.26 806	67	9.27 566	69	0.72 434	9.99 241	3	19
42	9.26 873	67	9.27 635	69	0.72 365̄	9.99 238	2	18
43	9.26 940	67	9.27 704	69	0.72 296	9.99 236	3	17
44	9.27 007	66	9.27 773	69	0.72 227	9.99 233	2	16
45	9.27 073	67	9.27 842	69	0.72 158	9.99 231	2	15
46	9.27 140	66	9.27 911	69	0.72 089	9.99 229	3	14
47	9.27 206	67	9.27 980	69	0.72 020	9.99 226	2	13
48	9.27 273	66	9.28 049	68	0.71 951	9.99 224	3	12
49	9.27 339	66	9.28 117	69	0.71 883	9.99 221	2	11
50	9.27 405̄	66	9.28 186	68	0.71 814	9.99 219	2	**10**
51	9.27 471	66	9.28 254	69	0.71 746	9.99 217	3	9
52	9.27 537	65	9.28 323	68	0.71 677	9.99 214	2	8
53	9.27 602	66	9.28 391	68	0.71 609	9.99 212	3	7
54	9.27 668	66	9.28 459	68	0.71 541	9.99 209	2	6
55	9.27 734	65	9.28 527	68	0.71 473	9.99 207	3	5
56	9.27 799	65	9.28 595̄	67	0.71 405	9.99 204	2	4
57	9.27 864	66	9.28 662	68	0.71 338	9.99 202	2	3
58	9.27 930	65	9.28 730	68	0.71 270	9.99 200	3	2
59	9.27 995̄	65	9.28 798	67	0.71 202	9.99 197	2	1
60	9.28 060		9.28 865		0.71 135̄	9.99 195̄		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

79°.

PP	70	69	68		67	66		65	3
.1	7.0	6.9	6.8	.1	6.7	6.6	.1	6.5	0.3
.2	14.0	13.8	13.6	.2	13.4	13.2	.2	13.0	0.6
.3	21.0	20.7	20.4	.3	20.1	19.8	.3	19.5	0.9
.4	28.0	27.6	27.2	.4	26.8	26.4	.4	26.0	1.2
.5	35.0	34.5	34.0	.5	33.5	33.0	.5	32.5	1.5
.6	42.0	41.4	40.8	.6	40.2	39.6	.6	39.0	1.8
.7	49.0	48.3	47.6	.7	46.9	46.2	.7	45.5	2.1
.8	56.0	55.2	54.4	.8	53.6	52.8	.8	52.0	2.4
.9	63.0	62.1	61.2	.9	60.3	59.4	.9	58.5	2.7

11°.

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.28 060	65	9.28 865	68	0.71 135̄	9.99 195̄	3	**60**
1	9.28 125̄	65	9.28 933	67	0.71 067	9.99 192	2	59
2	9.28 190	64	9.29 000	67	0.71 000	9.99 190	3	58
3	9.28 254	65	9.29 067	67	0.70 933	9.99 187	2	57
4	9.28 319	65	9.29 134	67	0.70 866	9.99 185̄	3	56
5	9.28 384	64	9.29 201	67	0.70 799	9.99 182	2	55
6	9.28 448	64	9.29 268	67	0.70 732	9.99 180	3	54
7	9.28 512	65	9.29 335̄	67	0.70 665	9.99 177	2	53
8	9.28 577	64	9.29 402	66	0.70 598	9.99 175̄	3	52
9	9.28 641	64	9.29 468	67	0.70 532	9.99 172	2	51
10	9.28 705̄	64	9.29 535̄	66	0.70 465	9.99 170	3	**50**
11	9.28 769	64	9.29 601	67	0.70 399	9.99 167	2	49
12	9.28 833	63	9.29 668	66	0.70 332	9.99 165̄	3	48
13	9.28 896	64	9.29 734	66	0.70 266	9.99 162	2	47
14	9.28 960	64	9.29 800	66	0.70 200	9.99 160	3	46
15	9.29 024	63	9.29 866	66	0.70 134	9.99 157	2	45
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155̄	3	44
17	9.29 150	64	9.29 998	66	0.70 002	9.99 152	2	43
18	9.29 214	63	9.30 064	66	0.69 936	9.99 150	3	42
19	9.29 277	63	9.30 130	65	0.69 870	9.99 147	2	41
20	9.29 340	63	9.30 195	66	0.69 805̄	9.99 145̄	3	**40**
21	9.29 403	63	9.30 261	65	0.69 739	9.99 142	2	39
22	9.29 466	63	9.30 326	65	0.69 674	9.99 140	3	38
23	9.29 529	62	9.30 391	66	0.69 609	9.99 137	2	37
24	9.29 591	63	9.30 457	65	0.69 543	9.99 135̄	3	36
25	9.29 654	62	9.30 522	65	0.69 478	9.99 132	2	35
26	9.29 716	63	9.30 587	65	0.69 413	9.99 130	3	34
27	9.29 779	62	9.30 652	65	0.69 348	9.99 127	3	33
28	9.29 841	62	9.30 717	65	0.69 283	9.99 124	2	32
29	9.29 903	63	9.30 782	64	0.69 218	9.99 122	3	31
30	9.29 966		9.30 846		0.69 154	9.99 119		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

78° 30′.

PP	68	67	66		65	64	63		62	3
.1	6.8	6.7	6.6	.1	6.5	6.4	6.3	.1	6.2	.3
.2	13.6	13.4	13.2	.2	13.0	12.8	12.6	.2	12.4	.6
.3	20.4	20.1	19.8	.3	19.5	19.2	18.9	.3	18.6	.9
.4	27.2	26.8	26.4	.4	26.0	25.6	25.2	.4	24.8	1.2
.5	34.0	33.5	33.0	.5	32.5	32.0	31.5	.5	31.0	1.5
.6	40.8	40.2	39.6	.6	39.0	38.4	37.8	.6	37.2	1.8
.7	47.6	46.9	46.2	.7	45.5	44.8	44.1	.7	43.4	2.1
.8	54.4	53.6	52.8	.8	52.0	51.2	50.4	.8	49.6	2.4
.9	61.2	60.3	59.4	.9	58.5	57.6	56.7	.9	55.8	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.29 966	62	9.30 846	65	0.69 154	9.99 119	2	**30**
31	9.30 028	62	9.30 911	64	0.69 089	9.99 117	3	29
32	9.30 090	61	9.30 975	65	0.69 025	9.99 114	2	28
33	9.30 151	62	9.31 040	64	0.68 960	9.99 112	3	27
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	3	26
35	9.30 275	61	9.31 168	65	0.68 832	9.99 106	2	25
36	9.30 336	62	9.31 233	64	0.68 767	9.99 104	3	24
37	9.30 398	61	9.31 297	64	0.68 703	9.99 101	2	23
38	9.30 459	62	9.31 361	64	0.68 639	9.99 099	3	22
39	9.30 521	61	9.31 425	64	0.68 575	9.99 096	3	21
40	9.30 582	61	9.31 489	63	0.68 511	9.99 093	2	**20**
41	9.30 643	61	9.31 552	64	0.68 448	9.99 091	3	19
42	9.30 704	61	9.31 616	63	0.68 384	9.99 088	2	18
43	9.30 765	61	9.31 679	64	0.68 321	9.99 086	3	17
44	9.30 826	61	9.31 743	63	0.68 257	9.99 083	3	16
45	9.30 887	60	9.31 806	64	0.68 194	9.99 080	2	15
46	9.30 947	61	9.31 870	63	0.68 130	9.99 078	3	14
47	9.31 008	60	9.31 933	63	0.68 067	9.99 075	3	13
48	9.31 068	61	9.31 996	63	0.68 004	9.99 072	2	12
49	9.31 129	60	9.32 059	63	0.67 941	9.99 070	3	11
50	9.31 189	61	9.32 122	63	0.67 878	9.99 067	3	**10**
51	9.31 250	60	9.32 185	63	0.67 815	9.99 064	2	9
52	9.31 310	60	9.32 248	63	0.67 752	9.99 062	3	8
53	9.31 370	60	9.32 311	62	0.67 689	9.99 059	3	7
54	9.31 430	60	9.32 373	63	0.67 627	9.99 056	2	6
55	9.31 490	59	9.32 436	62	0.67 564	9.99 054	3	5
56	9.31 549	60	9.32 498	63	0.67 502	9.99 051	3	4
57	9.31 609	60	9.32 561	62	0.67 439	9.99 048	2	3
58	9.31 669	59	9.32 623	62	0.67 377	9.99 046	3	2
59	9.31 728	60	9.32 685	62	0.67 315	9.99 043	3	1
60	9.31 788		9.32 747		0.67 253	9.99 040		**0**
	L. Cos.	**d.**	**L. Cotg.**	**d.**	**L. Tang.**	**L. Sin.**	**d.**	′

78°.

PP	65	64	63		62	61	60		59	3
.1	6.5	6.4	6.3	.1	6.2	6.1	6.0	.1	5.9	0.3
.2	13.0	12.8	12.6	.2	12.4	12.2	12.0	.2	11.8	0.6
.3	19.5	19.2	18.9	.3	18.6	18.3	18.0	.3	17.7	0.9
.4	26.0	25.6	25.2	.4	24.8	24.4	24.0	.4	23.6	1.2
.5	32.5	32.0	31.5	.5	31.0	30.5	30.0	.5	29.5	1.5
.6	39.0	38.4	37.8	.6	37.2	36.6	36.0	.6	35.4	1.8
.7	45.5	44.8	44.1	.7	43.4	42.7	42.0	.7	41.3	2.1
.8	52.0	51.2	50.4	.8	49.6	48.8	48.0	.8	47.2	2.4
.9	58.5	57.6	56.7	.9	55.8	54.9	54.0	.9	53.1	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.31 788	59	9.32 747	63	0.67 253	9.99 040	2	**60**
1	9.31 847	60	9.32 810	62	0.67 190	9.99 038	3	59
2	9.31 907	59	9.32 872	61	0.67 128	9.99 035	3	58
3	9.31 966	59	9.32 933	62	0.67 067	9.99 032	2	57
4	9.32 025	59	9.32 995	62	0.67 005	9.99 030	3	56
5	9.32 084	59	9.33 057	62	0.66 943	9.99 027	3	55
6	9.32 143	59	9.33 119	61	0.66 881	9.99 024	2	54
7	9.32 202	59	9.33 180	62	0.66 820	9.99 022	3	53
8	9.32 261	58	9.33 242	61	0.66 758	9.99 019	3	52
9	9.32 319	59	9.33 303	62	0.66 697	9.99 016	3	51
10	9.32 378	59	9.33 365	61	0.66 635	9.99 013	2	**50**
11	9.32 437	58	9.33 426	61	0.66 574	9.99 011	3	49
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	3	48
13	9.32 553	59	9.33 548	61	0.66 452	9.99 005	3	47
14	9.32 612	58	9.33 609	61	0.66 391	9.99 002	2	46
15	9.32 670	58	9.33 670	61	0.66 330	9.99 000	3	45
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	3	44
17	9.32 786	58	9.33 792	61	0.66 208	9.98 994	3	43
18	9.32 844	58	9.33 853	60	0.66 147	9.98 991	2	42
19	9.32 902	58	9.33 913	61	0.66 087	9.98 989	3	41
20	9.32 960	58	9.33 974	60	0.66 026	9.98 986	3	**40**
21	9.33 018	57	9.34 034	61	0.65 966	9.98 983	3	39
22	9.33 075	58	9.34 095	60	0.65 905	9.98 980	2	38
23	9.33 133	57	9.34 155	60	0.65 845	9.98 978	3	37
24	9.33 190	58	9.34 215	61	0.65 785	9.98 975	3	36
25	9.33 248	57	9.34 276	60	0.65 724	9.98 972	3	35
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	2	34
27	9.33 362	58	9.34 396	60	0.65 604	9.98 967	3	33
28	9.33 420	57	9.34 456	60	0.65 544	9.98 964	3	32
29	9.33 477	57	9.34 516	60	0.65 484	9.98 961	3	31
30	9.33 534		9.34 576		0.65 424	9.98 958		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

PP	63	62	61		60	59	58		57	3
.1	6.3	6.2	6.1	.1	6.0	5.9	5.8	.1	5.7	0.3
.2	12.6	12.4	12.2	.2	12.0	11.8	11.6	.2	11.4	0.6
.3	18.9	18.6	18.3	.3	18.0	17.7	17.4	.3	17.1	0.9
.4	25.2	24.8	24.4	.4	24.0	23.6	23.2	.4	22.8	1.2
.5	31.5	31.0	30.5	.5	30.0	29.5	29.0	.5	28.5	1.5
.6	37.8	37.2	36.6	.6	36.0	35.4	34.8	.6	34.2	1.8
.7	44.1	43.4	42.7	.7	42.0	41.3	40.6	.7	39.9	2.1
.8	50.4	49.6	48.8	.8	48.0	47.2	46.4	.8	45.6	2.4
.9	56.7	55.8	54.9	.9	54.0	53.1	52.2	.9	51.3	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.33 534	57	9.34 576	59	0.65 424	9.98 958	3	**30**
31	9.33 591	56	9.34 635	60	0.65 365	9.98 955	2	29
32	9.33 647	57	9.34 695	60	0.65 305	9.98 953	3	28
33	9.33 704	57	9.34 755	59	0.65 245	9.98 950	3	27
34	9.33 761	57	9.34 814	60	0.65 186	9.98 947	3	26
35	9.33 818	56	9.34 874	59	0.65 126	9.98 944	3	25
36	9.33 874	57	9.34 933	59	0.65 067	9.98 941	3	24
37	9.33 931	56	9.34 992	59	0.65 008	9.98 938	2	23
38	9.33 987	56	9.35 051	60	0.64 949	9.98 936	3	22
39	9.34 043	57	9.35 111	59	0.64 889	9.98 933	3	21
40	9.34 100	56	9.36 170	59	0.64 830	9.98 930	3	**20**
41	9.34 156	56	9.35 229	59	0.64 771	9.98 927	3	19
42	9.34 212	56	9.35 288	59	0.64 712	9.98 924	3	18
43	9.34 268	56	9.35 347	58	0.64 653	9.98 921	2	17
44	9.34 324	56	9.35 405	59	0.64 595	9.98 919	3	16
45	9.34 380	56	9.35 464	59	0.64 536	9.98 916	3	15
46	9.34 436	55	9.35 523	58	0.64 477	9.98 913	3	14
47	9.34 491	56	9.35 581	59	0.64 419	9.98 910	3	13
48	9.34 547	55	9.35 640	58	0.64 360	9.98 907	3	12
49	9.34 602	56	9.35 698	59	0.64 302	9.98 904	3	11
50	9.34 658	55	9.35 757	58	0.64 243	9.98 901	3	**10**
51	9.34 713	56	9.35 815	58	0.64 185	9.98 898	2	9
52	9.34 769	55	9.35 873	58	0.64 127	9.98 896	3	8
53	9.34 824	55	9.35 931	58	0.64 069	9.98 893	3	7
54	9.34 879	55	9.35 989	58	0.64 011	9.98 890	3	6
55	9.34 934	55	9.36 047	58	0.63 953	9.98 887	3	5
56	9.34 989	55	9.36 105	58	0.63 895	9.98 884	3	4
57	9.35 044	55	9.36 163	58	0.63 837	9.98 881	3	3
58	9.35 099	55	9.36 221	58	0.63 779	9.98 878	3	2
59	9.35 154	55	9.36 279	57	0.63 721	9.98 875	3	1
60	9.35 209		9.36 336		0.63 664	9.98 872		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

77°.

PP	60	59	58		57	56		55	3
.1	6.0	5.9	5.8	.1	5.7	5.6	.1	5.5	0.3
.2	12.0	11.8	11.6	.2	11.4	11.2	.2	11.0	0.6
.3	18.0	17.7	17.4	.3	17.1	16.8	.3	16.5	0.9
.4	24.0	23.6	23.2	.4	22.8	22.4	.4	22.0	1.2
.5	30.0	29.5	29.0	.5	28.5	28.0	.5	27.5	1.5
.6	36.0	35.4	34.8	.6	34.2	33.6	.6	33.0	1.8
.7	42.0	41.3	40.6	.7	39.9	39.2	.7	38.5	2.1
.8	48.0	47.2	46.4	.8	45.6	44.8	.8	44.0	2.4
.9	54.0	53.1	52.2	.9	51.3	50.4	.9	49.5	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.35 209	54	9.36 336	58	0.63 664	9.98 872	3	**60**
1	9.35 263	55	9.36 394	58	0.63 606	9.98 869	2	59
2	9.35 318	55	9.36 452	57	0.63 548	9.98 867	3	58
3	9.35 373	54	9.36 509	57	0.63 491	9.98 864	3	57
4	9.35 427	54	9.36 566	58	0.63 434	9.98 861	3	56
5	9.35 481	55	9.36 624	57	0.63 376	9.98 858	3	55
6	9.35 536	54	9.36 681	57	0.63 319	9.98 855	3	54
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	3	53
8	9.35 644	54	9.36 795	57	0.63 205	9.98 849	3	52
9	9.35 698	54	9.36 852	57	0.63 148	9.98 846	3	51
10	9.35 752	54	9.36 909	57	0.63 091	9.98 843	3	**50**
11	9.35 806	54	9.36 966	57	0.63 034	9.98 840	3	49
12	9.35 860	54	9.37 023	57	0.62 977	9.98 837	3	48
13	9.35 914	54	9.37 080	57	0.62 920	9.98 834	3	47
14	9.35 968	54	9.37 137	56	0.62 863	9.98 831	3	46
15	9.36 022	53	9.37 193	57	0.62 807	9.98 828	3	45
16	9.36 075	54	9.37 250	56	0.62 750	9.98 825	3	44
17	9.36 129	53	9.37 306	57	0.62 694	9.98 822	3	43
18	9.36 182	54	9.37 363	56	0.62 637	9.98 819	3	42
19	9.36 236	53	9.37 419	57	0.62 581	9.98 816	3	41
20	9.36 289	53	9.37 476	56	0.62 524	9.98 813	3	**40**
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	3	39
22	9.36 395	54	9.37 588	56	0.62 412	9.98 807	3	38
23	9.36 449	53	9.37 644	56	0.62 356	9.98 804	3	37
24	9.36 502	53	9.37 700	56	0.62 300	9.98 801	3	36
25	9.36 555	53	9.37 756	56	0.62 244	9.98 798	3	35
26	9.36 608	52	9.37 812	56	0.62 188	9.98 795	3	34
27	9.36 660	53	9.37 868	56	0.62 132	9.98 792	3	33
28	9.36 713	53	9.37 924	56	0.62 076	9.98 789	3	32
29	9.36 766	53	9.37 980	55	0.62 020	9.98 786	3	31
30	9.36 819		9.38 035		0.61 965	9.98 783		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

76° 30′.

PP	58	57	56		55	54		53	3
.1	5.8	5.7	5.6	.1	5.5	5.4	.1	5.3	.3
.2	11.6	11.4	11.2	.2	11.0	10.8	.2	10.6	.6
.3	17.4	17.1	16.8	.3	16.5	16.2	.3	15.9	.9
.4	23.2	22.8	22.4	.4	22.0	21.6	.4	21.2	1.2
.5	29.0	28.5	28.0	.5	27.5	27.0	.5	26.5	1.5
.6	34.8	34.2	33.6	.6	33.0	32.4	.6	31.8	1.8
.7	40.6	39.9	39.2	.7	38.5	37.8	.7	37.1	2.1
.8	46.4	45.6	44.8	.8	44.0	43.2	.8	42.4	2.4
.9	52.2	51.3	50.4	.9	49.5	48.6	.9	47.7	2.7

13-27

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.36 819	52	9.38 035	56	0.61 965	9.98 783	3	**30**
31	9.36 871	53	9.38 091	56	0.61 909	9.98 780	3	29
32	9.36 924	52	9.38 147	55	0.61 853	9.98 777	3	28
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	3	27
34	9.37 028	53	9.38 257	56	0.61 743	9.98 771	3	26
35	9.37 081	52	9.38 313	55	0.61 687	9.98 768	3	25
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	3	24
37	9.37 185	52	9.38 423	56	0.61 577	9.98 762	3	23
38	9.37 237	52	9.38 479	55	0.61 521	9.98 759	3	22
39	9.37 289	52	9.38 534	55	0.61 466	9.98 756	3	21
40	9.37 341	52	9.38 589	55	0.61 411	9.98 753	3	**20**
41	9.37 393	52	9.38 644	55	0.61 356	9.98 750	4	19
42	9.37 445	52	9.38 699	55	0.61 301	9.98 746	3	18
43	9.37 497	52	9.38 754	54	0.61 246	9.98 743	3	17
44	9.37 549	51	9.38 808	55	0.61 192	9.98 740	3	16
45	9.37 600	52	9.38 863	55	0.61 137	9.98 737	3	15
46	9.37 652	51	9.38 918	54	0.61 082	9.98 734	3	14
47	9.37 703	52	9.38 972	55	0.61 028	9.98 731	3	13
48	9.37 755	51	9.39 027	55	0.60 973	9.98 728	3	12
49	9.37 806	52	9.39 082	54	0.60 918	9.98 725	3	11
50	9.37 858	51	9.39 136	54	0.60 864	9.98 722	3	**10**
51	9.37 909	51	9.39 190	55	0.60 810	9.98 719	4	9
52	9.37 960	51	9.39 245	54	0.60 755	9.98 715	3	8
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	3	7
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	3	6
55	9.38 113	51	9.39 407	54	0.60 593	9.98 706	3	5
56	9.38 164	51	9.39 461	54	0.60 539	9.98 703	3	4
57	9.38 215	51	9.39 515	54	0.60 485	9.98 700	3	3
58	9.38 266	51	9.39 569	54	0.60 431	9.98 697	3	2
59	9.38 317	51	9.39 623	54	0.60 377	9.98 694	4	1
60	9.38 368		9.39 677		0.60 323	9.98 690		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

76°.

PP	56	55	54		53	52	51		4	3
.1	5.6	5.5	5.4	.1	5.3	5.2	5.1	.1	0.4	0.3
.2	11.2	11.0	10.8	.2	10.6	10.4	10.2	.2	0.8	0.6
.3	16.8	16.5	16.2	.3	15.9	15.6	15.3	.3	1.2	0.9
.4	22.4	22.0	21.6	.4	21.2	20.8	20.4	.4	1.6	1.2
.5	28.0	27.5	27.0	.5	26.5	26.0	25.5	.5	2.0	1.5
.6	33.6	33.0	32.4	.6	31.8	31.2	30.6	.6	2.4	1.8
.7	39.2	38.5	37.8	.7	37.1	36.4	35.7	.7	2.8	2.1
.8	44.8	44.0	43.2	.8	42.4	41.6	40.8	.8	3.2	2.4
.9	50.4	49.5	48.6	.9	47.7	46.8	45.9	.9	3.6	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.38 368		9.39 677		0.60 323	9.98 690		60
1	9.38 418	50	9.39 731	54	0.60 269	9.98 687	3	59
2	9.38 469	51	9.39 785̄	54	0.60 215	9.98 684	3	58
3	9.38 519	50	9.39 838	53	0.60 162	9.98 681	3	57
4	9.38 570	51	9.39 892	54	0.60 108	9.98 678	3	56
5	9.38 620	50	9.39 945	53	0.60 055̄	9.98 675̄	3	55
6	9.38 670	50	9.39 999	54	0.60 001	9.98 671	4	54
7	9.38 721	51	9.40 052	53	0.59 948	9.98 668	3	53
8	9.38 771	50	9.40 106	54	0.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	53	0.59 788	9.98 659	3	50
11	9.38 921	50	9.40 266	54	0.59 734	9.98 656	3	49
12	9.38 971	50	9.40 319	53	0.59 681	9.98 652	4	48
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47
14	9.39 071	50	9.40 425̄	53	0.59 575	9.98 646	3	46
15	9.39 121	50	9.40 478	53	0.59 522	9.98 643	3	45
16	9.39 170	49	9.40 531	53	0.59 469	9.98 640	3	44
17	9.39 220	50	9.40 584	53	0.59 416	9.98 636	4	43
18	9.39 270	50	9.40 636	52	0.59 364	9.98 633	3	42
19	9.39 319	49	9.40 689	53	0.59 311	9.98 630	3	41
20	9.39 369	50	9.40 742	53	0.59 258	9.98 627	3	40
21	9.39 418	49	9.40 795̄	53	0.59 205	9.98 623	4	39
22	9.39 467	49	9.40 847	52	0.59 153	9.98 620	3	38
23	9.39 517	50	9.40 900	53	0.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	52	0.59 048	9.98 614	3	36
25	9.39 615̄	49	9.41 005̄	53	0.58 995	9.98 610	4	35
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	52	0.58 839	9.98 601	3	32
29	9.39 811	49	9.41 214	53	0.58 786	9.98 597	4	31
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

75° 30′.

PP	54	53	52		51	50	49		4	3
.1	5.4	5.3	5.2	.1	5.1	5.0	4.9	.1	.4	.3
.2	10.8	10.6	10.4	.2	10.2	10.0	9.8	.2	.8	.6
.3	16.2	15.9	15.6	.3	15.3	15.0	14.7	.3	1.2	.9
.4	21.6	21.2	20.8	.4	20.4	20.0	19.6	.4	1.6	1.2
.5	27.0	26.5	26.0	.5	25.5	25.0	24.5	.5	2.0	1.5
.6	32.4	31.8	31.2	.6	30.6	30.0	29.4	.6	2.4	1.8
.7	37.8	37.1	36.4	.7	35.7	35.0	34.3	.7	2.8	2.1
.8	43.2	42.4	41.6	.8	40.8	40.0	39.2	.8	3.2	2.4
.9	48.6	47.7	46.8	.9	45.9	45.0	44.1	.9	3.6	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	**30**
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29
32	9.39 958	48	9.41 370	52	0.58 630	9.98 588	4	28
33	9.40 006	49	9.41 422	52	0.58 578	9.98 584	3	27
34	9.40 055	48	9.41 474	52	0.58 526	9.98 581	3	26
35	9.40 103	49	9.41 526	52	0.58 474	9.98 578	4	25
36	9.40 152	48	9.41 578	51	0.58 422	9.98 574	3	24
37	9.40 200	49	9.41 629	52	0.58 371	9.98 571	3	23
38	9.40 249	48	9.41 681	52	0.58 319	9.98 568	3	22
39	9.40 297	49	9.41 733	51	0.58 267	9.98 565	4	21
40	9.40 346	48	9.41 784	52	0.58 216	9.98 561	3	**20**
41	9.40 394	48	9.41 836	51	0.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	52	0.58 113	9.98 555	4	18
43	9.40 490	48	9.41 939	51	0.58 061	9.98 551	3	17
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	52	0.57 959	9.98 545	4	15
46	9.40 634	48	9.42 093	51	0.57 907	9.98 541	3	14
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13
48	9.40 730	48	9.42 195	51	0.57 805	9.98 535	4	12
49	9.40 778	47	9.42 246	51	0.57 754	9.98 531	3	11
50	9.40 825	48	9.42 297	51	0.57 703	9.98 528	3	**10**
51	9.40 873	48	9.42 348	51	0.57 652	9.98 525	4	9
52	9.40 921	47	9.42 399	51	0.57 601	9.98 521	3	8
53	9.40 968	48	9.42 450	51	0.57 550	9.98 518	3	7
54	9.41 016	47	9.42 501	51	0.57 499	9.98 515	4	6
55	9.41 063	48	9.42 552	51	0.57 448	9.98 511	3	5
56	9.41 111	47	9.42 603	50	0.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	51	0.57 347	9.98 505	4	3
58	9.41 205	47	9.42 704	51	0.57 296	9.98 501	3	2
59	9.41 252	48	9.42 755	50	0.57 245	9.98 498	4	1
60	9.41 300		9.42 805		0.57 195	9.98 494		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

75°.

PP	52	51	50		49	48	47		4	3
.1	5.2	5.1	5.0	.1	4.9	4.8	4.7	.1	0.4	0.3
.2	10.4	10.2	10.0	.2	9.8	9.6	9.4	.2	0.8	0.6
.3	15.6	15.3	15.0	.3	14.7	14.4	14.1	.3	1.2	0.9
.4	20.8	20.4	20.0	.4	19.6	19.2	18.8	.4	1.6	1.2
.5	26.0	25.5	25.0	.5	24.5	24.0	23.5	.5	2.0	1.5
.6	31.2	30.6	30.0	.6	29.4	28.8	28.2	.6	2.4	1.8
.7	36.4	35.7	35.0	.7	34.3	33.6	32.9	.7	2.8	2.1
.8	41.6	40.8	40.0	.8	39.2	38.4	37.6	.8	3.2	2.4
.9	46.8	45.9	45.0	.9	44.1	43.2	42.3	.9	3.6	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.41 300		9.42 805		0.57 195	9.98 494		60
1	9.41 347	47	9.42 856	51	0.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	50	0.57 094	9.98 488	3	58
3	9.41 441	47	9.42 957	51	0.57 043	9.98 484	4	57
4	9.41 488	47	9.43 007	50	0.56 993	9.98 481	3	56
5	9.41 535	47	9.43 057	50	0.56 943	9.98 477	4	55
6	9.41 582	47	9.43 108	51	0.56 892	9.98 474	3	54
7	9.41 628	46	9.43 158	50	0.56 842	9.98 471	3	53
8	9.41 675	47	9.43 208	50	0.56 792	9.98 467	4	52
9	9.41 722	47	9.43 258	50	0.56 742	9.98 464	3	51
10	9.41 768	46	9.43 308	50	0.56 692	9.98 460	4	50
11	9.41 815	47	9.43 358	50	0.56 642	9.98 457	3	49
12	9.41 861	46	9.43 408	50	0.56 592	9.98 453	4	48
13	9.41 908	47	9.43 458	50	0.56 542	9.98 450	3	47
14	9.41 954	46	9.43 508	50	0.56 492	9.98 447	3	46
15	9.42 001	47	9.43 558	50	0.56 442	9.98 443	4	45
16	9.42 047	46	9.43 607	49	0.56 393	9.98 440	3	44
17	9.42 093	46	9.43 657	50	0.56 343	9.98 436	4	43
18	9.42 140	47	9.43 707	50	0.56 293	9.98 433	3	42
19	9.42 186	46	9.43 756	49	0.56 244	9.98 429	4	41
20	9.42 232	46	9.43 806	50	0.56 194	9.98 426	3	40
21	9.42 278	46	9.43 855	49	0.56 145	9.98 422	4	39
22	9.42 324	46	9.43 905	50	0.56 095	9.98 419	3	38
23	9.42 370	46	9.43 954	49	0.56 046	9.98 415	4	37
24	9.42 416	46	9.44 004	50	0.55 996	9.98 412	3	36
25	9.42 461	45	9.44 053	49	0.55 947	9.98 409	3	35
26	9.42 507	46	9.44 102	49	0.55 898	9.98 405	4	34
27	9.42 553	46	9.44 151	49	0.55 849	9.98 402	3	33
28	9.42 599	46	9.44 201	50	0.55 799	9.98 398	4	32
29	9.42 644	45	9.44 250	49	0.55 750	9.98 395	3	31
30	9.42 690	46	9.44 299	49	0.55 701	9.98 391	4	30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

74° 30′.

PP	51	50	49		48	47	46		45	4	3
.1	5.1	5.0	4.9	.1	4.8	4.7	4.6	.1	4.5	0.4	0.3
.2	10.2	10.0	9.8	.2	9.6	9.4	9.2	.2	9.0	0.8	0.6
.3	15.3	15.0	14.7	.3	14.4	14.1	13.8	.3	13.5	1.2	0.9
.4	20.4	20.0	19.6	.4	19.2	18.8	18.4	.4	18.0	1.6	1.2
.5	25.5	25.0	24.5	.5	24.0	23.5	23.0	.5	22.5	2.0	1.5
.6	30.6	30.0	29.4	.6	28.8	28.2	27.6	.6	27.0	2.4	1.8
.7	35.7	35.0	34.3	.7	33.6	32.9	32.2	.7	31.5	2.8	2.1
.8	40.8	40.0	39.2	.8	38.4	37.6	36.8	.8	36.0	3.2	2.4
.9	45.9	45.0	44.1	.9	43.2	42.3	41.4	.9	40.5	3.6	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.42 690	45	9.44 299	49	0.55 701	9.98 391	3	30
31	9.42 735	46	9.44 348	49	0.55 652	9.98 388	4	29
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	3	28
33	9.42 826	46	9.44 446	49	0.55 554	9.98 381	4	27
34	9.42 872	45	9.44 495	49	0.55 505	9.98 377	4	26
35	9.42 917	45	9.44 544	48	0.55 456	9.98 373	3	25
36	9.42 962	46	9.44 592	49	0.55 408	9.98 370	4	24
37	9.43 008	45	9.44 641	49	0.55 359	9.98 366	3	23
38	9.43 053	45	9.44 690	48	0.55 310	9.98 363	4	22
39	9.43 098	45	9.44 738	49	0.55 262	9.98 359	3	21
40	9.43 143	45	9.44 787	49	0.55 213	9.98 356	4	20
41	9.43 188	45	9.44 836	48	0.55 164	9.98 352	3	19
42	9.43 233	45	9.44 884	49	0.55 116	9.98 349	4	18
43	9.43 278	45	9.44 933	48	0.55 067	9.98 345	3	17
44	9.43 323	44	9.44 981	48	0.55 019	9.98 342	4	16
45	9.43 367	45	9.45 029	49	0.54 971	9.98 338	4	15
46	9.43 412	45	9.45 078	48	0.54 922	9.98 334	3	14
47	9.43 457	45	9.45 126	48	0.54 874	9.98 331	4	13
48	9.43 502	44	9.45 174	48	0.54 826	9.98 327	3	12
49	9.43 546	45	9.45 222	49	0.54 778	9.98 324	4	11
50	9.43 591	44	9.45 271	48	0.54 729	9.98 320	3	10
51	9.43 635	45	9.45 319	48	0.54 681	9.98 317	4	9
52	9.43 680	44	9.45 367	48	0.54 633	9.98 313	4	8
53	9.43 724	45	9.45 415	48	0.54 585	9.98 309	3	7
54	9.43 769	44	9.45 463	48	0.54 537	9.98 306	4	6
55	9.43 813	44	9.45 511	48	0.54 489	9.98 302	3	5
56	9.43 857	44	9.45 559	47	0.54 441	9.98 299	4	4
57	9.43 901	45	9.45 606	48	0.54 394	9.98 295	4	3
58	9.43 946	44	9.45 654	48	0.54 346	9.98 291	3	2
59	9.43 990	44	9.45 702	48	0.54 298	9.98 288	4	1
60	9.44 034		9.45 750		0.54 250	9.98 284		0
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

74°.

PP	49	48	47		46	45	44		4	3
.1	4.9	4.8	4.7	.1	4.6	4.5	4.4	.1	0.4	0.3
.2	9.8	9.6	9.4	.2	9.2	9.0	8.8	.2	0.8	0.6
.3	14.7	14.4	14.1	.3	13.8	13.5	13.2	.3	1.2	0.9
.4	19.6	19.2	18.8	.4	18.4	18.0	17.6	.4	1.6	1.2
.5	24.5	24.0	23.5	.5	23.0	22.5	22.0	.5	2.0	1.5
.6	29.4	28.8	28.2	.6	27.6	27.0	26.4	.6	2.4	1.8
.7	34.3	33.6	32.9	.7	32.2	31.5	30.8	.7	2.8	2.1
.8	39.2	38.4	37.6	.8	36.8	36.0	35.2	.8	3.2	2.4
.9	44.1	43.2	42.3	.9	41.4	40.5	39.6	.9	3.6	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.44 034		9.45 750		0.54 250	9.98 284		60
1	9.44 078	44	9.45 797	47	0.54 203	9.98 281	3	59
2	9.44 122	44	9.45 845	48	0.54 155	9.98 277	4	58
3	9.44 166	44	9.45 892	47	0.54 108	9.98 273	4	57
4	9.44 210	44	9.45 940	48	0.54 060	9.98 270	3	56
5	9.44 253	43	9.45 987	47	0.54 013	9.98 266	4	55
6	9.44 297	44	9.46 035	48	0.53 965	9.98 262	4	54
7	9.44 341	44	9.46 082	47	0.53 918	9.98 259	3	53
8	9.44 385	44	9.46 130	48	0.53 870	9.98 255	4	52
9	9.44 428	43	9.46 177	47	0.53 823	9.98 251	4	51
10	9.44 472	44	9.46 224	47	0.53 776	9.98 248	3	50
11	9.44 516	44	9.46 271	47	0.53 729	9.98 244	4	49
12	9.44 559	43	9.46 319	48	0.53 681	9.98 240	4	48
13	9.44 602	43	9.46 366	47	0.53 634	9.98 237	3	47
14	9.44 646	44	9.46 413	47	0.53 587	9.98 233	4	46
15	9.44 689	43	9.46 460	47	0.53 540	9.98 229	4	45
16	9.44 733	44	9.46 507	47	0.53 493	9.98 226	3	44
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	4	43
18	9.44 819	43	9.46 601	47	0.53 399	9.98 218	4	42
19	9.44 862	43	9.46 648	47	0.53 352	9.98 215	3	41
20	9.44 905	43	9.46 694	46	0.53 306	9.98 211	4	40
21	9.44 948	43	9.46 741	47	0.53 259	9.98 207	4	39
22	9.44 992	44	9.46 788	47	0.53 212	9.98 204	3	38
23	9.45 035	43	9.46 835	47	0.53 165	9.98 200	4	37
24	9.45 077	42	9.46 881	46	0.53 119	9.98 196	4	36
25	9.45 120	43	9.46 928	47	0.53 072	9.98 192	4	35
26	9.45 163	43	9.46 975	47	0.53 025	9.98 189	3	34
27	9.45 206	43	9.47 021	46	0.52 979	9.98 185	4	33
28	9.45 249	43	9.47 068	47	0.52 932	9.98 181	4	32
29	9.45 292	43	9.47 114	46	0.52 886	9.98 177	4	31
30	9.45 334	42	9.47 160	46	0.52 840	9.98 174	3	30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

73° 30′.

PP	48	47	46		45	44	43		42	4	3
.1	4.8	4.7	4.6	.1	4.5	4.4	4.3	.1	4.2	0.4	0.3
.2	9.6	9.4	9.2	.2	9.0	8.8	8.6	.2	8.4	0.8	0.6
.3	14.4	14.1	13.8	.3	13.5	13.2	12.9	.3	12.6	1.2	0.9
.4	19.2	18.8	18.4	.4	18.0	17.6	17.2	.4	16.8	1.6	1.2
.5	24.0	23.5	23.0	.5	22.5	22.0	21.5	.5	21.0	2.0	1.5
.6	28.8	28.2	27.6	.6	27.0	26.4	25.8	.6	25.2	2.4	1.8
.7	33.6	32.9	32.2	.7	31.5	30.8	30.1	.7	29.4	2.8	2.1
.8	38.4	37.6	36.8	.8	36.0	35.2	34.4	.8	33.6	3.2	2.4
.9	43.2	42.3	41.4	.9	40.5	39.6	38.7	.9	37.8	3.6	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.45 344	43	9.47 160	47	0.52 840	9.98 174	4	30
31	9.45 377	42	9.47 207	46	0.52 793	9.98 170	4	29
32	9.45 419	43	9.47 253	46	0.52 747	9.98 166	4	28
33	9.45 462	42	9.47 299	47	0.52 701	9.98 162	3	27
34	9.45 504	43	9.47 346	46	0.52 654	9.98 159	4	26
35	9.45 547	42	9.47 392	46	0.52 608	9.98 155	4	25
36	9.45 589	43	9.47 438	46	0.52 562	9.98 151	4	24
37	9.45 632	42	9.47 484	46	0.52 516	9.98 147	3	23
38	9.45 674	42	9.47 530	46	0.52 470	9.98 144	4	22
39	9.45 716	42	9.47 576	46	0.52 424	9.98 140	4	21
40	9.45 758	43	9.47 622	46	0.52 378	9.98 136	4	20
41	9.45 801	42	9.47 668	46	0.52 332	9.98 132	3	19
42	9.45 843	42	9.47 714	46	0.52 286	9.98 129	4	18
43	9.45 885	42	9.47 760	46	0.52 240	9.98 125	4	17
44	9.45 927	42	9.47 806	46	0.52 194	9.98 121	4	16
45	9.45 969	42	9.47 852	45	0.52 148	9.98 117	4	15
46	9.46 011	42	9.47 897	46	0.52 103	9.98 113	3	14
47	9.46 053	42	9.47 943	46	0.52 057	9.98 110	4	13
48	9.46 095	41	9.47 989	46	0.52 011	9.98 106	4	12
49	9.46 136	42	9.48 035	45	0.51 965	9.98 102	4	11
50	9.46 178	42	9.48 080	46	0.51 920	9.98 098	4	10
51	9.46 220	42	9.48 126	45	0.51 874	9.98 094	4	9
52	9.46 262	41	9.48 171	46	0.51 829	9.98 090	3	8
53	9.46 303	42	9.48 217	45	0.51 783	9.98 087	4	7
54	9.46 345	41	9.48 262	45	0.51 738	9.98 083	4	6
55	9.46 386	42	9.48 307	46	0.51 693	9.98 079	4	5
56	9.46 428	41	9.48 353	45	0.51 647	9.98 075	4	4
57	9.46 469	42	9.48 398	45	0.51 602	9.98 071	4	3
58	9.46 511	41	9.48 443	46	0.51 557	9.98 067	4	2
59	9.46 552	42	9.48 489	45	0.51 511	9.98 063	3	1
60	9.46 594		9.48 534		0.51 466	9.98 060		0
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

73°.

PP	47	46	45		44	43	42		41	4	3
.1	4.7	4.6	4.5	.1	4.4	4.3	4.2	.1	4.1	0.4	0.3
.2	9.4	9.2	9.0	.2	8.8	8.6	8.4	.2	8.2	0.8	0.6
.3	14.1	13.8	13.5	.3	13.2	12.9	12.6	.3	12.3	1.2	0.9
.4	18.8	18.4	18.0	.4	17.6	17.2	16.8	.4	16.4	1.6	1.2
.5	23.5	23.0	22.5	.5	22.0	21.5	21.0	.5	20.5	2.0	1.5
.6	28.2	27.6	27.0	.6	26.4	25.8	25.2	.6	24.6	2.4	1.8
.7	32.9	32.2	31.5	.7	30.8	30.1	29.4	.7	28.7	2.8	2.1
.8	37.6	36.8	36.0	.8	35.2	34.4	33.6	.8	32.8	3.2	2.4
.9	42.3	41.4	40.5	.9	39.6	38.7	37.8	.9	36.9	3.6	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.46 594	41	9.48 534	45	0.51 466	9.98 060	4	60
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	0.51 331	9.98 048	4	57
4	9.46 758	42	9.48 714	45	0.51 286	9.98 044	4	56
5	9.46 800	41	9.48 759	45	0.51 241	9.98 040	4	55
6	9.46 841	41	9.48 804	45	0.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	0.51 151	9.98 032	3	53
8	9.46 923	41	9.48 894	45	0.51 106	9.98 029	4	52
9	9.46 964	41	9.48 939	45	0.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	0.51 016	9.98 021	4	50
11	9.47 045	41	9.49 029	44	0.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	0.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	0.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	44	0.50 837	9.98 005	4	46
15	9.47 209	40	9.49 207	45	0.50 793	9.98 001	4	45
16	9.47 249	41	9.49 252	44	0.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	45	0.50 704	9.97 993	4	43
18	9.47 330	41	9.49 341	44	0.50 659	9.97 989	3	42
19	9.47 371	40	9.49 385	45	0.50 615	9.97 986	4	41
20	9.47 411	41	9.49 430	44	0.50 570	9.97 982	4	40
21	9.47 452	40	9.49 474	45	0.50 526	9.97 978	4	39
22	9.47 492	41	9.49 519	44	0.50 481	9.97 974	4	38
23	9.47 533	40	9.49 563	44	0.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	45	0.50 393	9.97 966	4	36
25	9.47 613	41	9.49 652	44	0.50 348	9.97 962	4	35
26	9.47 654	40	9.49 696	44	0.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31
30	9.47 814		9.49 872		0.50 128	9.97 942		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

72° 30′.

PP	45	44	43
.1	4.5	4.4	4.3
.2	9.0	8.8	8.6
.3	13.5	13.2	12.9
.4	18.0	17.6	17.2
.5	22.5	22.0	21.5
.6	27.0	26.4	25.8
.7	31.5	30.8	30.1
.8	36.0	35.2	34.4
.9	40.5	39.6	38.7

	42	41	40
.1	4.2	4.1	4.0
.2	8.4	8.2	8.0
.3	12.6	12.3	12.0
.4	16.8	16.4	16.0
.5	21.0	20.5	20.0
.6	25.2	24.6	24.0
.7	29.4	28.7	28.0
.8	33.6	32.8	32.0
.9	37.8	36.9	36.0

	4	3
.1	0.4	0.3
.2	0.8	0.6
.3	1.2	0.9
.4	1.6	1.2
.5	2.0	1.5
.6	2.4	1.8
.7	2.8	2.1
.8	3.2	2.4
.9	3.6	2.7

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.47 814		9.49 872		0.50 128	9.97 942		30
31	9.47 854	40	9.49 916	44	0.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	0.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24
37	9.48 094	40	9.50 180	44	0.49 820	9.97 914	4	23
38	9.48 133	39	9.50 223	43	0.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21
40	9.48 213	40	9.50 311	44	0.49 689	9.97 902	4	20
41	9.48 252	39	9.50 355	44	0.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	43	0.49 602	9.97 894	4	18
43	9.48 332	40	9.50 442	44	0.49 558	9.97 890	4	17
44	9.48 371	39	9.50 485	43	0.49 515	9.97 886	4	16
45	9.48 411	40	9.50 529	44	0.49 471	9.97 882	4	15
46	9.48 450	39	9.50 572	43	0.49 428	9.97 878	4	14
47	9.48 490	40	9.50 616	44	0.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	43	0.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	44	0.49 297	9.97 866	4	11
50	9.48 607	39	9.50 746	43	0.49 254	9.97 861	5	10
51	9.48 647	40	9.50 789	43	0.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	44	0.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	0.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	43	0.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	44	0.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	0.48 865	9.97 825	4	1
60	9.48 998	39	9.51 178	43	0.48 822	9.97 821	4	0
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

72°.

PP	44	43	42		41	40	39		5	4
1	4.4	4.3	4.2	.1	4.1	4.0	3.9	.1	0.5	0.4
.2	8.8	8.6	8.4	.2	8.2	8.0	7.8	.2	1.0	0.8
.3	13.2	12.9	12.6	.3	12.3	12.0	11.7	.3	1.5	1.2
.4	17.6	17.2	16.8	.4	16.4	16.0	15.6	.4	2.0	1.6
.5	22.0	21.5	21.0	.5	20.5	20.0	19.5	.5	2.5	2.0
.6	26.4	25.8	25.2	.6	24.6	24.0	23.4	.6	3.0	2.4
.7	30.8	30.1	29.4	.7	28.7	28.0	27.3	.7	3.5	2.8
.8	35.2	34.4	33.6	.8	32.8	32.0	31.2	.8	4.0	3.2
.9	39.6	38.7	37.8	.9	36.9	36.0	35.1	.9	4.5	3.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.48 998	39	9.51 178	43	0.48 822	9.97 821	4	**60**
1	9.49 037	39	9.51 221	43	0.48 779	9.97 817	5	59
2	9.49 076	39	9.51 264	42	0.48 736	9.97 812	4	58
3	9.49 115	38	9.51 306	43	0.48 694	9.97 808	4	57
4	9.49 153	39	9.51 349	43	0.48 651	9.97 804	4	56
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	55
6	9.49 231	38	9.51 435	43	0.48 565	9.97 796	4	54
7	9.49 269	39	9.51 478	42	0.48 522	9.97 792	4	53
8	9.49 308	39	9.51 520	43	0.48 480	9.97 788	4	52
9	9.49 347	38	9.51 563	43	0.48 437	9.97 784	5	51
10	9.49 385	39	9.51 606	42	0.48 394	9.97 779	4	**50**
11	9.49 424	38	9.51 648	43	0.48 352	9.97 775	4	49
12	9.49 462	38	9.51 691	43	0.48 309	9.97 771	4	48
13	9.49 500	39	9.51 734	42	0.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	43	0.48 224	9.97 763	4	46
15	9.49 577	38	9.51 819	42	0.48 181	9.97 759	5	45
16	9.49 615	39	9.51 861	42	0.48 139	9.97 754	4	44
17	9.49 654	38	9.51 903	43	0.48 097	9.97 750	4	43
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	43	0.48 012	9.97 742	4	41
20	9.49 768	38	9.52 031	42	0.47 969	9.97 738	4	**40**
21	9.49 806	38	9.52 073	42	0.47 927	9.97 734	5	39
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	43	0.47 843	9.97 725	4	37
24	9.49 920	38	9.52 200	42	0.47 800	9.97 721	4	36
25	9.49 958	38	9.52 242	42	0.47 758	9.97 717	4	35
26	9.49 996	38	9.52 284	42	0.47 716	9.97 713	5	34
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32
29	9.50 110	38	9.52 410	42	0.47 590	9.97 700	4	31
30	9.50 148		9.52 452		0.47 548	9.97 696		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

71° 30′.

PP	43	42		39	38		5	4
.1	4.3	4.2	.1	3.9	3.8	.1	0.5	0.4
.2	8.6	8.4	.2	7.8	7.6	.2	1.0	0.8
.3	12.9	12.6	.3	11.7	11.4	.3	1.5	1.2
.4	17.2	16.8	.4	15.6	15.2	.4	2.0	1.6
.5	21.5	21.0	.5	19.5	19.0	.5	2.5	2.0
.6	25.8	25.2	.6	23.4	22.8	.6	3.0	2.4
.7	30.1	29.4	.7	27.3	26.6	.7	3.5	2.8
.8	34.4	33.6	.8	31.2	30.4	.8	4.0	3.2
.9	38.7	37.8	.9	35.1	34.2	.9	4.5	3.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.50 148	37	9.52 452	42	0.47 548	9.97 696	5	**30**
31	9.50 185	38	9.52 494	42	0.47 506	9.97 691	4	29
32	9.50 223	38	9.52 536	42	0.47 464	9.97 687	4	28
33	9.50 261	37	9.52 578	42	0.47 422	9.97 683	4	27
34	9.50 298	38	9.52 620	41	0.47 380	9.97 679	5	26
35	9.50 336	38	9.52 661	42	0.47 339	9.97 674	4	25
36	9.50 374	37	9.52 703	42	0.47 297	9.97 670	4	24
37	9.50 411	38	9.52 745	42	0.47 25$\bar{5}$	9.97 666	4	23
38	9.50 449	37	9.52 787	42	0.47 213	9.97 662	5	22
39	9.50 486	37	9.52 829	41	0.47 171	9.97 657	4	21
40	9.50 523	38	9.52 870	42	0.47 130	9.97 653	4	**20**
41	9.50 561	37	9.52 912	41	0.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	42	0.47 047	9.97 64$\bar{5}$	5	18
43	9.50 635	38	9.52 995	42	0.47 00$\bar{5}$	9.97 640	4	17
44	9.50 673	37	9.53 037	41	0.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	42	0.46 922	9.97 632	4	15
46	9.50 747	37	9.53 120	41	0.46 880	9.97 628	5	14
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	42	0.46 798	9.97 619	4	12
49	9.50 858	38	9.53 244	41	0.46 756	9.97 61$\bar{5}$	5	11
50	9.50 896	37	9.53 285	42	0.46 71$\bar{5}$	9.97 610	4	**10**
51	9.50 933	37	9.53 327	41	0.46 673	9.97 606	4	9
52	9.50 970	37	9.53 368	41	0.46 632	9.97 602	5	8
53	9.51 007	36	9.53 409	41	0.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	42	0.46 5$\bar{5}$0	9.97 593	4	6
55	9.51 080	37	9.53 492	41	0.46 508	9.97 589	5	5
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	4	4
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3
58	9.51 191	36	9.53 615	41	0.46 38$\bar{5}$	9.97 576	5	2
59	9.51 227	37	9.53 656	41	0.46 344	9.97 571	4	1
60	9.51 264		9.53 697		0.46 303	9.97 567		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

71°.

PP	42	41	38		37	36		5	4
.1	4.2	4.1	3.8	.1	3.7	3.6	.1	0.5	0.4
.2	8.4	8.2	7.6	.2	7.4	7.2	.2	1.0	0.8
.3	12.6	12.3	11.4	.3	11.1	10.8	.3	1.5	1.2
.4	16.8	16.4	15.2	.4	14.8	14.4	.4	2.0	1.6
.5	21.0	20.5	19.0	.5	18.5	18.0	.5	2.5	2.0
.6	25.2	24.6	22.8	.6	22.2	21.6	.6	3.0	2.4
.7	29.4	28.7	26.6	.7	25.9	25.2	.7	3.5	2.8
.8	33.6	32.8	30.4	.8	29.6	28.8	.8	4.0	3.2
.9	37.8	36.9	34.2	.9	33.3	32.4	.9	4.5	3.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	**60**
1	9.51 301	37	9.53 738	41	0.46 262	9.97 563	5	59
2	9.51 338	36	9.53 779	41	0.46 221	9.97 558	4	58
3	9.51 374	37	9.53 820	41	0.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	0.46 139	9.97 55̄0	5	56
5	9.51 447	37	9.53 902	41	0.46 098	9.97 545	4	55
6	9.51 484	36	9.53 943	41	0.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	0.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025̄	40	0.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	0.45 935̄	9.97 528	5	51
10	9.51 629	37	9.54 106	41	0.45 894	9.97 523	4	**50**
11	9.51 666	36	9.54 147	40	0.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	41	0.45 813	9.97 515̄	5	48
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	4	47
14	9.51 774	37	9.54 269	40	0.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	0.45 691	9.97 501	4	45
16	9.51 847	36	9.54 35̄0	40	0.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	0.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	0.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	**40**
21	9.52 027	36	9.54 552	41	0.45 448	9.97 475̄	5	39
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	5	37
24	9.52 135̄	36	9.54 673	41	0.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	0.45 286	9.97 457	4	35
26	9.52 207	35	9.54 754	40	0.45 246	9.97 453	5	34
27	9.52 242	36	9.54 794	41	0.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835̄	40	0.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875̄	40	0.45 125	9.97 439	4	31
30	9.52 35̄0		9.54 915̄		0.45 085	9.97 435̄		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

70° 30′.

PP	41	40	37		36	35		5	4
.1	4.1	4.0	3.7	.1	3.6	3.5	.1	0.5	0.4
.2	8.2	8.0	7.4	.2	7.2	7.0	.2	1.0	0.8
.3	12.3	12.0	11.1	.3	10.8	10.5	.3	1.5	1.2
.4	16.4	16.0	14.8	.4	14.4	14.0	.4	2.0	1.6
.5	20.5	20.0	18.5	.5	18.0	17.5	.5	2.5	2.0
.6	24.6	24.0	22.2	.6	21.6	21.0	.6	3.0	2.4
.7	28.7	28.0	25.9	.7	25.2	24.5	.7	3.5	2.8
.8	32.8	32.0	29.6	.8	28.8	28.0	.8	4.0	3.2
.9	36.9	36.0	33.3	.9	32.4	31.5	.9	4.5	3.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.52 35̄0	35	9.54 915̄	40	0.45 085	9.97 435̄	5	**30**
31	9.52 385	36	9.54 955	40	0.45 045̄	9.97 430	4	29
32	9.52 421	35	9.54 995	40	0.45 005̄	9.97 426	5	28
33	9.52 456	36	9.55 035	40	0.44 965̄	9.97 421	4	27
34	9.52 492	35	9.55 075	40	0.44 925̄	9.97 417	5	26
35	9.52 527	36	9.55 115	40	0.44 885̄	9.97 412	4	25
36	9.52 563	35	9.55 155	40	0.44 845̄	9.97 408	5	24
37	9.52 598	36	9.55 195	40	0.44 805̄	9.97 403	4	23
38	9.52 634	35	9.55 235	40	0.44 765̄	9.97 399	5	22
39	9.52 669	36	9.55 275	40	0.44 725̄	9.97 394	4	21
40	9.52 705̄	35	9.55 315̄	40	0.44 685	9.97 390	5	**20**
41	9.52 740	35	9.55 355̄	40	0.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395̄	39	0.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	0.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	0.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	4	15
46	9.52 916	35	9.55 554	39	0.44 446	9.97 363	5	14
47	9.52 951	35	9.55 593	40	0.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	0.44 367	9.97 353	4	12
49	9.53 021	35	9.55 673	39	0.44 327	9.97 349	5	11
50	9.53 056	36	9.55 712	40	0.44 288	9.97 344	4	**10**
51	9.53 092	34	9.55 752	39	0.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	40	0.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	39	0.44 169	9.97 331	5	7
54	9.53 196	35	9.55 870	40	0.44 130	9.97 326	4	6
55	9.53 231	35	9.55 910	39	0.44 090	9.97 322	5	5
56	9.53 266	35	9.55 949	40	0.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	0.44 011	9.97 312	4	3
58	9.53 336	34	9.56 028	39	0.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	0.43 933	9.97 303	4	1
60	9.53 405		9.56 107		0.43 893	9.97 299		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

70°.

PP	40	39	36		35	34		5	4
.1	4.0	3.9	3.6	.1	3.5	3.4	.1	0.5	0.4
.2	8.0	7.8	7.2	.2	7.0	6.8	.2	1.0	0.8
.3	12.0	11.7	10.8	.3	10.5	10.2	.3	1.5	1.2
.4	16.0	15.6	14.4	.4	14.0	13.6	.4	2.0	1.6
.5	20.0	19.5	18.0	.5	17.5	17.0	.5	2.5	2.0
.6	24.0	23.4	21.6	.6	21.0	20.4	.6	3.0	2.4
.7	28.0	27.3	25.2	.7	24.5	23.8	.7	3.5	2.8
.8	32.0	31.2	28.8	.8	28.0	27.2	.8	4.0	3.2
.9	36.0	35.1	32.4	.9	31.5	30.6	.9	4.5	3.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.53 405	35	9.56 107	39	0.43 893	9.97 299	5	60
1	9.53 440	35	9.56 146	39	0.43 854	9.97 294	5	59
2	9.53 475	34	9.56 185	39	0.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	40	0.43 776	9.97 285	5	57
4	9.53 544	34	9.56 264	39	0.43 736	9.97 280	4	56
5	9.53 578	35	9.56 303	39	0.43 697	9.97 276	5	55
6	9.53 613	34	9.56 342	39	0.43 658	9.97 271	5	54
7	9.53 647	35	9.56 381	39	0.43 619	9.97 266	4	53
8	9.53 682	34	9.56 420	39	0.43 580	9.97 262	5	52
9	9.53 716	35	9.56 459	39	0.43 541	9.97 257	5	51
10	9.53 751	34	9.56 498	39	0.43 502	9.97 252	4	50
11	9.53 785	34	9.56 537	39	0.43 463	9.97 248	5	49
12	9.53 819	35	9.56 576	39	0.43 424	9.97 243	5	48
13	9.53 854	34	9.56 615	39	0.43 385	9.97 238	4	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	5	46
15	9.53 922	35	9.56 693	39	0.43 307	9.97 229	5	45
16	9.53 957	34	9.56 732	39	0.43 268	9.97 224	4	44
17	9.53 991	34	9.56 771	39	0.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	39	0.43 190	9.97 215	5	42
19	9.54 059	34	9.56 849	38	0.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	39	0.43 113	9.97 206	5	40
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	5	39
22	9.54 161	34	9.56 965	39	0.43 035	9.97 196	4	38
23	9.54 195	34	9.57 004	38	0.42 996	9.97 192	5	37
24	9.54 229	34	9.57 042	39	0.42 958	9.97 187	5	36
25	9.54 263	34	9.57 081	39	0.42 919	9.97 182	4	35
26	9.54 297	34	9.57 120	38	0.42 880	9.97 178	5	34
27	9.54 331	34	9.57 158	39	0.42 842	9.97 173	5	33
28	9.54 365	34	9.57 197	38	0.42 803	9.97 168	5	32
29	9.54 399	34	9.57 235	39	0.42 765	9.97 163	4	31
30	9.54 433		9.57 274		0.42 726	9.97 159		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

69° 30′.

PP	40	39	38		35	34		5	4
.1	4.0	3.9	3.8	.1	3.5	3.4	.1	0.5	0.4
.2	8.0	7.8	7.6	.2	7.0	6.8	.2	1.0	0.8
.3	12.0	11.7	11.4	.3	10.5	10.2	.3	1.5	1.2
.4	16.0	15.6	15.2	.4	14.0	13.6	.4	2.0	1.6
.5	20.0	19.5	19.0	.5	17.5	17.0	.5	2.5	2.0
.6	24.0	23.4	22.8	.6	21.0	20.4	.6	3.0	2.4
.7	28.0	27.3	26.6	.7	24.5	23.8	.7	3.5	2.8
.8	32.0	31.2	30.4	.8	28.0	27.2	.8	4.0	3.2
.9	36.0	35.1	34.2	.9	31.5	30.6	.9	4.5	3.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.54 433	33	9.57 274	38	0.42 726	9.97 159	5	30
31	9.54 466	34	9.57 312	39	0.42 688	9.97 154	5	29
32	9.54 500	34	9.57 351	38	0.42 649	9.97 149	4	28
33	9.54 534	33	9.57 389	39	0.42 611	8.97 145	5	27
34	9.54 567	34	9.57 428	38	0.42 572	9.97 140	5	26
35	9.54 601	34	9.57 466	38	0.42 534	9.97 135	5	25
36	9.54 635	33	9.57 504	39	0.42 496	9.97 130	4	24
37	9.54 668	34	9.57 543	38	0.42 457	9.97 126	5	23
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	34	9.57 619	39	0.42 381	9.97 116	5	21
40	9.54 769	33	9.57 658	38	0.42 342	9.97 111	4	20
41	9.54 802	34	9.57 696	38	0.42 304	9.97 107	5	19
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	34	9.57 772	38	0.42 228	9.97 097	5	17
44	9.54 903	33	9.57 810	39	0.42 190	9.97 092	5	16
45	9.54 936	33	9.57 849	38	0.42 151	9.97 087	4	15
46	9.54 969	34	9.57 887	38	0.42 113	9.97 083	5	14
47	9.55 003	33	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	5	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	34	9.58 039	38	0.41 961	9.97 063	4	10
51	9.55 136	33	9.58 077	38	0.41 923	9.97 059	5	9
52	9.55 169	33	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	38	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 191	38	0.41 809	9.97 044	5	6
55	9.55 268	33	9.58 229	38	0.41 771	9.97 039	4	5
56	9.55 301	33	9.58 267	37	0.41 733	9.97 035	5	4
57	9.55 334	33	9.58 304	38	0.41 696	9.97 030	5	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	38	0.41 620	9.97 020	5	1
60	9.55 433		9.58 418		0.41 582	9.97 015		0
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

69°.

PP	39	38	37		34	33		5	4
.1	3.9	3.8	3.7	.1	3.4	3.3	.1	0.5	0.4
.2	7.8	7.6	7.4	.2	6.8	6.6	.2	1.0	0.8
.3	11.7	11.4	11.1	.3	10.2	9.9	.3	1.5	1.2
.4	15.6	15.2	14.8	.4	13.6	13.2	.4	2.0	1.6
.5	19.5	19.0	18.5	.5	17.0	16.5	.5	2.5	2.0
.6	23.4	22.8	22.2	.6	20.4	19.8	.6	3.0	2.4
.7	27.3	26.6	25.9	.7	23.8	23.1	.7	3.5	2.8
.8	31.2	30.4	29.6	.8	27.2	26.4	.8	4.0	3.2
.9	35.1	34.2	33.3	.9	30.6	29.7	.9	4.5	3.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.55 433	33	9.58 418	37	0.41 582	9.97 015	5	60
1	9.55 466	33	9.58 455	38	0.41 545	9.97 010	5	59
2	9.55 499	33	9.58 493	38	0.41 507	9.97 005	4	58
3	9.55 532	32	9.58 531	38	0.41 469	9.97 001	5	57
4	9.55 564	33	9.58 569	37	0.41 431	9.96 996	5	56
5	9.55 597	33	9.58 606	38	0.41 394	9.96 991	5	55
6	9.55 630	33	9.58 644	37	0.41 356	9.96 986	5	54
7	9.55 663	32	9.58 681	38	0.41 319	9.96 981	5	53
8	9.55 695	33	9.58 719	38	0.41 281	9.96 976	5	52
9	9.55 728	33	9.58 757	37	0.41 243	9.96 971	5	51
10	9.55 761	32	9.58 794	38	0.41 206	9.96 966	4	50
11	9.55 793	33	9.58 832	37	0.41 168	9.96 962	5	49
12	9.55 826	32	9.58 869	38	0.41 131	9.96 957	5	48
13	9.55 858	33	9.58 907	37	0.41 093	9.96 952	5	47
14	9.55 891	32	9.58 944	37	0.41 056	9.96 947	5	46
15	9.55 923	33	9.58 981	38	0.41 019	9.96 942	5	45
16	9.55 956	32	9.59 019	37	0.40 981	9.96 937	5	44
17	9.55 988	33	9.59 056	38	0.40 944	9.96 932	5	43
18	9.56 021	32	9.59 094	37	0.40 906	9.96 927	5	42
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41
20	9.56 085	33	9.59 168	37	0.40 832	9.96 917	5	40
21	9.56 118	32	9.59 205	38	0.40 795	9.96 912	5	39
22	9.56 150	32	9.59 243	37	0.40 757	9.96 907	4	38
23	9.56 182	33	9.59 280	37	0.40 720	9.96 903	5	37
24	9.56 215	32	9.59 317	37	0.40 683	9.96 898	5	36
25	9.56 247	32	9.59 354	37	0.40 646	9.96 893	5	35
26	9.56 279	32	9.59 391	38	0.40 609	9.96 888	5	34
27	9.56 311	32	9.59 429	37	0.40 571	9.96 883	5	33
28	9.56 343	32	9.59 466	37	0.40 534	9.96 878	5	32
29	9.56 375	33	9.59 503	37	0.40 497	9.96 873	5	31
30	9.56 408		9.59 540		0.40 460	9.96 868		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

68° 30′.

PP	38	37		33	32		5	4
.1	3.8	3.7	.1	3.3	3.2	.1	0.5	0.4
.2	7.6	7.4	.2	6.6	6.4	.2	1.0	0.8
.3	11.4	11.1	.3	9.9	9.6	.3	1.5	1.2
.4	15.2	14.8	.4	13.2	12.8	.4	2.0	1.6
.5	19.0	18.5	.5	16.5	16.0	.5	2.5	2.0
.6	22.8	22.2	.6	19.8	19.2	.6	3.0	2.4
.7	26.6	25.9	.7	23.1	22.4	.7	3.5	2.8
.8	30.4	29.6	.8	26.4	25.6	.8	4.0	3.2
.9	34.2	33.3	.9	29.7	28.8	.9	4.5	3.6

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.56 408	32	9.59 540	37	0.40 460	9.96 868	5	**30**
31	9.56 440	32	9.59 577	37	0.40 423	9.96 863	5	29
32	9.56 472	32	9.59 614	37	0.40 386	9.96 858	5	28
33	9.56 504	32	9.59 651	37	0.40 349	9.96 853	5	27
34	9.56 536	32	9.59 688	37	0.40 312	9.96 848	5	26
35	9.56 568	31	9.59 725	37	0.40 275	9.96 843	5	25
36	9.56 599	32	9.59 762	37	0.40 238	9.96 838	5	24
37	9.56 631	32	9.59 799	36	0.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	37	0.40 165	9.96 828	5	22
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21
40	9.56 727	32	9.59 909	37	0.40 091	9.96 818	5	**20**
41	9.56 759	31	9.59 946	37	0.40 054	9.96 813	5	19
42	9.56 790	32	9.59 983	36	0.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	37	0.39 981	9.96 803	5	17
44	9.56 854	32	9.60 056	37	0.39 944	9.96 798	5	16
45	9.56 886	31	9.60 093	37	0.39 907	9.96 793	5	15
46	9.56 917	32	9.60 130	36	0.39 870	9.96 788	5	14
47	9.56 949	31	9.60 166	37	0.39 834	9.96 783	5	13
48	9.56 980	32	9.60 203	37	0.39 797	9.96 778	6	12
49	9.57 012	32	9.60 240	36	0.39 760	9.96 772	5	11
50	9.57 044	31	9.60 276	37	0.39 724	9.96 767	5	**10**
51	9.57 075	32	9.60 313	36	0.39 687	9.96 762	5	9
52	9.57 107	31	9.60 349	37	0.39 651	9.96 757	5	8
53	9.57 138	31	9.60 386	36	0.39 614	9.96 752	5	7
54	9.57 169	32	9.60 422	37	0.39 578	9.96 747	5	6
55	9.57 201	31	9.60 459	36	0.39 541	9.96 742	5	5
56	9.57 232	32	9.60 495	37	0.39 505	9.96 737	5	4
57	9.57 264	31	9.60 532	36	0.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	37	0.39 432	9.96 727	5	2
59	9.57 326	32	9.60 605	36	0.39 395	9.96 722	5	1
60	9.57 358		9.60 641		0.39 359	9.96 717		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

68°.

PP	37	36		32	31		6	5
.1	3.7	3.6	.1	3.2	3.1	.1	0.6	0.5
.2	7.4	7.2	.2	6.4	6.2	.2	1.2	1.0
.3	11.1	10.8	.3	9.6	9.3	.3	1.8	1.5
.4	14.8	14.4	.4	12.8	12.4	.4	2.4	2.0
.5	18.5	18.0	.5	16.0	15.5	.5	3.0	2.5
.6	22.2	21.6	.6	19.2	18.6	.6	3.6	3.0
.7	25.9	25.2	.7	22.4	21.7	.7	4.2	3.5
.8	29.6	28.8	.8	25.6	24.8	.8	4.8	4.0
.9	33.3	32.4	.9	28.8	27.9	.9	5.4	4.5

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.57 358	31	9.60 641	36	0.39 359	9.96 717	6	60
1	9.57 389	31	9.60 677	37	0.39 323	9.96 711	5	59
2	9.57 420	31	9.60 714	36	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	32	9.60 786	37	0.39 214	9.96 696	5	56
5	9.57 514	31	9.60 823	36	0.39 177	9.96 691	5	55
6	9.57 545	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 676	6	52
9	9.57 638	31	9.60 967	37	0.39 033	9.96 670	5	51
10	9.57 669	31	9.61 004	36	0.38 996	9.96 665	5	50
11	9.57 700	31	9.61 040	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 793	31	9.61 148	36	0.38 852	9.96 645	5	46
15	9.57 824	31	9.61 184	36	0.38 816	9.96 640	6	45
16	9.57 855	30	9.61 220	36	0.38 780	9.96 634	5	44
17	9.57 885	31	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 978	30	9.61 364	36	0.38 636	9.96 614	6	40
21	9.58 008	31	9.61 400	36	0.38 600	9.96 608	5	39
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37
24	9.58 101	30	9.61 508	36	0.38 492	9.96 593	5	36
25	9.58 131	31	9.61 544	35	0.38 456	9.96 588	6	35
26	9.58 162	30	9.61 579	36	0.38 421	9.96 582	5	34
27	9.58 192	31	9.61 615	36	0.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	36	0.38 349	9.96 572	5	32
29	9.58 253	31	9.61 687	35	0.38 313	9.96 567	5	31
30	9.58 284		9.61 722		0.38 278	9.96 562		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

67° 30′.

PP	37	36	35		32	31	30		6	5
.1	3.7	3.6	3.5	.1	3.2	3.1	3.0	.1	0.6	0.5
.2	7.4	7.2	7.0	.2	6.4	6.2	6.0	.2	1.2	1.0
.3	11.1	10.8	10.5	.3	9.6	9.3	9.0	.3	1.8	1.5
.4	14.8	14.4	14.0	.4	12.8	12.4	12.0	.4	2.4	2.0
.5	18.5	18.0	17.5	.5	16.0	15.5	15.0	.5	3.0	2.5
.6	22.2	21.6	21.0	.6	19.2	18.6	18.0	.6	3.6	3.0
.7	25.9	25.2	24.5	.7	22.4	21.7	21.0	.7	4.2	3.5
.8	29.6	28.8	28.0	.8	25.6	24.8	24.0	.8	4.8	4.0
.9	33.3	32.4	31.5	.9	28.8	27.9	27.0	.9	5.4	4.5

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.58 284	30	9.61 722	36	0.38 278	9.96 562	6	**30**
31	9.58 314	31	9.61 758	36	0.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	36	0.38 206	9.96 551	5	28
33	9.58 375	31	9.61 830	35	0.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	36	0.38 135	9.96 541	6	26
35	9.58 436	31	9.61 901	35	0.38 099	9.96 535	5	25
36	9.58 467	30	9.61 936	36	0.38 064	9.96 530	5	24
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 008	35	0.37 992	9.96 520	6	22
39	9.58 557	31	9.62 043	36	0.37 957	9.96 514	5	21
40	9.58 588	30	9.62 079	35	0.37 921	9.96 509	5	**20**
41	9.58 618	30	9.62 114	36	0.37 886	9.96 504	6	19
42	9.58 648	30	9.62 150	35	0.37 850	9.96 498	5	18
43	9.58 678	31	9.62 185	36	0.37 815	9.96 493	5	17
44	9.58 709	30	9.62 221	35	0.37 779	9.96 488	5	16
45	9.58 739	30	9.62 256	36	0.37 744	9.96 483	6	15
46	9.58 769	30	9.62 292	35	0.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	36	0.37 638	9.96 467	6	12
49	9.58 859	30	9.62 398	35	0.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	**10**
51	9.58 919	30	9.62 468	36	0.37 532	9.96 451	6	9
52	9.58 949	30	9.62 504	35	0.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	0.37 426	9.96 435	6	6
55	9.59 039	30	9.62 609	36	0.37 391	9.96 429	5	5
56	9.59 069	29	9.62 645	35	0.37 355	9.96 424	5	4
57	9.59 098	30	9.62 680	35	0.37 320	9.96 419	6	3
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	5	2
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188		9.62 785		0.37 215	9.96 403		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

67°.

PP	36	35	31		30	29		6	5
.1	3.6	3.5	3.1	.1	3.0	2.9	.1	0.6	0.5
.2	7.2	7.0	6.2	2	6.0	5.8	.2	1.2	1.0
.3	10.8	10.5	9.3	.3	9.0	8.7	.3	1.8	1.5
.4	14.4	14.0	12.4	.4	12.0	11.6	.4	2.4	2.0
.5	18.0	17.5	15.5	.5	15.0	14.5	.5	3.0	2.5
.6	21.6	21.0	18.6	6	18.0	17.4	.6	3.6	3.0
.7	25.2	24.5	21.7	.7	21.0	20.3	.7	4.2	3.5
.8	28.8	28.0	24.8	.8	24.0	23.2	.8	4.8	4.0
.9	32.4	31.5	27.9	.9	27.0	26.1	.9	5.4	4.5

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.59 188	30	9.62 785	35	0.37 21$\bar{5}$	9.96 403	6	**60**
1	9.59 218	29	9.62 820	35	0.37 180	9.96 397	5	59
2	9.59 247	30	9.62 855	35	0.37 14$\bar{5}$	9.96 392	5	58
3	9.59 277	30	9.62 890	36	0.37 110	9.96 387	6	57
4	9.59 307	29	9.62 926	35	0.37 074	9.96 381	5	56
5	9.59 336	30	9.62 961	35	0.37 039	9.96 376	6	55
6	9.59 366	30	9.62 996	35	0.37 004	9.96 370	5	54
7	9.59 396	29	9.63 031	35	0.36 969	9.96 36$\bar{5}$	5	53
8	9.59 425	30	9.63 066	35	0.36 934	9.96 360	6	52
9	9.59 45$\bar{5}$	29	9.63 101	34	0.36 899	9.96 354	5	51
10	9.59 484	30	9.63 135	35	0.36 86$\bar{5}$	9.96 349	6	**50**
11	9.59 514	29	9.63 170	35	0.36 830	9.96 343	5	49
12	9.59 543	30	9.63 205	35	0.36 79$\bar{5}$	9.96 338	5	48
13	9.59 573	29	9.63 240	35	0.36 760	9.96 333	6	47
14	9.59 602	30	9.63 275	35	0.36 72$\bar{5}$	9.96 327	5	46
15	9.59 632	29	9.63 310	35	0.36 690	9.96 322	6	45
16	9.59 661	29	9.63 34$\bar{5}$	34	0.36 655	9.96 316	5	44
17	9.59 690	30	9.63 379	35	0.36 621	9.96 311	6	43
18	9.59 720	29	9.63 414	35	0.36 586	9.96 305	5	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	6	41
20	9.59 778	30	9.63 484	35	0.36 516	9.96 294	5	**40**
21	9.59 808	29	9.63 519	34	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	35	0.36 447	9.96 284	6	38
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	5	37
24	9.59 895	29	9.63 623	34	0.36 377	9.96 273	6	36
25	9.59 924	30	9.63 657	35	0.36 343	9.96 267	5	35
26	9.59 954	29	9.63 692	34	0.36 308	9.96 262	6	34
27	9.59 983	29	9.63 726	35	0.36 274	9.96 256	5	33
28	9.60 012	29	9.63 761	35	0.36 239	9.96 251	6	32
29	9.60 041	29	9.63 796	34	0.36 204	9.96 245	5	31
30	9.60 070		9.63 830		0.36 170	9.96 240		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

66° 30′.

PP	36	35	34		30	29		6	5
.1	3.6	3.5	3.4	.1	3.0	2.9	.1	0.6	0.5
.2	7.2	7.0	6.8	.2	6.0	5.8	.2	1.2	1.0
.3	10.8	10.5	10.2	.3	9.0	8.7	.3	1.8	1.5
.4	14.4	14.0	13.6	.4	12.0	11.6	.4	2.4	2.0
.5	18.0	17.5	17.0	.5	15.0	14.5	.5	3.0	2.5
.6	21.6	21.0	20.4	.6	18.0	17.4	.6	3.6	3.0
.7	25.2	24.5	23.8	.7	21.0	20.3	.7	4.2	3.5
.8	28.8	28.0	27.2	.8	24.0	23.2	.8	4.8	4.0
.9	32.4	31.5	30.6	.9	27.0	26.1	.9	5.4	4.5

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.60 070	29	9.63 830	35	0.36 170	9.96 240	6	30
31	9.60 099	29	9.63 865̄	34	0.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	35	0.36 101	9.96 229	6	28
33	9.60 157	29	9.63 934	34	0.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	35	0.36 032	9.96 218	6	26
35	9.60 215̄	29	9.64 003	34	0.35 997	9.96 212	5	25
36	9.60 244	29	9.64 037	35	0.35 963	9.96 207	6	24
37	9.60 273	29	9.64 072	34	0.35 928	9.96 201	5	23
38	9.60 302	29	9.64 106	34	0.35 894	9.96 196	6	22
39	9.60 331	28	9.64 140	35	0.35 860	9.96 190	5	21
40	9.60 359	29	9.64 175̄	34	0.35 825	9.96 185̄	6	20
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	29	9.64 243	35	0.35 757	9.96 174	6	18
43	9.60 446	28	9.64 278	34	0.35 722	9.96 168	6	17
44	9.60 474	29	9.64 312	34	0.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	35	0.35 654	9.96 157	6	15
46	9.60 532	29	9.64 381	34	0.35 619	9.96 151	5	14
47	9.60 561	28	9.64 415̄	34	0.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	5	12
49	9.60 618	28	9.64 483	34	0.35 517	9.96 135̄	6	11
50	9.60 646	29	9.64 517	35	0.35 483	9.96 129	6	10
51	9.60 675	29	9.64 552	34	0.35 448	9.96 123	5	9
52	9.60 704	28	9.64 586	34	0.35 414	9.96 118	6	8
53	9.60 732	29	9.64 620	34	0.35 380	9.96 112	5	7
54	9.60 761	28	9.64 654	34	0.35 346	9.96 107	6	6
55	9.60 789	29	9.64 688	34	0.35 312	9.96 101	6	5
56	9.60 818	28	9.64 722	34	0.35 278	9.96 095	5	4
57	9.60 846	29	9.64 756	34	0.35 244	9.96 090	6	3
58	9.60 875̄	28	9.64 790	34	0.35 210	9.96 084	5	2
59	9.60 903	28	9.64 824	34	0.35 176	9.96 079	6	1
60	9.60 931		9.64 858		0.35 142	9.96 073		0
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

66°.

PP	35	34		29	28		6	5
.1	3.5	3.4	.1	2.9	2.8	.1	0.6	0.5
.2	7.0	6.8	.2	5.8	5.6	.2	1.2	1.0
.3	10.5	10.2	.3	8.7	8.4	.3	1.8	1.5
.4	14.0	13.6	.4	11.6	11.2	.4	2.4	2.0
.5	17.5	17.0	.5	14.5	14.0	.5	3.0	2.5
.6	21.0	20.4	.6	17.4	16.8	.6	3.6	3.0
.7	24.5	23.8	.7	20.3	19.6	.7	4.2	3.5
.8	28.0	27.2	.8	23.2	22.4	.8	4.8	4.0
.9	31.5	30.6	.9	26.1	25.2	.9	5.4	4.5

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.60 931	29	9.64 858	34	0.35 142	9.96 073	6	**60**
1	9.60 960	28	9.64 892	34	0.35 108	9.96 067	5	59
2	9.60 988	28	9.64 926	34	0.35 074	9.96 062	6	58
3	9.61 016	29	9.64 960	34	0.35 040	9.96 056	6	57
4	9.61 045	28	9.64 994	34	0.35 006	9.96 050	5	56
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	6	55
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	5	54
7	9.61 129	29	9.65 096	34	0.34 904	9.96 034	6	53
8	9.61 158	28	9.65 130	34	0.34 870	9.96 028	6	52
9	9.61 186	28	9.65 164	33	0.34 836	9.96 022	5	51
10	9.61 214	28	9.65 197	34	0.34 803	9.96 017	6	**50**
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	5	48
13	9.61 298	28	9.65 299	34	0.34 701	9.96 000	6	47
14	9.61 326	28	9.65 333	33	0.34 667	9.95 994	6	46
15	9.61 354	28	9.65 366	34	0.34 634	9.95 988	6	45
16	9.61 382	29	9.65 400	34	0.34 600	9.95 982	5	44
17	9.61 411	27	9.65 434	33	0.34 566	9.95 977	6	43
18	9.61 438	28	9.65 467	34	0.34 533	9.95 971	6	42
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	5	41
20	9.61 494	28	9.65 535	33	0.34 465	9.95 960	6	**40**
21	9.61 522	28	9.65 568	34	0.34 432	9.95 954	6	39
22	9.61 550	28	9.65 602	34	0.34 398	9.95 948	6	38
23	9.61 578	28	9.65 636	33	0.34 364	9.95 942	5	37
24	9.61 606	28	9.65 669	34	0.34 331	9.95 937	6	36
25	9.61 634	28	9.65 703	33	0.34 297	9.95 931	6	35
26	9.61 662	27	9.65 736	34	0.34 264	9.95 925	5	34
27	9.61 689	28	9.65 770	33	0.34 230	9.95 920	6	33
28	9.61 717	28	9.65 803	34	0.34 197	9.95 914	6	32
29	9.61 745	28	9.65 837	33	0.34 163	9.95 908	6	31
30	9.61 773		9.65 870		0.34 130	9.95 902		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

65° 30′.

PP	34	33	29		28	27		6	5
.1	3.4	3.3	2.9	.1	2.8	2.7	.1	0.6	0.5
.2	6.8	6.6	5.8	.2	5.6	5.4	.2	1.2	1.0
.3	10.2	9.9	8.7	.3	8.4	8.1	.3	1.8	1.5
.4	13.6	13.2	11.6	.4	11.2	10.8	.4	2.4	2.0
.5	17.0	16.5	14.5	.5	14.0	13.5	.5	3.0	2.5
.6	20.4	19.8	17.4	.6	16.8	16.2	.6	3.6	3.0
.7	23.8	23.1	20.3	.7	19.6	18.9	.7	4.2	3.5
.8	27.2	26.4	23.2	.8	22.4	21.6	.8	4.8	4.0
.9	30.6	29.7	26.1	.9	25.2	24.3	.9	5.4	4.5

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.61 773	27	9.65 870	34	0.34 130	9.95 902	5	**30**
31	9.61 800	28	9.65 904	33	0.34 096	9.95 897	6	29
32	9.61 828	28	9.65 937	34	0.34 063	9.95 891	6	28
33	9.61 856	27	9.65 971	33	0.34 029	9.95 885	6	27
34	9.61 883	28	9.66 004	34	0.33 996	9.95 879	6	26
35	9.61 911	28	9.66 038	33	0.33 962	9.95 873	5	25
36	9.61 939	27	9.66 071	33	0.33 929	9.95 868	6	24
37	9.61 966	28	9.66 104	34	0.33 896	9.95 862	6	23
38	9.61 994	27	9.66 138	33	0.33 862	9.95 856	6	22
39	9.62 021	28	9.66 171	33	0.33 829	9.95 850	6	21
40	9.62 049	27	9.66 204	34	0.33 796	9.95 844	5	**20**
41	9.62 076	28	9.66 238	33	0.33 762	9.95 839	6	19
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18
43	9.62 131	28	9.66 304	33	0.33 696	9.95 827	6	17
44	9.62 159	27	9.66 337	34	0.33 663	9.95 821	6	16
45	9.62 186	28	9.66 371	33	0.33 629	9.95 815	5	15
46	9.62 214	27	9.66 404	33	0.33 596	9.95 810	6	14
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13
48	9.62 268	28	9.66 470	33	0.33 530	9.95 798	6	12
49	9.62 296	27	9.66 503	34	0.33 497	9.95 792	6	11
50	9.62 323	27	9.66 537	33	0.33 463	9.95 786	6	**10**
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	5	9
52	9.62 377	28	9.66 603	33	0.33 397	9.95 775	6	8
53	9.62 405	27	9.66 636	33	0.33 364	9.95 769	6	7
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	5
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4
57	9.62 513	28	9.66 768	33	0.33 232	9.95 745	6	3
58	9.62 541	27	9.66 801	33	0.33 199	9.95 739	6	2
59	9.62 568	27	9.66 834	33	0.33 166	9.95 733	5	1
60	9.62 595		9.66 867		0.33 133	9.95 728		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

65°.

PP	34	33		28	27		6	5
1	3.4	3.3	.1	2.8	2.7	.1	0.6	0.5
.2	6.8	6.6	.2	5.6	5.4	.2	1.2	1.0
.3	10.2	9.9	3	8.4	8.1	.3	1.8	1.5
.4	13.6	13.2	.4	11.2	10.8	.4	2.4	2.0
.5	17.0	16.5	5	14.0	13.5	.5	3.0	2.5
.6	20.4	19.8	.6	16.8	16.2	6	3.6	3.0
.7	23.8	23.1	.7	19.6	18.9	.7	4.2	3.5
.8	27.2	26.4	.8	22.4	21.6	.8	4.8	4.0
.9	30.6	29.7	.9	25.2	24.3	.9	5.4	4.5

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.62 595	27	9.66 867	33	0.33 133	9.95 728	6	**60**
1	9.62 622	27	9.66 900	33	0.33 100	9.95 722	6	59
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54
7	9.62 784	27	9.67 098	33	0.32 902	9.95 686	6	53
8	9.62 811	27	9.67 131	32	0.32 869	9.95 680	6	52
9	9.62 838	27	9.67 163	33	0.32 837	9.95 674	6	51
10	9.62 865	27	9.67 196	33	0.32 804	9.95 668	5	**50**
11	9.62 892	26	9.67 229	33	0.32 771	9.95 663	6	49
12	9.62 918	27	9.67 262	33	0.32 738	9.95 657	6	48
13	9.62 945	27	9.67 295	32	0.32 705	9.95 651	6	47
14	9.62 972	27	9.67 327	33	0.32 673	9.95 645	6	46
15	9.62 999	27	9.67 360	33	0.32 640	9.95 639	6	45
16	9.63 026	26	9.67 393	33	0.32 607	9.95 633	6	44
17	9.63 052	27	9.67 426	32	0.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	33	0.32 542	9.95 621	6	42
19	9.63 106	27	9.67 491	33	0.32 509	9.95 615	6	41
20	9.63 133	26	9.67 524	32	0.32 476	9.95 609	6	**40**
21	9.63 159	27	9.67 556	33	0.32 444	9.95 603	6	39
22	9.63 186	27	9.67 589	33	0.32 411	9.95 597	6	38
23	9.63 213	26	9.67 622	32	0.32 378	9.95 591	6	37
24	9.63 239	27	9.67 654	33	0.32 346	9.95 585	6	36
25	9.63 266	26	9.67 687	32	0.32 313	9.95 579	6	35
26	9.63 292	27	9.67 719	33	0.32 281	9.95 573	6	34
27	9.63 319	26	9.67 752	33	0.32 248	9.95 567	6	33
28	9.63 345	27	9.67 785	32	0.32 215	9.95 561	6	32
29	9.63 372	26	9.67 817	33	0.32 183	9.95 555	6	31
30	9.63 398		9.67 850		0.32 150	9.95 549		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

64° 30′.

PP	33	32		27	26		6	5
.1	3.3	3.2	.1	2.7	2.6	.1	0.6	0.5
.2	6.6	6.4	.2	5.4	5.2	.2	1.2	1.0
.3	9.9	9.6	.3	8.1	7.8	.3	1.8	1.5
.4	13.2	12.8	.4	10.8	10.4	.4	2.4	2.0
.5	16.5	16.0	.5	13.5	13.0	.5	3.0	2.5
.6	19.8	19.2	6	16.2	15.6	.6	3.6	3.0
.7	23.1	22.4	.7	18.9	18.2	.7	4.2	3.5
.8	26.4	25.6	.8	21.6	20.8	.8	4.8	4.0
.9	29.7	28.8	.9	24.3	23.4	.9	5.4	4.5

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.63 398	27	9.67 850	32	0.32 150	9.95 549	6	**30**
31	9.63 425	26	9.67 882	33	0.32 118	9.95 543	6	29
32	9.63 451	27	9.67 915	32	0.32 085	9.95 537	6	28
33	9.63 478	26	9.67 947	33	0.32 053	9.95 531	6	27
34	9.63 504	27	9.67 980	32	0.32 020	9.95 525	6	26
35	9.63 531	26	9.68 012	32	0.31 988	9.95 519	6	25
36	9.63 557	26	9.68 044	33	0.31 956	9.95 513	6	24
37	9.63 583	27	9.68 077	32	0.31 923	9.95 507	7	23
38	9.63 610	26	9.68 109	33	0.31 891	9.95 500	6	22
39	9.63 636	26	9.68 142	32	0.31 858	9.95 494	6	21
40	9.63 662	27	9.68 174	32	0.31 826	9.95 488	6	**20**
41	9.63 689	26	9.68 206	33	0.31 794	9.95 482	6	19
42	9.63 715	26	9.68 239	32	0.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17
44	9.63 767	27	9.68 303	33	0.31 697	9.95 464	6	16
45	9.63 794	26	9.68 336	32	0.31 664	9.95 458	6	15
46	9.63 820	26	9.68 368	32	0.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	0.31 600	9.95 446	6	13
48	9.63 872	26	9.68 432	33	0.31 568	9.95 440	6	12
49	9.63 898	26	9.68 465	32	0.31 535	9.95 434	7	11
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	6	**10**
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	33	0.31 407	9.95 409	6	7
54	9.64 028	26	9.68 626	32	0.31 374	9.95 403	6	6
55	9.64 054	26	9.68 658	32	0.31 342	9.95 397	6	5
56	9.64 080	26	9.68 690	32	0.31 310	9.95 391	7	4
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	6	3
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2
59	9.64 158	26	9.68 786	32	0.31 214	9.95 372	6	1
60	9.64 184		9.68 818		0.31 182	9.95 366		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

64°.

PP	33	32		27	26		7	6
.1	3.3	3.2	.1	2.7	2.6	.1	0.7	0.6
.2	6.6	6.4	.2	5.4	5.2	.2	1.4	1.2
.3	9.9	9.6	.3	8.1	7.8	.3	2.1	1.8
.4	13.2	12.8	.4	10.8	10.4	.4	2.8	2.4
.5	16.5	16.0	.5	13.5	13.0	.5	3.5	3.0
.6	19.8	19.2	.6	16.2	15.6	.6	4.2	3.6
.7	23.1	22.4	.7	18.9	18.2	.7	4.9	4.2
.8	26.4	25.6	.8	21.6	20.8	.8	5.6	4.8
.9	29.7	28.8	.9	24.3	23.4	.9	6.3	5.4

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	60
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	0.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	0.31 086	9.95 348	7	57
4	9.64 288	25	9.68 946	32	0.31 054	9.95 341	6	56
5	9.64 313	26	9.68 978	32	0.31 022	9.95 335	6	55
6	9.64 339	26	9.69 010	32	0.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	0.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	0.30 926	9.95 317	7	52
9	9.64 417	25	9.69 106	32	0.30 894	9.95 310	6	51
10	9.64 442	26	9.69 138	32	0.30 862	9.95 304	6	50
11	9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49
12	9.64 494	25	9.69 202	32	0.30 798	9.95 292	6	48
13	9.64 519	26	9.69 234	32	0.30 766	9.95 286	7	47
14	9.64 545	26	9.69 266	32	0.30 734	9.95 279	6	46
15	9.64 571	25	9.69 298	31	0.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	32	0.30 671	9.95 267	6	44
17	9.64 622	25	9.69 361	32	0.30 639	9.95 261	7	43
18	9.64 647	26	9.69 393	32	0.30 607	9.95 254	6	42
19	9.64 673	25	9.69 425	32	0.30 575	9.95 248	6	41
20	9.64 698	26	9.69 457	31	0.30 543	9.95 242	6	40
21	9.64 724	25	9.69 488	32	0.30 512	9.95 236	7	39
22	9.64 749	26	9.69 520	32	0.30 480	9.95 229	6	38
23	9.64 775	25	9.69 552	32	0.30 448	9.95 223	6	37
24	9.64 800	26	9.69 584	31	0.30 416	9.95 217	6	36
25	9.64 826	25	9.69 615	32	0.30 385	9.95 211	7	35
26	9.64 851	26	9.69 647	32	0.30 353	9.95 204	6	34
27	9.64 877	25	9.69 679	31	0.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	32	0.30 290	9.95 192	7	32
29	9.64 927	26	9.69 742	32	0.30 258	9.95 185	6	31
30	9.64 953		9.69 774		0.30 226	9.95 179		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

63° 30′.

PP	32	31		26	25		7	6
.1	3.2	3.1	.1	2.6	2.5	.1	0.7	0.6
.2	6.4	6.2	.2	5.2	5.0	.2	1.4	1.2
.3	9.6	9.3	.3	7.8	7.5	.3	2.1	1.8
.4	12.8	12.4	.4	10.4	10.0	.4	2.8	2.4
.5	16.0	15.5	.5	13.0	12.5	.5	3.5	3.0
.6	19.2	18.6	.6	15.6	15.0	.6	4.2	3.6
.7	22.4	21.7	.7	18.2	17.5	.7	4.9	4.2
.8	25.6	24.8	.8	20.8	20.0	.8	5.6	4.8
.9	28.8	27.9	.9	23.4	22.5	.9	6.3	5.4

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.64 953	25	9.69 774	31	0.30 226	9.95 179	6	**30**
31	9.64 978	25	9.69 805	32	0.30 195̄	9.95 173	6	29
32	9.65 003	26	9.69 837	31	0.30 163	9.95 167	7	28
33	9.65 029	25	9.69 868	32	0.30 132	9.95 160	6	27
34	9.65 054	25	9.69 900	32	0.30 100	9.95 154	6	26
35	9.65 079	25	9.69 932	31	0.30 068	9.95 148	7	25
36	9.65 104	26	9.69 963	32	0.30 037	9.95 141	6	24
37	9.65 130	25	9.69 995̄	31	0.30 005	9.95 135̄	6	23
38	9.65 155̄	25	9.70 026	32	0.29 974	9.95 129	7	22
39	9.65 180	25	9.70 058	31	0.29 942	9.95 122	6	21
40	9.65 205	25	9.70 089	32	0.29 911	9.95 116	6	**20**
41	9.65 230	25	9.70 121	31	0.29 879	9.95 110	7	19
42	9.65 255	26	9.70 152	32	0.29 848	9.95 103	6	18
43	9.65 281	25	9.70 184	31	0.29 816	9.95 097	7	17
44	9.65 306	25	9.70 215	32	0.29 785̄	9.95 090	6	16
45	9.65 331	25	9.70 247	31	0.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	7	14
47	9.65 381	25	9.70 309	32	0.29 691	9.95 071	6	13
48	9.65 406	25	9.70 341	31	0.29 659	9.95 065̄	6	12
49	9.65 431	25	9.70 372	32	0.29 628	9.95 059	7	11
50	9.65 456	25	9.70 404	31	0.29 596	9.95 052	6	**10**
51	9.65 481	25	9.70 435̄	31	0.29 565	9.95 046	7	9
52	9.65 506	25	9.70 466	32	0.29 534	9.95 039	6	8
53	9.65 531	25	9.70 498	31	0.29 502	9.95 033	6	7
54	9.65 556	24	9.70 529	31	0.29 471	9.95 027	7	6
55	9.65 580	25	9.70 560	32	0.29 440	9.95 020	6	5
56	9.65 605	25	9.70 592	31	0.29 408	9.95 014	7	4
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	6	3
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2
59	9.65 680	25	9.70 685	32	0.29 315̄	9.94 995̄	7	1
60	9.65 705̄		9.70 717		0.29 283	9.94 988		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

63°.

PP	32	31	26		25	24		7	6
.1	3.2	3.1	2.6	.1	2.5	2.4	.1	0.7	0.6
.2	6.4	6.2	5.2	2	5.0	4.8	.2	1.4	1.2
.3	9.6	9.3	7.8	.3	7.5	7.2	.3	2.1	1.8
.4	12.8	12.4	10.4	.4	10.0	9.6	.4	2.8	2.4
.5	16.0	15.5	13.0	.5	12.5	12.0	5	3.5	3.0
.6	19.2	18.6	15.6	.6	15.0	14.4	.6	4.2	3.6
.7	22.4	21.7	18.2	.7	17.5	16.8	.7	4.9	4.2
.8	25.6	24.8	20.8	8	20.0	19.2	.8	5.6	4.8
.9	28.8	27.9	23.4	.9	22.5	21.6	.9	6.3	5.4

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.65 705̄	24	9.70 717	31	0.29 283	9.94 988	6	**60**
1	9.65 729	25	9.70 748	31	0.29 252	9.94 982	7	59
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	6	58
3	9.65 779	25	9.70 810	31	0.29 190	9.94 969	7	57
4	9.65 804	24	9.70 841	32	0.29 159	9.94 962	6	56
5	9.65 828	25	9.70 873	31	0.29 127	9.94 956	7	55
6	9.65 853	25	9.70 904	31	0.29 096	9.94 949	6	54
7	9.65 878	24	9.70 935̄	31	0.29 065	9.94 943	7	53
8	9.65 902	25	9.70 966	31	0.29 034	9.94 936	6	52
9	9.65 927	25	9.70 997	31	0.29 003	9.94 930	7	51
10	9.65 952	24	9.71 028	31	0.28 972	9.94 923	6	**50**
11	9.65 976	25	9.71 059	31	0.28 941	9.94 917	6	49
12	9.66 001	24	9.71 090	31	0.28 910	9.94 911	7	48
13	9.66 025	25	9.71 121	32	0.28 879	9.94 904	6	47
14	9.66 050	25	9.71 153	31	0.28 847	9.94 898	7	46
15	9.66 075̄	24	9.71 184	31	0.28 816	9.94 891	6	45
16	9.66 099	25	9.71 215̄	31	0.28 785	9.94 885̄	7	44
17	9.66 124	24	9.71 246	31	0.28 754	9.94 878	7	43
18	9.66 148	25	9.71 277	31	0.28 723	9.94 871	6	42
19	9.66 173	24	9.71 308	31	0.28 692	9.94 865̄	7	41
20	9.66 197	24	9.71 339	31	0.28 661	9.94 858	6	**40**
21	9.66 221	25	9.71 370	31	0.28 630	9.94 852	7	39
22	9.66 246	24	9.71 401	30	0.28 599	9.94 845	6	38
23	9.66 270	25	9.71 431	31	0.28 569	9.94 839	7	37
24	9.66 295̄	24	9.71 462	31	0.28 538	9.94 832	6	36
25	9.66 319	24	9.71 493	31	0.28 507	9.94 826	7	35
26	9.66 343	25	9.71 524	31	0.28 476	9.94 819	6	34
27	9.66 368	24	9.71 555	31	0.28 445̄	9.94 813	7	33
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	7	32
29	9.66 416	25	9.71 617	31	0.28 383	9.94 799	6	31
30	9.66 441		9.71 648		0.28 352	9.94 793		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

62° 30′.

PP	32	31	30		25	24		7	6
.1	3.2	3.1	3.0	.1	2.5	2.4	.1	0.7	0.6
.2	6.4	6.2	6.0	.2	5.0	4.8	.2	1.4	1.2
.3	9.6	9.3	9.0	.3	7.5	7.2	.3	2.1	1.8
.4	12.8	12.4	12.0	.4	10.0	9.6	.4	2.8	2.4
.5	16.0	15.5	15.0	.5	12.5	12.0	.5	3.5	3.0
.6	19.2	18.6	18.0	.6	15.0	14.4	.6	4.2	3.6
.7	22.4	21.7	21.0	.7	17.5	16.8	.7	4.9	4.2
.8	25.6	24.8	24.0	.8	20.0	19.2	.8	5.6	4.8
.9	28.8	27.9	27.0	.9	22.5	21.6	.9	6.3	5.4

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.66 441	24	9.71 648	31	0.28 352	9.94 793	7	**30**
31	9.66 465	24	9.71 679	30	0.28 321	9.94 786	6	29
32	9.66 489	24	9.71 709	31	0.28 291	9.94 780	7	28
33	9.66 513	24	9.71 740	31	0.28 260	9.94 773	6	27
34	9.66 537	25	9.71 771	31	0.28 229	9.94 767	7	26
35	9.66 562	24	9.71 802	31	0.28 198	9.94 760	7	25
36	9.66 586	24	9.71 833	30	0.28 167	9.94 753	6	24
37	9.66 610	24	9.71 863	31	0.28 137	9.94 747	7	23
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	6	22
39	9.66 658	24	9.71 925	30	0.28 075	9.94 734	7	21
40	9.66 682	24	9.71 955	31	0.28 045	9.94 727	7	**20**
41	9.66 706	25	9.71 986	31	0.28 014	9.94 720	6	19
42	9.66 731	24	9.72 017	31	0.27 983	9.94 714	7	18
43	9.66 755	24	9.72 048	30	0.27 952	9.94 707	7	17
44	9.66 779	24	9.72 078	31	0.27 922	9.94 700	6	16
45	9.66 803	24	9.72 109	31	0.27 891	9.94 694	7	15
46	9.66 827	24	9.72 140	30	0.27 860	9.94 687	7	14
47	9.66 851	24	9.72 170	31	0.27 830	9.94 680	6	13
48	9.66 875	24	9.72 201	30	0.27 799	9.94 674	7	12
49	9.66 899	23	9.72 231	31	0.27 769	9.94 667	7	11
50	9.66 922	24	9.72 262	31	0.27 738	9.94 660	6	**10**
51	9.66 946	24	9.72 293	30	0.27 707	9.94 654	7	9
52	9.66 970	24	9.72 323	31	0.27 677	9.94 647	7	8
53	9.66 994	24	9.72 354	30	0.27 646	9.94 640	6	7
54	9.67 018	24	9.72 384	31	0.27 616	9.94 634	7	6
55	9.67 042	24	9.72 415	30	0.27 585	9.94 627	7	5
56	9.67 066	24	9.72 445	31	0.27 555	9.94 620	6	4
57	9.67 090	23	9.72 476	30	0.27 524	9.94 614	7	3
58	9.67 113	24	9.72 506	31	0.27 494	9.94 607	7	2
59	9.67 137	24	9.72 537	30	0.27 463	9.94 600	7	1
60	9.67 161		9.72 567		0.27 433	9.94 593		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

62°.

PP	31	30	25		24	23		7	6
.1	3.1	3.0	2.5	.1	2.4	2.3	.1	0.7	0.6
.2	6.2	6.0	5.0	.2	4.8	4.6	.2	1.4	1.2
.3	9.3	9.0	7.5	.3	7.2	6.9	.3	2.1	1.8
.4	12.4	12.0	10.0	.4	9.6	9.2	.4	2.8	2.4
.5	15.5	15.0	12.5	.5	12.0	11.5	.5	3.5	3.0
.6	18.6	18.0	15.0	.6	14.4	13.8	.6	4.2	3.6
.7	21.7	21.0	17.5	.7	16.8	16.1	.7	4.9	4.2
.8	24.8	24.0	20.0	.8	19.2	18.4	.8	5.6	4.8
.9	27.9	27.0	22.5	.9	21.6	20.7	.9	6.3	5.4

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.67 161	24	9.72 567	31	0.27 433	9.94 593	6	**60**
1	9.67 18$\bar{5}$	23	9.72 598	30	0.27 402	9.94 587	7	59
2	9.67 208	24	9.72 628	31	0.27 372	9.94 580	7	58
3	9.67 232	24	9.72 659	30	0.27 341	9.94 573	6	57
4	9.67 256	24	9.72 689	31	0.27 311	9.94 567	7	56
5	9.67 280	23	9.72 720	30	0.27 280	9.94 560	7	55
6	9.67 303	24	9.72 750	30	0.27 2$\bar{5}$0	9.94 553	7	54
7	9.67 327	23	9.72 780	31	0.27 220	9.94 546	6	53
8	9.67 350	24	9.72 811	30	0.27 189	9.94 540	7	52
9	9.67 374	24	9.72 841	31	0.27 159	9.94 533	7	51
10	9.67 398	23	9.72 872	30	0.27 128	9.94 526	7	**50**
11	9.67 421	24	9.72 902	30	0.27 098	9.94 519	6	49
12	9.67 44$\bar{5}$	23	9.72 932	31	0.27 068	9.94 513	7	48
13	9.67 468	24	9.72 963	30	0.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	0.27 007	9.94 499	7	46
15	9.67 515	24	9.73 023	31	0.26 977	9.94 492	7	45
16	9.67 539	23	9.73 054	30	0.26 946	9.94 485	6	44
17	9.67 562	24	9.73 084	30	0.26 916	9.94 479	7	43
18	9.67 586	23	9.73 114	30	0.26 886	9.94 472	7	42
19	9.67 609	24	9.73 144	31	0.26 856	9.94 465	7	41
20	9.67 633	23	9.73 17$\bar{5}$	30	0.26 825	9.94 458	7	**40**
21	9.67 656	24	9.73 20$\bar{5}$	30	0.26 795	9.94 451	6	39
22	9.67 680	23	9.73 235	30	0.26 76$\bar{5}$	9.94 44$\bar{5}$	7	38
23	9.67 703	23	9.73 265	30	0.26 73$\bar{5}$	9.94 438	7	37
24	9.67 726	24	9.73 295	31	0.26 70$\bar{5}$	9.94 431	7	36
25	9.67 7$\bar{5}$0	23	9.73 326	30	0.26 674	9.94 424	7	35
26	9.67 773	23	9.73 356	30	0.26 644	9.94 417	7	34
27	9.67 796	24	9.73 386	30	0.26 614	9.94 410	6	33
28	9.67 820	23	9.73 416	30	0.26 584	9.94 404	7	32
29	9.67 843	23	9.73 446	30	0.26 554	9.94 397	7	31
30	9.67 866		9.73 476		0.26 524	9.94 390		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

61° 30′.

PP	31	30		24	23		7	6
.1	3.1	3.0	.1	2.4	2.3	.1	0.7	0.6
.2	6.2	6.0	.2	4.8	4.6	.2	1.4	1.2
.3	9.3	9.0	.3	7.2	6.9	.3	2.1	1.8
.4	12.4	12.0	.4	9.6	9.2	.4	2.8	2.4
.5	15.5	15.0	.5	12.0	11.5	.5	3.5	3.0
.6	18.6	18.0	.6	14.4	13.8	.6	4.2	3.6
.7	21.7	21.0	.7	16.8	16.1	.7	4.9	4.2
.8	24.8	24.0	.8	19.2	18.4	.8	5.6	4.8
.9	27.9	27.0	.9	21.6	20.7	.9	6.3	5.4

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.67 866	24	9.73 476	31	0.26 524	9.94 390	7	30
31	9.67 890	23	9.73 507	30	0.26 493	9.94 383	7	29
32	9.67 913	23	9.73 537	30	0.26 463	9.94 376	7	28
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26
35	9.67 982	24	9.73 627	30	0.26 373	9.94 355	6	25
36	9.68 006	23	9.73 657	30	0.26 343	9.94 349	7	24
37	9.68 029	23	9.73 687	30	0.26 313	9.94 342	7	23
38	9.68 052	23	9.73 717	30	0.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	0.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	20
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	7	18
43	9.68 167	23	9.73 867	30	0.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16
45	9.68 213	24	9.73 927	30	0.26 073	9.94 286	7	15
46	9.68 237	23	9.73 957	30	0.26 043	9.94 279	6	14
47	9.68 260	23	9.73 987	30	0.26 013	9.94 273	7	13
48	9.68 283	22	9.74 017	30	0.25 983	9.94 266	7	12
49	9.68 305	23	9.74 047	30	0.25 953	9.94 259	7	11
50	9.68 328	23	9.74 077	30	0.25 923	9.94 252	7	10
51	9.68 351	23	9.74 107	30	0.25 893	9.94 245	7	9
52	9.68 374	23	9.74 137	29	0.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	30	0.25 834	9.94 231	7	7
54	9.68 420	23	9.74 196	30	0.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	5
56	9.68 466	23	9.74 256	30	0.25 744	9.94 210	7	4
57	9.68 489	23	9.74 286	30	0.25 714	9.94 203	7	3
58	9.68 512	22	9.74 316	29	0.25 684	9.94 196	7	2
59	9.68 534	23	9.74 345	30	0.25 655	9.94 189	7	1
60	9.68 557		9.74 375		0.25 625	9.94 182		0
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

61°.

PP	31	30	29		24	23	22		7	6
.1	3.1	3.0	2.9	.1	2.4	2.3	2.2	.1	0.7	0.6
.2	6.2	6.0	5.8	.2	4.8	4.6	4.4	.2	1.4	1.2
.3	9.3	9.0	8.7	.3	7.2	6.9	6.6	.3	2.1	1.8
.4	12.4	12.0	11.6	.4	9.6	9.2	8.8	.4	2.8	2.4
.5	15.5	15.0	14.5	.5	12.0	11.5	11.0	.5	3.5	3.0
.6	18.6	18.0	17.4	.6	14.4	13.8	13.2	.6	4.2	3.6
.7	21.7	21.0	20.3	.7	16.8	16.1	15.4	.7	4.9	4.2
.8	24.8	24.0	23.2	.8	19.2	18.4	17.6	.8	5.6	4.8
.9	27.9	27.0	26.1	.9	21.6	20.7	19.8	.9	6.3	5.4

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.68 557	23	9.74 375	30	0.25 625̄	9.94 182	7	**60**
1	9.68 580	23	9.74 405̄	30	0.25 595	9.94 175̄	7	59
2	9.68 603	22	9.74 435̄	30	0.25 565	9.94 168	7	58
3	9.68 625	23	9.74 465̄	29	0.25 535	9.94 161	7	57
4	9.68 648	23	9.74 494	30	0.25 506	9.94 154	7	56
5	9.68 671	23	9.74 524	30	0.25 476	9.94 147	7	55
6	9.68 694	22	9.74 554	29	0.25 446	9.94 140	7	54
7	9.68 716	23	9.74 583	30	0.25 417	9.94 133	7	53
8	9.68 739	23	9.74 613	30	0.25 387	9.94 126	7	52
9	9.68 762	22	9.74 643	30	0.25 357	9.94 119	7	51
10	9.68 784	23	9.74 673	29	0.25 327	9.94 112	7	**50**
11	9.68 807	22	9.74 702	30	0.25 298	9.94 105̄	7	49
12	9.68 829	23	9.74 732	30	0.25 268	9.94 098	8	48
13	9.68 852	23	9.74 762	29	0.25 238	9.94 090	7	47
14	9.68 875̄	22	9.74 791	30	0.25 209	9.94 083	7	46
15	9.68 897	23	9.74 821	30	0.25 179	9.94 076	7	45
16	9.68 920	22	9.74 851	29	0.25 149	9.94 069	7	44
17	9.68 942	23	9.74 880	30	0.25 120	9.94 062	7	43
18	9.68 965̄	22	9.74 910	29	0.25 090	9.94 055	7	42
19	9.68 987	23	9.74 939	30	0.25 061	9.94 048	7	41
20	9.69 010	22	9.74 969	29	0.25 031	9.94 041	7	**40**
21	9.69 032	23	9.74 998	30	0.25 002	9.94 034	7	39
22	9.69 055̄	22	9.75 028	30	0.24 972	9.94 027	7	38
23	9.69 077	23	9.75 058	29	0.24 942	9.94 020	8	37
24	9.69 100	22	9.75 087	30	0.24 913	9.94 012	7	36
25	9.69 122	22	9.75 117	29	0.24 883	9.94 005	7	35
26	9.69 144	23	9.75 146	30	0.24 854	9.93 998	7	34
27	9.69 167	22	9.75 176	29	0.24 824	9.93 991	7	33
28	9.69 189	23	9.75 205	30	0.24 795̄	9.93 984	7	32
29	9.69 212	22	9.75 235̄	29	0.24 765	9.93 977	7	31
30	9.69 234		9.75 264		0.24 736	9.93 970		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

60° 30′.

PP	30	29		23	22		8	7
.1	3.0	2.9	.1	2.3	2.2	.1	0.8	0.7
.2	6.0	5.8	.2	4.6	4.4	.2	1.6	1.4
.3	9.0	8.7	.3	6.9	6.6	.3	2.4	2.1
.4	12.0	11.6	.4	9.2	8.8	.4	3.2	2.8
.5	15.0	14.5	.5	11.5	11.0	.5	4.0	3.5
.6	18.0	17.4	.6	13.8	13.2	.6	4.8	4.2
.7	21.0	20.3	.7	16.1	15.4	.7	5.6	4.9
.8	24.0	23.2	.8	18.4	17.6	.8	6.4	5.6
.9	27.0	26.1	.9	20.7	19.8	.9	7.2	6.3

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.69 234	22	9.75 264	30	0.24 736	9.93 970	7	**30**
31	9.69 256	23	9.75 294	29	0.24 706	9.93 963	8	29
32	9.69 279	22	9.75 323	30	0.24 677	9.93 955	7	28
33	9.69 301	22	9.75 353	29	0.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26
35	9.69 345	23	9.75 411	30	0.24 589	9.93 934	7	25
36	9.69 368	22	9.75 441	29	0.24 559	9.93 927	7	24
37	9.69 390	22	9.75 470	30	0.24 530	9.93 920	8	23
38	9.69 412	22	9.75 500	29	0.24 500	9.93 912	7	22
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21
40	9.69 456	23	9.75 558	30	0.24 442	9.93 898	7	**20**
41	9.69 479	22	9.75 588	29	0.24 412	9.93 891	7	19
42	9.69 501	22	9.75 617	30	0.24 383	9.93 884	8	18
43	9.69 523	22	9.75 647	29	0.24 353	9.93 876	7	17
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	30	0.24 29$\bar{5}$	9.93 862	7	15
46	9.69 589	22	9.75 73$\bar{5}$	29	0.24 265	9.93 85$\bar{5}$	8	14
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	7	13
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	30	0.24 178	9.93 833	7	11
50	9.69 677	22	9.75 852	29	0.24 148	9.93 826	7	**10**
51	9.69 699	22	9.75 881	29	0.24 119	9.93 819	8	9
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	7	8
53	9.69 743	22	9.75 939	30	0.24 061	9.93 804	7	7
54	9.69 765	22	9.75 969	29	0.24 031	9.93 797	8	6
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	7	5
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4
57	9.69 831	22	9.76 056	30	0.23 944	9.93 77$\bar{5}$	7	3
58	9.69 853	22	9.76 086	29	0.23 914	9.93 768	8	2
59	9.69 875	22	9.76 11$\bar{5}$	29	0.23 885	9.93 760	7	1
60	9.69 897		9.76 144		0.23 856	9.93 753		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

60°.

PP	30	29		23	22		8	7
.1	3.0	2.9	.1	2.3	2.2	.1	0.8	0.7
.2	6.0	5.8	.2	4.6	4.4	.2	1.6	1.4
.3	9.0	8.7	.3	6.9	6.6	.3	2.4	2.1
.4	12.0	11.6	.4	9.2	8.8	.4	3.2	2.8
.5	15.0	14.5	.5	11.5	11.0	.5	4.0	3.5
.6	18.0	17.4	.6	13.8	13.2	.6	4.8	4.2
.7	21.0	20.3	.7	16.1	15.4	.7	5.6	4.9
.8	24.0	23.2	.8	18.4	17.6	.8	6.4	5.6
.9	27.0	26.1	.9	20.7	19.8	.9	7.2	6.3

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	60
1	9.69 919	22	9.76 173	29	0.23 827	9.93 746	8	59
2	9.69 941	22	9.76 202	29	0.23 798	9.93 738	7	58
3	9.69 963	21	9.76 231	30	0.23 769	9.93 731	7	57
4	9.69 984	22	9.76 261	29	0.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	0.23 710	9.93 717	8	55
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54
7	9.70 05$\bar{0}$	22	9.76 348	29	0.23 652	9.93 702	7	53
8	9.70 072	21	9.76 377	29	0.23 623	9.93 695	8	52
9	9.70 093	22	9.76 406	29	0.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	0.23 56$\bar{5}$	9.93 680	7	50
11	9.70 137	22	9.76 464	29	0.23 536	9.93 673	8	49
12	9.70 159	21	9.76 493	29	0.23 507	9.93 665	7	48
13	9.70 180	22	9.76 522	29	0.23 478	9.93 658	8	47
14	9.70 202	22	9.76 551	29	0.23 449	9.93 650	7	46
15	9.70 224	21	9.76 580	29	0.23 420	9.93 643	7	45
16	9.70 245	22	9.76 609	30	0.23 391	9.93 636	8	44
17	9.70 267	21	9.76 639	29	0.23 361	9.93 628	7	43
18	9.70 288	22	9.76 668	29	0.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	28	0.23 303	9.93 614	8	41
20	9.70 332	21	9.76 725	29	0.23 27$\bar{5}$	9.93 606	7	40
21	9.70 353	22	9.76 754	29	0.23 246	9.93 599	8	39
22	9.70 37$\bar{5}$	21	9.76 783	29	0.23 217	9.93 591	7	38
23	9.70 396	22	9.76 812	29	0.23 188	9.93 584	7	37
24	9.70 418	21	9.76 841	29	0.23 159	9.93 577	8	36
25	9.70 439	22	9.76 870	29	0.23 130	9.93 569	7	35
26	9.70 461	21	9.76 899	29	0.23 101	9.93 562	8	34
27	9.70 482	22	9.76 928	29	0.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	8	32
29	9.70 525	22	9.76 986	29	0.23 014	9.93 539	7	31
30	9.70 547		9.77 01$\bar{5}$		0.22 985	9.93 532		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

59° 30′.

PP	30	29	28		22	21		8	7
.1	3.0	2.9	2.8	.1	2.2	2.1	.1	0.8	0.7
.2	6.0	5.8	5.6	.2	4.4	4.2	.2	1.6	1.4
.3	9.0	8.7	8.4	.3	6.6	6.3	.3	2.4	2.1
.4	12.0	11.6	11.2	.4	8.8	8.4	.4	3.2	2.8
.5	15.0	14.5	14.0	.5	11.0	10.5	.5	4.0	3.5
.6	18.0	17.4	16.8	.6	13.2	12.6	.6	4.8	4.2
.7	21.0	20.3	19.6	.7	15.4	14.7	.7	5.6	4.9
.8	24.0	23.2	22.4	.8	17.6	16.8	.8	6.4	5.6
.9	27.0	26.1	25.2	.9	19.8	18.9	.9	7.2	6.3

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.70 547	21	9.77 015̄	29	0.22 985	9.93 532	7	**30**
31	9.70 568	22	9.77 044	29	0.22 956	9.93 525̄	8	29
32	9.70 590	21	9.77 073	28	0.22 927	9.93 517	7	28
33	9.70 611	22	9.77 101	29	0.22 899	9.93 510	8	27
34	9.70 633	21	9.77 130	29	0.22 870	9.93 502	7	26
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495̄	8	25
36	9.70 675	22	9.77 188	29	0.22 812	9.93 487	7	24
37	9.70 697	21	9.77 217	29	0.22 783	9.93 480	8	23
38	9.70 718	21	9.77 246	28	0.22 754	9.93 472	7	22
39	9.70 739	22	9.77 274	29	0.22 726	9.93 465̄	8	21
40	9.70 761	21	9.77 303	29	0.22 697	9.93 457	7	**20**
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450̄	8	19
42	9.70 803	21	9.77 361	29	0.22 639	9.93 442	7	18
43	9.70 824	22	9.77 390	28	0.22 610	9.93 435̄	8	17
44	9.70 846	21	9.77 418	29	0.22 582	9.93 427	7	16
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	8	15
46	9.70 888	21	9.77 476	29	0.22 524	9.93 412	7	14
47	9.70 909	22	9.77 505̄	28	0.22 495	9.93 405̄	8	13
48	9.70 931	21	9.77 533	29	0.22 467	9.93 397	7	12
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	28	0.22 409	9.93 382	7	**10**
51	9.70 994	21	9.77 619	29	0.22 381	9.93 375̄	8	9
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	7	8
53	9.71 036	22	9.77 677	29	0.22 323	9.93 360	8	7
54	9.71 058	21	9.77 706	28	0.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	29	0.22 266	9.93 344	7	5
56	9.71 100	21	9.77 763	28	0.22 237	9.93 337	8	4
57	9.71 121	21	9.77 791	29	0.22 209	9.93 329	7	3
58	9.71 142	21	9.77 820	29	0.22 180	9.93 322	8	2
59	9.71 163	21	9.77 849	28	0.22 151	9.93 314	7	1
60	9.71 184		9.77 877		0.22 123	9.93 307		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

59°.

PP	29	28		22	21		8	7
.1	2.9	2.8	.1	2.2	2.1	.1	0.8	0.7
.2	5.8	5.6	.2	4.4	4.2	.2	1.6	1.4
.3	8.7	8.4	.3	6.6	6.3	.3	2.4	2.1
.4	11.6	11.2	.4	8.8	8.4	.4	3.2	2.8
.5	14.5	14.0	.5	11.0	10.5	.5	4.0	3.5
.6	17.4	16.8	.6	13.2	12.6	.6	4.8	4.2
.7	20.3	19.6	.7	15.4	14.7	.7	5.6	4.9
.8	23.2	22.4	.8	17.6	16.8	.8	6.4	5.6
.9	26.1	25.2	.9	19.8	18.9	.9	7.2	6.3

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.71 184	21	9.77 877	29	0.22 123	9.93 307	8	**60**
1	9.71 205	21	9.77 906	29	0.22 094	9.93 299	8	59
2	9.71 226	21	9.77 935	28	0.22 065	9.93 291	7	58
3	9.71 247	21	9.77 963	29	0.22 037	9.93 284	8	57
4	9.71 268	21	9.77 992	28	0.22 008	9.93 276	7	56
5	9.71 289	21	9.78 020	29	0.21 980	9.93 269	8	55
6	9.71 310	21	9.78 049	28	0.21 951	9.93 261	8	54
7	9.71 331	21	9.78 077	29	0.21 923	9.93 253	7	53
8	9.71 352	21	9.78 106	29	0.21 894	9.93 246	8	52
9	9.71 373	20	9.78 135	28	0.21 865	9.93 238	8	51
10	9.71 393	21	9.78 163	29	0.21 837	9.93 230	7	**50**
11	9.71 414	21	9.78 192	28	0.21 808	9.93 223	8	49
12	9.71 435	21	9.78 220	29	0.21 780	9.93 215	8	48
13	9.71 456	21	9.78 249	28	0.21 751	9.93 207	7	47
14	9.71 477	21	9.78 277	29	0.21 723	9.93 200	8	46
15	9.71 498	21	9.78 306	28	0.21 694	9.93 192	8	45
16	9.71 519	20	9.78 334	29	0.21 666	9.93 184	7	44
17	9.71 539	21	9.78 363	28	0.21 637	9.93 177	8	43
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	8	42
19	9.71 581	21	9.78 419	29	0.21 581	9.93 161	7	41
20	9.71 602	20	9.78 448	28	0.21 552	9.93 154	8	**40**
21	9.71 622	21	9.78 476	29	0.21 524	9.93 146	8	39
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	7	38
23	9.71 664	21	9.78 533	29	0.21 467	9.93 131	8	37
24	9.71 685	20	9.78 562	28	0.21 438	9.93 123	8	36
25	9.71 705	21	9.78 590	28	0.21 410	9.93 115	7	35
26	9.71 726	21	9.78 618	29	0.21 382	9.93 108	8	34
27	9.71 747	20	9.78 647	28	0.21 353	9.93 100	8	33
28	9.71 767	21	9.78 675	29	0.21 325	9.93 092	8	32
29	9.71 788	21	9.78 704	28	0.21 296	9.93 084	7	31
30	9.71 809		9.78 732		0.21 268	9.93 077		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

58° 30′.

PP	29	28		21	20		8	7
.1	2.9	2.8	.1	2.1	2.0	.1	0.8	0.7
.2	5.8	5.6	.2	4.2	4.0	.2	1.6	1.4
.3	8.7	8.4	.3	6.3	6.0	.3	2.4	2.1
.4	11.6	11.2	.4	8.4	8.0	.4	3.2	2.8
.5	14.5	14.0	.5	10.5	10.0	.5	4.0	3.5
.6	17.4	16.8	.6	12.6	12.0	.6	4.8	4.2
.7	20.3	19.6	.7	14.7	14.0	.7	5.6	4.9
.8	23.2	22.4	.8	16.8	16.0	.8	6.4	5.6
.9	26.1	25.2	.9	18.9	18.0	.9	7.2	6.3

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.71 809	20	9.78 732	28	0.21 268	9.93 077	8	**30**
31	9.71 829	21	9.78 760	29	0.21 240	9.93 069	8	29
32	9.71 85$\bar{0}$	20	9.78 789	28	0.21 211	9.93 061	8	28
33	9.71 870	21	9.78 817	28	0.21 183	9.93 053	7	27
34	9.71 891	20	9.78 845	29	0.21 15$\bar{5}$	9.93 046	8	26
35	9.71 911	21	9.78 874	28	0.21 126	9.93 038	8	25
36	9.71 932	20	9.78 902	28	0.21 098	9.93 030	8	24
37	9.71 952	21	9.78 930	29	0.21 070	9.93 022	8	23
38	9.71 973	21	9.78 959	28	0.21 041	9.93 014	7	22
39	9.71 994	20	9.78 987	28	0.21 013	9.93 007	8	21
40	9.72 014	20	9.79 015	28	0.20 98$\bar{5}$	9.92 999	8	**20**
41	9.72 034	21	9.79 043	29	0.20 957	9.92 991	8	19
42	9.72 05$\bar{5}$	20	9.79 072	28	0.20 928	9.92 983	7	18
43	9.72 075	21	9.79 100	28	0.20 900	9.92 976	8	17
44	9.72 096	20	9.79 128	28	0.20 872	9.92 968	8	16
45	9.72 116	21	9.79 156	29	0.20 844	9.92 960	8	15
46	9.72 137	20	9.79 18$\bar{5}$	28	0.20 815	9.92 952	8	14
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	8	13
48	9.72 177	21	9.79 241	28	0.20 759	9.92 936	7	12
49	9.72 198	20	9.79 269	28	0.20 731	9.92 929	8	11
50	9.72 218	20	9.79 297	29	0.20 703	9.92 921	8	**10**
51	9.72 238	21	9.79 326	28	0.20 674	9.92 913	8	9
52	9.72 259	20	9.79 354	28	0.20 646	9.92 905	8	8
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	8	7
54	9.72 299	21	9.79 410	28	0.20 590	9.92 889	8	6
55	9.72 320	20	9.79 438	28	0.20 562	9.92 881	7	5
56	9.72 340	20	9.79 466	29	0.20 534	9.92 874	8	4
57	9.72 360	21	9.79 49$\bar{5}$	28	0.20 505	9.92 866	8	3
58	9.72 381	20	9.79 523	28	0.20 477	9.92 858	8	2
59	9.72 401	20	9.79 551	28	0.20 449	9.92 85$\bar{0}$	8	1
60	9.72 421		9.79 579		0.20 421	9.92 842		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

58°.

PP	29	28		21	20		8	7
.1	2.9	2.8	.1	2.1	2.0	.1	0.8	0.7
.2	5.8	5.6	.2	4.2	4.0	.2	1.6	1.4
.3	8.7	8.4	.3	6.3	6.0	.3	2.4	2.1
.4	11.6	11.2	.4	8.4	8.0	.4	3.2	2.8
.5	14.5	14.0	.5	10.5	10.0	.5	4.0	3.5
.6	17.4	16.8	.6	12.6	12.0	.6	4.8	4.2
.7	20.3	19.6	.7	14.7	14.0	.7	5.6	4.9
.8	23.2	22.4	.8	16.8	16.0	.8	6.4	5.6
.9	26.1	25.2	.9	18.9	18.0	.9	7.2	6.3

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	60
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	8	59
2	9.72 461	21	9.79 635	28	0.20 365̄	9.92 826	8	58
3	9.72 482	20	9.79 663	28	0.20 337	9.92 818	8	57
4	9.72 502	20	9.79 691	28	0.20 309	9.92 810	7	56
5	9.72 522	20	9.79 719	28	0.20 281	9.92 803	8	55
6	9.72 542	20	9.79 747	29	0.20 253	9.92 795̄	8	54
7	9.72 562	20	9.79 776	28	0.20 224	9.92 787	8	53
8	9.72 582	20	9.79 804	28	0.20 196	9.92 779	8	52
9	9.72 602	20	9.79 832	28	0.20 168	9.92 771	8	51
10	9.72 622	21	9.79 860	28	0.20 140	9.92 763	8	50
11	9.72 643	20	9.79 888	28	0.20 112	9.92 755̄	8	49
12	9.72 663	20	9.79 916	28	0.20 084	9.92 747	8	48
13	9.72 683	20	9.79 944	28	0.20 056	9.92 739	8	47
14	9.72 703	20	9.79 972	28	0.20 028	9.92 731	8	46
15	9.72 723	20	9.80 000	28	0.20 000	9.92 723	8	45
16	9.72 743	20	9.80 028	28	0.19 972	9.92 715	8	44
17	9.72 763	20	9.80 056	28	0.19 944	9.92 707	8	43
18	9.72 783	20	9.80 084	28	0.19 916	9.92 699	8	42
19	9.72 803	20	9.80 112	28	0.19 888	9.92 691	8	41
20	9.72 823	20	9.80 140	28	0.19 860	9.92 683	8	40
21	9.72 843	20	9.80 168	27	0.19 832	9.92 675	8	39
22	9.72 863	20	9.80 195	28	0.19 805̄	9.92 667	8	38
23	9.72 883	19	9.80 223	28	0.19 777	9.92 659	8	37
24	9.72 902	20	9.80 251	28	0.19 749	9.92 651	8	36
25	9.72 922	20	9.80 279	28	0.19 721	9.92 643	8	35
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	0.19 665̄	9.92 627	8	33
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32
29	9.73 002	20	9.80 391	28	0.19 609	9.92 611	8	31
30	9.73 022		9.80 419		0.19 581	9.92 603		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

57° 30′.

PP	29	28	27		21	20	19		8	7
.1	2.9	2.8	2.7	.1	2.1	2.0	1.9	.1	0.8	0.7
.2	5.8	5.6	5.4	.2	4.2	4.0	3.8	.2	1.6	1.4
.3	8.7	8.4	8.1	.3	6.3	6.0	5.7	.3	2.4	2.1
.4	11.6	11.2	10.8	.4	8.4	8.0	7.6	.4	3.2	2.8
.5	14.5	14.0	13.5	.5	10.5	10.0	9.5	.5	4.0	3.5
.6	17.4	16.8	16.2	.6	12.6	12.0	11.4	.6	4.8	4.2
.7	20.3	19.6	18.9	.7	14.7	14.0	13.3	.7	5.6	4.9
.8	23.2	22.4	21.6	.8	16.8	16.0	15.2	.8	6.4	5.6
.9	26.1	25.2	24.3	.9	18.9	18.0	17.1	.9	7.2	6.3

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.73 022	19	9.80 419	28	0.19 581	9.92 603	8	**30**
31	9.73 041	20	9.80 447	27	0.19 553	9.92 595̄	8	29
32	9.73 061	20	9.80 474	28	0.19 526	9.92 587	8	28
33	9.73 081	20	9.80 502	28	0.19 498	9.92 579	8	27
34	9.73 101	20	9.80 530	28	0.19 470	9.92 571	8	26
35	9.73 121	19	9.80 558	28	0.19 442	9.92 563	8	25
36	9.73 140	20	9.80 586	28	0.19 414	9.92 555̄	9	24
37	9.73 160	20	9.80 614	28	0.19 386	9.92 546	8	23
38	9.73 180	20	9.80 642	27	0.19 358	9.92 538	8	22
39	9.73 200	19	9.80 669	28	0.19 331	9.92 530	8	21
40	9.73 219	20	9.80 697	28	0.19 303	9.92 522	8	**20**
41	9.73 239	20	9.80 725̄	28	0.19 275	9.92 514	8	19
42	9.73 259	19	9.80 753	28	0.19 247	9.92 506	8	18
43	9.73 278	20	9.80 781	27	0.19 219	9.92 498	8	17
44	9.73 298	20	9.80 808	28	0.19 192	9.92 490	8	16
45	9.73 318	19	9.80 836	28	0.19 164	9.92 482	9	15
46	9.73 337	20	9.80 864	28	0.19 136	9.92 473	8	14
47	9.73 357	20	9.80 892	27	0.19 108	9.92 465	8	13
48	9.73 377	19	9.80 919	28	0.19 081	9.92 457	8	12
49	9.73 396	20	9.80 947	28	0.19 053	9.92 449	8	11
50	9.73 416	19	9.80 975̄	28	0.19 025	9.92 441	8	**10**
51	9.73 435	20	9.81 003	27	0.18 997	9.92 433	8	9
52	9.73 455̄	19	9.81 030	28	0.18 970	9.92 425̄	9	8
53	9.73 474	20	9.81 058	28	0.18 942	9.92 416	8	7
54	9.73 494	19	9.81 086	27	0.18 914	9.92 408	8	6
55	9.73 513	20	9.81 113	28	0.18 887	9.92 400	8	5
56	9.73 533	19	9.81 141	28	0.18 859	9.92 392	8	4
57	9.73 552	20	9.81 169	27	0.18 831	9.92 384	8	3
58	9.73 572	19	9.81 196	28	0.18 804	9.92 376	9	2
59	9.73 591	20	9.81 224	28	0.18 776	9.92 367	8	1
60	9.73 611		9.81 252		0.18 748	9.92 359		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

57°.

PP	28	27		20	19		9	8
.1	2.8	2.7	.1	2.0	1.9	.1	0.9	0.8
.2	5.6	5.4	.2	4.0	3.8	.2	1.8	1.6
.3	8.4	8.1	.3	6.0	5.7	.3	2.7	2.4
.4	11 2	10.8	.4	8.0	7.6	.4	3.6	3.2
.5	14.0	13.5	.5	10.0	9.5	.5	4.5	4.0
.6	16.8	16.2	.6	12.0	11.4	.6	5.4	4.8
.7	19.6	18.9	.7	14.0	13.3	.7	6.3	5.6
.8	22.4	21.6	.8	16.0	15.2	.8	7.2	6.4
.9	25.2	24.3	.9	18.0	17.1	.9	8.1	7.2

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.73 611	19	9.81 252	27	0.18 748	9.92 359	8	**60**
1	9.73 630	20	9.81 279	28	0.18 721	9.92 351	8	59
2	9.73 650	19	9.81 307	28	0.18 693	9.92 343	8	58
3	9.73 669	20	9.81 335	27	0.18 665	9.92 335	9	57
4	9.73 689	19	9.81 362	28	0.18 638	9.92 326	8	56
5	9.73 708	19	9.81 390	28	0.18 610	9.92 318	8	55
6	9.73 727	20	9.81 418	27	0.18 582	9.92 310	8	54
7	9.73 747	19	9.81 445	28	0.18 555	9.92 302	9	53
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	8	52
9	9.73 785	20	9.81 500	28	0.18 500	9.92 285	8	51
10	9.73 805	19	9.81 528	28	0.18 472	9.92 277	8	**50**
11	9.73 824	19	9.81 556	27	0.18 444	9.92 269	9	49
12	9.73 843	20	9.81 583	28	0.18 417	9.92 260	8	48
13	9.73 863	19	9.81 611	27	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	9	46
15	9.73 901	20	9.81 666	27	0.18 334	9.92 235	8	45
16	9.73 921	19	9.81 693	28	0.18 307	9.92 227	8	44
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	28	0.18 252	9.92 211	9	42
19	9.73 978	19	9.81 776	27	0.18 224	9.92 202	8	41
20	9.73 997	20	9.81 803	28	0.18 197	9.92 194	8	**40**
21	9.74 017	19	9.81 831	27	0.18 169	9.92 186	9	39
22	9.74 036	19	9.81 858	28	0.18 142	9.92 177	8	38
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	28	0.18 087	9.92 161	9	36
25	9.74 093	20	9.81 941	27	0.18 059	9.92 152	8	35
26	9.74 113	19	9.81 968	28	0.18 032	9.92 144	8	34
27	9.74 132	19	9.81 996	27	0.18 004	9.92 136	9	33
28	9.74 151	19	9.82 023	28	0.17 977	9.92 127	8	32
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31
30	9.74 189		9.82 078		0.17 922	9.92 111		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

56° 30′.

PP	28	27		20	19		9	8
.1	2.8	2.7	.1	2.0	1.9	.1	0.9	0.8
.2	5.6	5.4	.2	4.0	3.8	.2	1.8	1.6
.3	8.4	8.1	.3	6.0	5.7	.3	2.7	2.4
.4	11.2	10.8	.4	8.0	7.6	.4	3.6	3.2
.5	14.0	13.5	.5	10.0	9.5	.5	4.5	4.0
.6	16.8	16.2	.6	12.0	11.4	.6	5.4	4.8
.7	19.6	18.9	.7	14.0	13.3	.7	6.3	5.6
.8	22.4	21.6	.8	16.0	15.2	.8	7.2	6.4
.9	25.2	24.3	.9	18.0	17.1	.9	8.1	7.2

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.74 189	19	9.82 078	28	0.17 922	9.92 111	9	30
31	9.74 208	19	9.82 106	27	0.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	28	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 161	27	0.17 839	9.92 086	9	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	28	0.17 785̄	9.92 069	9	25
36	9.74 303	19	9.82 243	27	0.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	28	0.17 730	9.92 052	8	23
38	9.74 341	19	9.82 298	27	0.17 702	9.92 044	9	22
39	9.74 360	19	9.82 325	27	0.17 675̄	9.92 035	8	21
40	9.74 379	19	9.82 352	28	0.17 648	9.92 027	9	20
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	28	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 435̄	27	0.17 565	9.92 002	9	17
44	9.74 455̄	19	9.82 462	27	0.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	28	0.17 511	9.91 985̄	9	15
46	9.74 493	19	9.82 517	27	0.17 483	9.91 976	8	14
47	9.74 512	19	9.82 544	27	0.17 456	9.91 968	9	13
48	9.74 531	18	9.82 571	28	0.17 429	9.91 959	8	12
49	9.74 549	19	9.82 599	27	0.17 401	9.91 951	9	11
50	9.74 568	19	9.82 626	27	0.17 374	9.91 942	8	10
51	9.74 587	19	9.82 653	28	0.17 347	9.91 934	9	9
52	9.74 606	19	9.82 681	27	0.17 319	9.91 925	8	8
53	9.74 625̄	19	9.82 708	27	0.17 292	9.91 917	9	7
54	9.74 644	18	9.82 735	27	0.17 265̄	9.91 908	8	6
55	9.74 662	19	9.82 762	28	0.17 238	9.91 900	9	5
56	9.74 681	19	9.82 790	27	0.17 210	9.91 891	8	4
57	9.74 700	19	9.82 817	27	0.17 183	9.91 883	9	3
58	9.74 719	18	9.82 844	27	0.17 156	9.91 874	8	2
59	9.74 737	19	9.82 871	28	0.17 129	9.91 866	9	1
60	9.74 756		9.82 899		0.17 101	9.91 857		0
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

56°.

PP	28	27		19	18		9	8
.1	2.8	2.7	.1	1.9	1.8	.1	0.9	0.8
.2	5.6	5.4	.2	3.8	3.6	.2	1.8	1.6
.3	8.4	8.1	.3	5.7	5.4	.3	2.7	2.4
.4	11.2	10.8	.4	7.6	7.2	.4	3.6	3.2
.5	14.0	13.5	.5	9.5	9.0	.5	4.5	4.0
.6	16.8	16.2	.6	11.4	10.8	.6	5.4	4.8
.7	19.6	18.9	.7	13.3	12.6	.7	6.3	5.6
.8	22.4	21.6	.8	15.2	14.4	.8	7.2	6.4
.9	25.2	24.3	.9	17.1	16.2	.9	8.1	7.2

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.74 756	19	9.82 899	27	0.17 101	9.91 857	8	**60**
1	9.74 775̄	19	9.82 926	27	0.17 074	9.91 849	9	59
2	9.74 794	18	9.82 953	27	0.17 047	9.91 840	8	58
3	9.74 812	19	9.82 980	28	0.17 020	9.91 832	9	57
4	9.74 831	19	9.83 008	27	0.16 992	9.91 823	8	56
5	9.74 850̄	18	9.83 035̄	27	0.16 965	9.91 815̄	9	55
6	9.74 868	19	9.83 062	27	0.16 938	9.91 806	8	54
7	9.74 887	19	9.83 089	28	0.16 911	9.91 798	9	53
8	9.74 906	18	9.83 117	27	0.16 883	9.91 789	8	52
9	9.74 924	19	9.83 144	27	0.16 856	9.91 781	9	51
10	9.74 943	18	9.83 171	27	0.16 829	9.91 772	9	**50**
11	9.74 961	19	9.83 198	27	0.16 802	9.91 763	8	49
12	9.74 980	19	9.83 225	27	0.16 775̄	9.91 755̄	9	48
13	9.74 999	18	9.83 252	28	0.16 748	9.91 746	8	47
14	9.75 017	19	9.83 280	27	0.16 720	9.91 738	9	46
15	9.75 036	18	9.83 307	27	0.16 693	9.91 729	9	45
16	9.75 054	19	9.83 334	27	0.16 666	9.91 720	8	44
17	9.75 073	18	9.83 361	27	0.16 639	9.91 712	9	43
18	9.75 091	19	9.83 388	27	0.16 612	9.91 703	8	42
19	9.75 110	18	9.83 415	27	0.16 585̄	9.91 695̄	9	41
20	9.75 128	19	9.83 442	28	0.16 558	9.91 686	9	**40**
21	9.75 147	18	9.83 470	27	0.16 530	9.91 677	8	39
22	9.75 165	19	9.83 497	27	0.16 503	9.91 669	9	38
23	9.75 184	18	9.83 524	27	0.16 476	9.91 660	9	37
24	9.75 202	19	9.83 551	27	0.16 449	9.91 651	8	36
25	9.75 221	18	9.83 578	27	0.16 422	9.91 643	9	35
26	9.75 239	19	9.83 605	27	0.16 395̄	9.91 634	9	34
27	9.75 258	18	9.83 632	27	0.16 368	9.91 625	8	33
28	9.75 276	18	9.83 659	27	0.16 341	9.91 617	9	32
29	9.75 294	19	9.83 686	27	0.16 314	9.91 608	9	31
30	9.75 313		9.83 713		0.16 287	9.91 599		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

55° 30′.

PP	28	27		19	18		9	8
.1	2.8	2.7	.1	1.9	1.8	.1	0.9	0.8
.2	5.6	5.4	.2	3.8	3.6	.2	1.8	1.6
.3	8.4	8.1	.3	5.7	5.4	.3	2.7	2.4
.4	11.2	10.8	.4	7.6	7.2	.4	3.6	3.2
.5	14.0	13.5	.5	9.5	9.0	.5	4.5	4.0
.6	16.8	16.2	.6	11.4	10.8	.6	5.4	4.8
.7	19.6	18.9	.7	13.3	12.6	.7	6.3	5.6
.8	22.4	21.6	.8	15.2	14.4	.8	7.2	6.4
.9	25.2	24.3	.9	17.1	16.2	.9	8.1	7.2

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.75 313	18	9.83 713	27	0.16 287	9.91 599	8	**30**
31	9.75 331	19	9.83 740	28	0.16 260	9.91 591	9	29
32	9.75 350	18	9.83 768	27	0.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	0.16 205	9.91 573	8	27
34	9.75 386	19	9.83 822	27	0.16 178	9.91 565	9	26
35	9.75 405	18	9.83 849	27	0.16 151	9.91 556	9	25
36	9.75 423	18	9.83 876	27	0.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	0.16 097	9.91 538	8	23
38	9.75 459	19	9.83 930	27	0.16 070	9.91 530	9	22
39	9.75 478	18	9.83 957	27	0.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	0.16 016	9.91 512	8	**20**
41	9.75 514	19	9.84 011	27	0.15 989	9.91 504	9	19
42	9.75 533	18	9.84 038	27	0.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	0.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	0.15 908	9.91 477	8	16
45	9.75 587	18	9.84 119	27	0.15 881	9.91 469	9	15
46	9.75 605	19	9.84 146	27	0.15 854	9.91 460	9	14
47	9.75 624	18	9.84 173	27	0.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	0.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	0.15 773	9.91 433	8	11
50	9.75 678	18	9.84 254	26	0.15 746	9.91 425	9	**10**
51	9.75 696	18	9.84 280	27	0.15 720	9.91 416	9	9
52	9.75 714	19	9.84 307	27	0.15 693	9.91 407	9	8
53	9.75 733	18	9.84 334	27	0.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	0.15 639	9.91 389	8	6
55	9.75 769	18	9.84 388	27	0.15 612	9.91 381	9	5
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	0.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	0.15 504	9.91 345	9	1
60	9.75 859		9.84 523		0.15 477	9.91 336		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

55°.

PP	28	27	26		19	18		9	8
.1	2.8	2.7	2.6	.1	1.9	1.8	.1	0.9	0.8
.2	5.6	5.4	5.2	.2	3.8	3.6	.2	1.8	1.6
.3	8.4	8.1	7.8	.3	5.7	5.4	.3	2.7	2.4
.4	11.2	10.8	10.4	.4	7.6	7.2	.4	3.6	3.2
.5	14.0	13.5	13.0	.5	9.5	9.0	.5	4.5	4.0
.6	16.8	16.2	15.6	.6	11.4	10.8	.6	5.4	4.8
.7	19.6	18.9	18.2	.7	13.3	12.6	.7	6.3	5.6
.8	22.4	21.6	20.8	.8	15.2	14.4	.8	7.2	6.4
.9	25.2	24.3	23.4	.9	17.1	16.2	.9	8.1	7.2

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.75 859	18	9.84 523	27	0.15 477	9.91 336	8	**60**
1	9.75 877	18	9.84 550	26	0.15 450	9.91 328	9	59
2	9.75 895	18	9.84 576	27	0.15 424	9.91 319	9	58
3	9.75 913	18	9.84 603	27	0.15 397	9.91 310	9	57
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	55
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	8	53
8	9.76 003	18	9.84 738	26	0.15 262	9.91 266	9	52
9	9.76 021	18	9.84 764	27	0.15 236	9.91 257	9	51
10	9.76 039	18	9.84 791	27	0.15 209	9.91 248	9	**50**
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48
13	9.76 093	18	9.84 872	27	0.15 128	9.91 221	9	47
14	9.76 111	18	9.84 899	26	0.15 101	9.91 212	9	46
15	9.76 129	17	9.84 925	27	0.15 075	9.91 203	9	45
16	9.76 146	18	9.84 952	27	0.15 048	9.91 194	9	44
17	9.76 164	18	9.84 979	27	0.15 021	9.91 185	9	43
18	9.76 182	18	9.85 006	27	0.14 994	9.91 176	9	42
19	9.76 200	18	9.85 033	26	0.14 967	9.91 167	9	41
20	9.76 218	18	9.85 059	27	0.14 941	9.91 158	9	**40**
21	9.76 236	17	9.85 086	27	0.14 914	9.91 149	8	39
22	9.76 253	18	9.85 113	27	0.14 887	9.91 141	9	38
23	9.76 271	18	9.85 140	26	0.14 860	9.91 132	9	37
24	9.76 289	18	9.85 166	27	0.14 834	9.91 123	9	36
25	9.76 307	17	9.85 193	27	0.14 807	9.91 114	9	35
26	9.76 324	18	9.85 220	27	0.14 780	9.91 105	9	34
27	9.76 342	18	9.85 247	26	0.14 753	9.91 096	9	33
28	9.76 360	18	9.85 273	27	0.14 727	9.91 087	9	32
29	9.76 378	17	9.85 300	27	0.14 700	9.91 078	9	31
30	9.76 395		9.85 327		0.14 673	9.91 069		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

54° 30′.

PP	27	26		18	17		9	8
.1	2.7	2.6	.1	1.8	1.7	.1	0.9	0.8
.2	5.4	5.2	.2	3.6	3.4	.2	1.8	1.6
.3	8.1	7.8	.3	5.4	5.1	.3	2.7	2.4
.4	10.8	10.4	.4	7.2	6.8	.4	3.6	3.2
.5	13.5	13.0	.5	9.0	8.5	.5	4.5	4.0
.6	16.2	15.6	.6	10.8	10.2	.6	5.4	4.8
.7	18.9	18.2	.7	12.6	11.9	.7	6.3	5.6
.8	21.6	20.8	.8	14.4	13.6	.8	7.2	6.4
.9	24.3	23.4	.9	16.2	15.3	.9	8.1	7.2

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.76 395	18	9.85 327	27	0.14 673	9.91 069	9	30
31	9.76 413	18	9.85 354	26	0.14 646	9.91 060	9	29
32	9.76 431	17	9.85 380	27	0.14 620	9.91 051	9	28
33	9.76 448	18	9.85 407	27	0.14 593	9.91 042	9	27
34	9.76 466	18	9.85 434	26	0.14 566	9.91 033	10	26
35	9.76 484	17	9.85 460	27	0.14 540	9.91 023	9	25
36	9.76 501	18	9.85 487	27	0.14 513	9.91 014	9	24
37	9.76 519	18	9.85 514	26	0.14 486	9.91 005	9	23
38	9.76 537	17	9.85 540	27	0.14 460	9.90 996	9	22
39	9.76 554	18	9.85 567	27	0.14 433	9.90 987	9	21
40	9.76 572	18	9.85 594	26	0.14 406	9.90 978	9	20
41	9.76 590	17	9.85 620	27	0.14 380	9.90 969	9	19
42	9.76 607	18	9.85 647	27	0.14 353	9.90 960	9	18
43	9.76 625̄	17	9.85 674	26	0.14 326	9.90 951	9	17
44	9.76 642	18	9.85 700	27	0.14 300	9.90 942	9	16
45	9.76 660	17	9.85 727	27	0.14 273	9.90 933	9	15
46	9.76 677	18	9.85 754	26	0.14 246	9.90 924	9	14
47	9.76 695̄	17	9.85 780	27	0.14 220	9.90 915̄	9	13
48	9.76 712	18	9.85 807	27	0.14 193	9.90 906	10	12
49	9.76 730	17	9.85 834	26	0.14 166	9.90 896	9	11
50	9.76 747	18	9.85 860	27	0.14 140	9.90 887	9	10
51	9.76 765̄	17	9.85 887	26	0.14 113	9.90 878	9	9
52	9.76 782	18	9.85 913	27	0.14 087	9.90 869	9	8
53	9.76 800	17	9.85 940	27	0.14 060	9.90 860	9	7
54	9.76 817	18	9.85 967	26	0.14 033	9.90 851	9	6
55	9.76 835̄	17	9.85 993	27	0.14 007	9.90 842	10	5
56	9.76 852	18	9.86 020	26	0.13 980	9.90 832	9	4
57	9.76 870	17	9.86 046	27	0.13 954	9.90 823	9	3
58	9.76 887	17	9.86 073	27	0.13 927	9.90 814	9	2
59	9.76 904	18	9.86 100	26	0.13 900	9.90 805̄	9	1
60	9.76 922		9.86 126		0.13 874	9.90 796		0
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

54°.

PP	27	26		18	17		10	9
.1	2.7	2.6	.1	1.8	1.7	.1	1.0	0.9
.2	5.4	5.2	.2	3.6	3.4	.2	2.0	1.8
.3	8.1	7.8	.3	5.4	5.1	.3	3.0	2.7
.4	10.8	10.4	.4	7.2	6.8	.4	4.0	3.6
.5	13.5	13.0	.5	9.0	8.5	.5	5.0	4.5
.6	16.2	15.6	.6	10.8	10.2	.6	6.0	5.4
.7	18.9	18.2	.7	12.6	11.9	.7	7.0	6.3
.8	21.6	20.8	.8	14.4	13.6	.8	8.0	7.2
.9	24.3	23.4	.9	16.2	15.3	.9	9.0	8.1

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.76 922	17	9.86 126	27	0.13 874	9.90 796	9	60
1	9.76 939	18	9.86 153	26	0.13 847	9.90 787	10	59
2	9.76 957	17	9.86 179	27	0.13 821	9.90 777	9	58
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57
4	9.76 991	18	9.86 232	27	0.13 768	9.90 759	9	56
5	9.77 009	17	9.86 259	26	0.13 741	9.90 750	9	55
6	9.77 026	17	9.86 285	27	0.13 715	9.90 741	10	54
7	9.77 043	18	9.86 312	26	0.13 688	9.90 731	9	53
8	9.77 061	17	9.86 338	27	0.13 662	9.90 722	9	52
9	9.77 078	17	9.86 365	27	0.13 635	9.90 713	9	51
10	9.77 095	17	9.86 392	26	0.13 608	9.90 704	10	50
11	9.77 112	18	9.86 418	27	0.13 582	9.90 694	9	49
12	9.77 130	17	9.86 445	26	0.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	27	0.13 529	9.90 676	9	47
14	9.77 164	17	9.86 498	26	0.13 502	9.90 667	10	46
15	9.77 181	18	9.86 524	27	0.13 476	9.90 657	9	45
16	9.77 199	17	9.86 551	26	0.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	27	0.13 397	9.90 630	10	42
19	9.77 250	18	9.86 630	26	0.13 370	9.90 620	9	41
20	9.77 268	17	9.86 656	27	0.13 344	9.90 611	9	40
21	9.77 285	17	9.86 683	26	0.13 317	9.90 602	10	39
22	9.77 302	17	9.86 709	27	0.13 291	9.90 592	9	38
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	27	0.13 238	9.90 574	9	36
25	9.77 353	17	9.86 789	26	0.13 211	9.90 565	10	35
26	9.77 370	17	9.86 815	27	0.13 185	9.90 555	9	34
27	9.77 387	18	9.86 842	26	0.13 158	9.90 546	9	33
28	9.77 405	17	9.86 868	26	0.13 132	9.90 537	10	32
29	9.77 422	17	9.86 894	27	0.13 106	9.90 527	9	31
30	9.77 439		9.86 921		0.13 079	9.90 518		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

53° 30′.

PP	27	26		18	17		10	9
.1	2.7	2.6	.1	1.8	1.7	.1	1.0	0.9
.2	5.4	5.2	.2	3.6	3.4	2	2.0	1.8
.3	8.1	7.8	.3	5.4	5.1	.3	3.0	2.7
.4	10.8	10.4	.4	7.2	6.8	.4	4.0	3.6
.5	13.5	13.0	.5	9.0	8.5	.5	5.0	4.5
.6	16.2	15.6	.6	10.8	10.2	.6	6.0	5.4
.7	18.9	18.2	.7	12.6	11.9	.7	7.0	6.3
.8	21.6	20.8	.8	14.4	13.6	.8	8.0	7.2
.9	24.3	23.4	.9	16.2	15.3	.9	9.0	8.1

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	**30**
31	9.77 456	17	9.86 947	27	0.13 053	9.90 509	10	29
32	9.77 473	17	9.86 974	26	0.13 026	9.90 499	9	28
33	9.77 490	17	9.87 000	27	0.13 000	9.90 490	10	27
34	9.77 507	17	9.87 027	26	0.12 973	9.90 480	9	26
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	25
36	9.77 541	17	9.87 079	27	0.12 921	9.90 462	10	24
37	9.77 558	17	9.87 106	26	0.12 894	9.90 452	9	23
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	27	0.12 842	9.90 434	10	21
40	9.77 609	17	9.87 18$\bar{5}$	26	0.12 815	9.90 424	9	**20**
41	9.77 626	17	9.87 211	27	0.12 789	9.90 41$\bar{5}$	10	19
42	9.77 643	17	9.87 238	26	0.12 762	9.90 405	9	18
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	10	17
44	9.77 677	17	9.87 290	27	0.12 710	9.90 386	9	16
45	9.77 694	17	9.87 317	26	0.12 683	9.90 377	9	15
46	9.77 711	17	9.87 343	26	0.12 657	9.90 368	10	14
47	9.77 728	16	9.87 369	27	0.12 631	9.90 358	9	13
48	9.77 744	17	9.87 396	26	0.12 604	9.90 349	10	12
49	9.77 761	17	9.87 422	26	0.12 578	9.90 339	9	11
50	9.77 778	17	9.87 448	27	0.12 552	9.90 330	10	**10**
51	9.77 795	17	9.87 47$\bar{5}$	26	0.12 525	9.90 320	9	9
52	9.77 812	17	9.87 501	26	0.12 499	9.90 311	10	8
53	9.77 829	17	9.87 527	27	0.12 473	9.90 301	9	7
54	9.77 846	16	9.87 554	26	0.12 446	9.90 292	10	6
55	9.77 862	17	9.87 580	26	0.12 420	9.90 282	9	5
56	9.77 879	17	9.87 606	27	0.12 394	9.90 273	10	4
57	9.77 896	17	9.87 633	26	0.12 367	9.90 263	9	3
58	9.77 913	17	9.87 659	26	0.12 341	9.90 254	10	2
59	9.77 930	16	9.87 685	26	0.12 31$\bar{5}$	9.90 244	9	1
60	9.77 946		9.87 711		0.12 289	9.90 23$\bar{5}$		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

53°.

PP	27	26		17	16		10	9
.1	2.7	2.6	.1	1.7	1.6	.1	1.0	0.9
.2	5.4	5.2	.2	3.4	3.2	.2	2.0	1.8
.3	8.1	7.8	.3	5.1	4.8	.3	3.0	2.7
.4	10.8	10.4	.4	6.8	6.4	.4	4.0	3.6
.5	13.5	13.0	.5	8.5	8.0	.5	5.0	4.5
.6	16.2	15.6	.6	10.2	9.6	.6	6.0	5.4
.7	18.9	18.2	.7	11.9	11.2	.7	7.0	6.3
.8	21.6	20.8	.8	13.6	12.8	.8	8.0	7.2
.9	24.3	23.4	.9	15.3	14.4	.9	9.0	8.1

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.77 946	17	9.87 711	27	0.12 289	9.90 235	10	60
1	9.77 963	17	9.87 738	26	0.12 262	9.90 225	9	59
2	9.77 980	17	9.87 764	26	0.12 236	9.90 216	10	58
3	9.77 997	16	9.87 790	27	0.12 210	9.90 206	9	57
4	9.78 013	17	9.87 817	26	0.12 183	9.90 197	10	56
5	9.78 030	17	9.87 843	26	0.12 157	9.90 187	9	55
6	9.78 047	16	9.87 869	26	0.12 131	9.90 178	10	54
7	9.78 063	17	9.87 895	27	0.12 105	9.90 168	9	53
8	9.78 080	17	9.87 922	26	0.12 078	9.90 159	10	52
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	10	51
10	9.78 113	17	9.87 974	26	0.12 026	9.90 139	9	50
11	9.78 130	17	9.88 000	27	0.12 000	9.90 130	10	49
12	9.78 147	16	9.88 027	26	0.11 973	9.90 120	9	48
13	9.78 163	17	9.88 053	26	0.11 947	9.90 111	10	47
14	9.78 180	17	9.88 079	26	0.11 921	9.90 101	10	46
15	9.78 197	16	9.88 105	26	0.11 895	9.90 091	9	45
16	9.78 213	17	9.88 131	27	0.11 869	9.90 082	10	44
17	9.78 230	16	9.88 158	26	0.11 842	9.90 072	9	43
18	9.78 246	17	9.88 184	26	0.11 816	9.90 063	10	42
19	9.78 263	17	9.88 210	26	0.11 790	9.90 053	10	41
20	9.78 280	16	9.88 236	26	0.11 764	9.90 043	9	40
21	9.78 296	17	9.88 262	27	0.11 738	9.90 034	10	39
22	9.78 313	16	9.88 289	26	0.11 711	9.90 024	10	38
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	9	37
24	9.78 346	16	9.88 341	26	0.11 659	9.90 005	10	36
25	9.78 362	17	9.88 367	26	0.11 633	9.89 995	10	35
26	9.78 379	16	9.88 393	27	0.11 607	9.89 985	9	34
27	9.78 395	17	9.88 420	26	0.11 580	9.89 976	10	33
28	9.78 412	16	9.88 446	26	0.11 554	9.89 966	10	32
29	9.78 428	17	9.88 472	26	0.11 528	9.89 956	9	31
30	9.78 445		9.88 498		0.11 502	9.89 947		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

52° 30′.

PP	27	26		17	16		10	9
.1	2.7	2.6	.1	1.7	1.6	.1	1.0	0.9
.2	5.4	5.2	.2	3.4	3.2	.2	2.0	1.8
.3	8.1	7.8	.3	5.1	4.8	.3	3.0	2.7
.4	10.8	10.4	.4	6.8	6.4	.4	4.0	3.6
.5	13.5	13.0	.5	8.5	8.0	.5	5.0	4.5
.6	16.2	15.6	.6	10.2	9.6	.6	6.0	5.4
.7	18.9	18.2	.7	11.9	11.2	.7	7.0	6.3
.8	21.6	20.8	.8	13.6	12.8	.8	8.0	7.2
.9	24.3	23.4	.9	15.3	14.4	.9	9.0	8.1

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.78 445	16	9.88 498	26	0.11 502	9.89 947	10	**30**
31	9.78 461	17	9.88 524	26	0.11 476	9.89 937	10	29
32	9.78 478	16	9.88 550	27	0.11 450	9.89 927	9	28
33	9.78 494	16	9.88 577	26	0.11 423	9.89 918	10	27
34	9.78 510	17	9.88 603	26	0.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	10	25
36	9.78 543	17	9.88 655	26	0.11 345	9.89 888	9	24
37	9.78 560	16	9.88 681	26	0.11 319	9.89 879	10	23
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22
39	9.78 592	17	9.88 733	26	0.11 267	9.89 859	10	21
40	9.78 609	16	9.88 759	27	0.11 241	9.89 849	9	**20**
41	9.78 625	17	9.88 786	26	0.11 214	9.89 840	10	19
42	9.78 642	16	9.88 812	26	0.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17
44	9.78 674	17	9.88 864	26	0.11 136	9.89 810	9	16
45	9.78 691	16	9.88 890	26	0.11 110	9.89 801	10	15
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13
48	9.78 739	17	9.88 968	26	0.11 032	9.89 771	10	12
49	9.78 756	16	9.88 994	26	0.11 006	9.89 761	9	11
50	9.78 772	16	9.89 020	26	0.10 980	9.89 752	10	**10**
51	9.78 788	17	9.89 046	27	0.10 954	9.89 742	10	9
52	9.78 805	16	9.89 073	26	0.10 927	9.89 732	10	8
53	9.78 821	16	9.89 099	26	0.10 901	9.89 722	10	7
54	9.78 837	16	9.89 125	26	0.10 875	9.89 712	10	6
55	9.78 853	16	9.89 151	26	0.10 849	9.89 702	9	5
56	9.78 869	17	9.89 177	26	0.10 823	9.89 693	10	4
57	9.78 886	16	9.89 203	26	0.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1
60	9.78 934		9.89 281		0.10 719	9.89 653		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

52°.

PP	27	26		17	16		10	9
.1	2.7	2.6	.1	1.7	1.6	.1	1.0	0.9
.2	5.4	5.2	.2	3.4	3.2	.2	2.0	1.8
.3	8.1	7.8	.3	5.1	4.8	.3	3.0	2.7
.4	10.8	10.4	.4	6.8	6.4	.4	4.0	3.6
.5	13.5	13.0	.5	8.5	8.0	.5	5.0	4.5
.6	16.2	15.6	.6	10.2	9.6	.6	6.0	5.4
.7	18.9	18.2	.7	11.9	11.2	.7	7.0	6.3
.8	21.6	20.8	.8	13.6	12.8	.8	8.0	7.2
.9	24.3	23.4	.9	15.3	14.4	.9	9.0	8.1

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	60
1	9.78 950	17	9.89 307	26	0.10 693	9.89 643	10	59
2	9.78 967	16	9.89 333	26	0.10 667	9.89 633	9	58
3	9.78 983	16	9.89 359	26	0.10 641	9.89 624	10	57
4	9.78 999	16	9.89 385	26	0.10 615̄	9.89 614	10	56
5	9.79 015̄	16	9.89 411	26	0.10 589	9.89 604	10	55
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	0.10 485̄	9.89 564	10	51
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	10	50
11	9.79 111	17	9.89 567	26	0.10 433	9.89 544	10	49
12	9.79 128	16	9.89 593	26	0.10 407	9.89 534	10	48
13	9.79 144	16	9.89 619	26	0.10 381	9.89 524	10	47
14	9.79 160	16	9.89 645	26	0.10 355̄	9.89 514	10	46
15	9.79 176	16	9.89 671	26	0.10 329	9.89 504	9	45
16	9.79 192	16	9.89 697	26	0.10 303	9.89 495̄	10	44
17	9.79 208	16	9.89 723	26	0.10 277	9.89 485̄	10	43
18	9.79 224	16	9.89 749	26	0.10 251	9.89 475̄	10	42
19	9.79 240	16	9.89 775	26	0.10 225̄	9.89 465̄	10	41
20	9.79 256	16	9.89 801	26	0.10 199	9.89 455̄	10	40
21	9.79 272	16	9.89 827	26	0.10 173	9.89 445̄	10	39
22	9.79 288	16	9.89 853	26	0.10 147	9.89 435̄	10	38
23	9.79 304	15	9.89 879	26	0.10 121	9.89 425̄	10	37
24	9.79 319	16	9.89 905̄	26	0.10 095	9.89 415̄	10	36
25	9.79 335	16	9.89 931	26	0.10 069	9.89 405̄	10	35
26	9.79 351	16	9.89 957	26	0.10 043	9.89 395̄	10	34
27	9.79 367	16	9.89 983	26	0.10 017	9.89 385̄	10	33
28	9.79 383	16	9.90 009	26	0.09 991	9.89 375̄	11	32
29	9.79 399	16	9.90 035̄	26	0.09 965	9.89 364	10	31
30	9.79 415̄		9.90 061		0.09 939	9.89 354		30
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

51° 30′.

PP	26	17	16		15	11		10	9
.1	2.6	1.7	1.6	.1	1.5	1.1	.1	1.0	0.9
.2	5.2	3.4	3.2	.2	3.0	2.2	.2	2.0	1.8
.3	7.8	5.1	4.8	.3	4.5	3.3	.3	3.0	2.7
.4	10.4	6.8	6.4	.4	6.0	4.4	.4	4.0	3.6
.5	13.0	8.5	8.0	.5	7.5	5.5	.5	5.0	4.5
.6	15.6	10.2	9.6	.6	9.0	6.6	.6	6.0	5.4
.7	18.2	11.9	11.2	.7	10.5	7.7	.7	7.0	6.3
.8	20.8	13.6	12.8	.8	12.0	8.8	.8	8.0	7.2
.9	23.4	15.3	14.4	.9	13.5	9.9	.9	9.0	8.1

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.79 41$\bar{5}$	16	9.90 061	25	0.09 939	9.89 354	10	**30**
31	9.79 431	16	9.90 086	26	0.09 914	9.89 344	10	29
32	9.79 447	16	9.90 112	26	0.09 888	9.89 334	10	28
33	9.79 463	15	9.90 138	26	0.09 862	9.89 324	10	27
34	9.79 478	16	9.90 164	26	0.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	25
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24
37	9.79 526	16	9.90 242	26	0.09 758	9.89 284	10	23
38	9.79 542	16	9.90 268	26	0.09 732	9.89 274	10	22
39	9.79 558	15	9.90 294	26	0.09 706	9.89 264	10	21
40	9.79 573	16	9.90 320	26	0.09 680	9.89 254	10	**20**
41	9.79 589	16	9.90 346	25	0.09 654	9.89 244	11	19
42	9.79 60$\bar{5}$	16	9.90 371	26	0.09 629	9.89 233	10	18
43	9.79 621	15	9.90 397	26	0.09 603	9.89 223	10	17
44	9.79 636	16	9.90 423	26	0.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	15
46	9.79 668	16	9.90 47$\bar{5}$	26	0.09 525	9.89 193	10	14
47	9.79 684	15	9.90 501	26	0.09 499	9.89 183	10	13
48	9.79 699	16	9.90 527	26	0.09 473	9.89 173	11	12
49	9.79 715	16	9.90 553	25	0.09 447	9.89 162	10	11
50	9.79 731	15	9.90 578	26	0.09 422	9.89 152	10	**10**
51	9.79 746	16	9.90 604	26	0.09 396	9.89 142	10	9
52	9.79 762	16	9.90 630	26	0.09 370	9.89 132	10	8
53	9.79 778	15	9.90 656	26	0.09 344	9.89 122	10	7
54	9.79 793	16	9.90 682	26	0.09 318	9.89 112	11	6
55	9.79 809	16	9.90 708	26	0.09 292	9.89 101	10	5
56	9.79 82$\bar{5}$	15	9.90 734	25	0.09 266	9.89 091	10	4
57	9.79 840	16	9.90 759	26	0.09 241	9.89 081	10	3
58	9.79 856	16	9.90 785	26	0.09 21$\bar{5}$	9.89 071	11	2
59	9.79 872	15	9.90 811	26	0.09 189	9.89 060	10	1
60	9.79 887		9.90 837		0.09 163	9.89 050		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

51°.

PP	26	25		16	15		11	10
.1	2.6	2.5	.1	1.6	1.5	.1	1.1	1.0
.2	5.2	5.0	.2	3.2	3.0	.2	2.2	2.0
.3	7.8	7.5	.3	4.8	4.5	.3	3.3	3.0
.4	10.4	10.0	.4	6.4	6.0	.4	4.4	4.0
.5	13.0	12.5	.5	8.0	7.5	.5	5.5	5.0
.6	15.6	15.0	.6	9.6	9.0	.6	6.6	6.0
.7	18.2	17.5	.7	11.2	10.5	.7	7.7	7.0
.8	20.8	20.0	.8	12.8	12.0	.8	8.8	8.0
.9	23.4	22.5	.9	14.4	13.5	.9	9.9	9.0

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.79 887	16	9.90 837	26	0.09 163	9.89 050	10	**60**
1	9.79 903	15	9.90 863	26	0.09 137	9.89 040	10	59
2	9.79 918	16	9.90 889	25	0.09 111	9.89 030	10	58
3	9.79 934	16	9.90 914	26	0.09 086	9.89 020	11	57
4	9.79 950	15	9.90 940	26	0.09 060	9.89 009	10	56
5	9.79 965	16	9.90 966	26	0.09 034	9.88 999	10	55
6	9.79 981	15	9.90 992	26	0.09 008	9.88 989	11	54
7	9.79 996	16	9.91 018	25	0.08 982	9.88 978	10	53
8	9.80 012	15	9.91 043	26	0.08 957	9.88 968	10	52
9	9.80 027	16	9.91 069	26	0.08 931	9.88 958	10	51
10	9.80 043	15	9.91 095	26	0.08 905	9.88 948	11	**50**
11	9.80 058	16	9.91 121	26	0.08 879	9.88 937	10	49
12	9.80 074	15	9.91 147	25	0.08 853	9.88 927	10	48
13	9.80 089	16	9.91 172	26	0.08 828	9.88 917	11	47
14	9.80 105	15	9.91 198	26	0.08 802	9.88 906	10	46
15	9.80 120	16	9.91 224	26	0.08 776	9.88 896	10	45
16	9.80 136	15	9.91 250	26	0.08 750	9.88 886	11	44
17	9.80 151	15	9.91 276	25	0.08 724	9.88 875	10	43
18	9.80 166	16	9.91 301	26	0.08 699	9.88 865	10	42
19	9.80 182	15	9.91 327	26	0.08 673	9.88 855	11	41
20	9.80 197	16	9.91 353	26	0.08 647	9.88 844	10	**40**
21	9.80 213	15	9.91 379	25	0.08 621	9.88 834	10	39
22	9.80 228	16	9.91 404	26	0.08 596	9.88 824	11	38
23	9.80 244	15	9.91 430	26	0.08 570	9.88 813	10	37
24	9.80 259	15	9.91 456	26	0.08 544	9.88 803	10	36
25	9.80 274	16	9.91 482	25	0.08 518	9.88 793	11	35
26	9.80 290	15	9.91 507	26	0.08 493	9.88 782	10	34
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	11	33
28	9.80 320	16	9.91 559	26	0.08 441	9.88 761	10	32
29	9.80 336	15	9.91 585	25	0.08 415	9.88 751	10	31
30	9.80 351		9.91 610		0.08 390	9.88 741		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

50° 30′.

PP	26	25		16	15		11	10
.1	2.6	2.5	.1	1.6	1.5	.1	1.1	1.0
.2	5.2	5.0	.2	3.2	3.0	.2	2.2	2.0
.3	7.8	7.5	.3	4.8	4.5	.3	3.3	3.0
.4	10.4	10.0	.4	6.4	6.0	.4	4.4	4.0
.5	13.0	12.5	.5	8.0	7.5	.5	5.5	5.0
.6	15.6	15.0	.6	9.6	9.0	.6	6.6	6.0
.7	18.2	17.5	.7	11.2	10.5	.7	7.7	7.0
.8	20.8	20.0	.8	12.8	12.0	.8	8.8	8.0
.9	23.4	22.5	.9	14.4	13.5	.9	9.9	9.0

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.80 351	15	9.91 610	26	0.08 390	9.88 741	11	**30**
31	9.80 366	16	9.91 636	26	0.08 364	9.88 730	10	29
32	9.80 382	15	9.91 662	26	0.08 338	9.88 720	11	28
33	9.80 397	15	9.91 688	25	0.08 312	9.88 709	10	27
34	9.80 412	16	9.91 713	26	0.08 287	9.88 699	11	26
35	9.80 428	15	9.91 739	26	0.08 261	9.88 688	10	25
36	9.80 443	15	9.91 76$\bar{5}$	26	0.08 235	9.88 678	10	24
37	9.80 458	15	9.91 791	25	0.08 209	9.88 668	11	23
38	9.80 473	16	9.91 816	26	0.08 184	9.88 657	10	22
39	9.80 489	15	9.91 842	26	0.08 158	9.88 647	11	21
40	9.80 504	15	9.91 868	25	0.08 132	9.88 636	10	**20**
41	9.80 519	15	9.91 893	26	0.08 107	9.88 626	11	19
42	9.80 534	16	9.91 919	26	0.08 081	9.88 615	10	18
43	9.80 $\bar{5}$50	15	9.91 94$\bar{5}$	26	0.08 055	9.88 60$\bar{5}$	11	17
44	9.80 56$\bar{5}$	15	9.91 971	25	0.08 029	9.88 594	10	16
45	9.80 580	15	9.91 996	26	0.08 004	9.88 584	11	15
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	10	14
47	9.80 610	15	9.92 048	25	0.07 952	9.88 563	11	13
48	9.80 625	16	9.92 073	26	0.07 927	9.88 552	10	12
49	9.80 641	15	9.92 099	26	0.07 901	9.88 542	11	11
50	9.80 656	15	9.92 12$\bar{5}$	25	0.07 875	9.88 531	10	**10**
51	9.80 671	15	9.92 150	26	0.07 8$\bar{5}$0	9.88 521	11	9
52	9.80 686	15	9.92 176	26	0.07 824	9.88 510	11	8
53	9.80 701	15	9.92 202	25	0.07 798	9.88 499	10	7
54	9.80 716	15	9.92 227	26	0.07 773	9.88 489	11	6
55	9.80 731	15	9.92 253	26	0.07 747	9.88 478	10	5
56	9.80 746	16	9.92 279	25	0.07 721	9.88 468	11	4
57	9.80 762	15	9.92 304	26	0.07 696	9.88 457	10	3
58	9.80 777	15	9.92 330	26	0.07 670	9.88 447	11	2
59	9.80 792	15	9.92 356	25	0.07 644	9.88 436	11	1
60	9.80 807		9.92 381		0.07 619	9.88 425		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

50°.

PP	26	25		16	15		11	10
.1	2.6	2.5	.1	1.6	1.5	.1	1.1	1.0
.2	5.2	5.0	.2	3.2	3.0	.2	2.2	2.0
.3	7.8	7.5	.3	4.8	4.5	.3	3.3	3.0
.4	10.4	10.0	.4	6.4	6.0	.4	4.4	4.0
.5	13.0	12.5	.5	8.0	7.5	.5	5.5	5.0
.6	15.6	15.0	.6	9.6	9.0	.6	6.6	6.0
.7	18.2	17.5	.7	11.2	10.5	.7	7.7	7.0
.8	20.8	20.0	.8	12.8	12.0	.8	8.8	8.0
.9	23.4	22.5	.9	14.4	13.5	.9	9.9	9.0

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.80 807	15	9.92 381	26	0.07 619	9.88 425	10	**60**
1	9.80 822	15	9.92 407	26	0.07 593	9.88 41$\bar{5}$	11	59
2	9.80 837	15	9.92 433	25	0.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	26	0.07 542	9.88 394	11	57
4	9.80 867	15	9.92 484	26	0.07 516	9.88 383	11	56
5	9.80 882	15	9.92 510	25	0.07 490	9.88 372	10	55
6	9.80 897	15	9.92 535	26	0.07 46$\bar{5}$	9.88 362	11	54
7	9.80 912	15	9.92 561	26	0.07 439	9.88 351	11	53
8	9.80 927	15	9.92 587	25	0.07 413	9.88 340	10	52
9	9.80 942	15	9.92 612	26	0.07 388	9.88 330	11	51
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	11	**50**
11	9.80 972	15	9.92 663	26	0.07 337	9.88 308	10	49
12	9.80 987	15	9.92 689	26	0.07 311	9.88 298	11	48
13	9.81 002	15	9.92 71$\bar{5}$	25	0.07 285	9.88 287	11	47
14	9.81 017	15	9.92 740	26	0.07 260	9.88 276	10	46
15	9.81 032	15	9.92 766	26	0.07 234	9.88 266	11	45
16	9.81 047	14	9.92 792	25	0.07 208	9.88 25$\bar{5}$	11	44
17	9.81 061	15	9.92 817	26	0.07 183	9.88 244	10	43
18	9.81 076	15	9.92 843	25	0.07 157	9.88 234	11	42
19	9.81 091	15	9.92 868	26	0.07 132	9.88 223	11	41
20	9.81 106	15	9.92 894	26	0.07 106	9.88 212	11	**40**
21	9.81 121	15	9.92 920	25	0.07 080	9.88 201	10	39
22	9.81 136	15	9.92 945	26	0.07 05$\bar{5}$	9.88 191	11	38
23	9.81 151	15	9.92 971	25	0.07 029	9.88 180	11	37
24	9.81 166	14	9.92 996	26	0.07 004	9.88 169	11	36
25	9.81 180	15	9.93 022	26	0.06 978	9.88 158	10	35
26	9.81 195	15	9.93 048	25	0.06 952	9.88 148	11	34
27	9.81 210	15	9.93 073	26	0.06 927	9.88 137	11	33
28	9.81 22$\bar{5}$	15	9.93 099	25	0.06 901	9.88 126	11	32
29	9.81 240	14	9.93 124	26	0.06 876	9.88 115	10	31
30	9.81 254		9.93 1$\bar{5}$0		0.06 850	9.88 10$\bar{5}$		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

49° 30′.

PP	26	25		15	14		11	10
.1	2.6	2.5	.1	1.5	1.4	.1	1.1	1.0
.2	5.2	5.0	.2	3.0	2.8	.2	2.2	2.0
.3	7.8	7.5	.3	4.5	4.2	.3	3.3	3.0
.4	10.4	10.0	.4	6.0	5.6	.4	4.4	4.0
.5	13.0	12.5	.5	7.5	7.0	.5	5.5	5.0
.6	15.6	15.0	.6	9.0	8.4	.6	6.6	6.0
.7	18.2	17.5	.7	10.5	9.8	.7	7.7	7.0
.8	20.8	20.0	.8	12.0	11.2	.8	8.8	8.0
.9	23.4	22.5	.9	13.5	12.6	.9	9.9	9.0

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.81 254		9.93 150		0.06 850	9.88 105		**30**
31	9.81 269	15	9.93 175	25	0.06 825	9.88 094	11	29
32	9.81 284	15	9.93 201	26	0.06 799	9.88 083	11	28
33	9.81 299	15	9.93 227	26	0.06 773	9.88 072	11	27
34	9.81 314	15	9.93 252	25	0.06 748	9.88 061	11	26
35	9.81 328	14	9.93 278	26	0.06 722	9.88 051	10	25
36	9.81 343	15	9.93 303	25	0.06 697	9.88 040	11	24
37	9.81 358	15	9.93 329	26	0.06 671	9.88 029	11	23
38	9.81 372	14	9.93 354	25	0.06 646	9.88 018	11	22
39	9.81 387	15	9.93 380	26	0.06 620	9.88 007	11	21
40	9.81 402	15	9.93 406	26	0.06 594	9.87 996	11	**20**
41	9.81 417	15	9.93 431	25	0.06 569	9.87 985	11	19
42	9.81 431	14	9.93 457	26	0.06 543	9.87 975	10	18
43	9.81 446	15	9.93 482	25	0.06 518	9.87 964	11	17
44	9.81 461	15	9.93 508	26	0.06 492	9.87 953	11	16
45	9.81 475	14	9.93 533	25	0.06 467	9.87 942	11	15
46	9.81 490	15	9.93 559	26	0.06 441	9.87 931	11	14
47	9.81 505	15	9.93 584	25	0.06 416	9.87 920	11	13
48	9.81 519	14	9.93 610	26	0.06 390	9.87 909	11	12
49	9.81 534	15	9.93 636	26	0.06 364	9.87 898	11	11
50	9.81 549	15	9.93 661	25	0.06 339	9.87 887	11	**10**
51	9.81 563	14	9.93 687	26	0.06 313	9.87 877	10	9
52	9.81 578	15	9.93 712	25	0.06 288	9.87 866	11	8
53	9.81 592	14	9.93 738	26	0.06 262	9.87 855	11	7
54	9.81 607	15	9.93 763	25	0.06 237	9.87 844	11	6
55	9.81 622	15	9.93 789	26	0.06 211	9.87 833	11	5
56	9.81 636	14	9.93 814	25	0.06 186	9.87 822	11	4
57	9.81 651	15	9.93 840	26	0.06 160	9.87 811	11	3
58	9.81 665	14	9.93 865	25	0.06 135	9.87 800	11	2
59	9.81 680	15	9.93 891	26	0.06 109	9.87 789	11	1
60	9.81 694	14	9.93 916	25	0.06 084	9.87 778	11	**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

49°.

PP	26	25		15	14		11	10
.1	2.6	2.5	.1	1.5	1.4	.1	1.1	1.0
.2	5.2	5.0	.2	3.0	2.8	.2	2.2	2.0
.3	7.8	7.5	.3	4.5	4.2	.3	3.3	3.0
.4	10.4	10.0	.4	6.0	5.6	.4	4.4	4.0
.5	13.0	12.5	.5	7.5	7.0	.5	5.5	5.0
.6	15.6	15.0	.6	9.0	8.4	.6	6.6	6.0
.7	18.2	17.5	.7	10.5	9.8	.7	7.7	7.0
.8	20.8	20.0	.8	12.0	11.2	.8	8.8	8.0
.9	23.4	22.5	.9	13.5	12.6	.9	9.9	9.0

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.81 694	15	9.93 916	26	0.06 084	9.87 778	11	**60**
1	9.81 709	14	9.93 942	25	0.06 058	9.87 767	11	59
2	9.81 723	15	9.93 967	26	0.06 033	9.87 756	11	58
3	9.81 738	14	9.93 993	25	0.06 007	9.87 745	11	57
4	9.81 752	15	9.94 018	26	0.05 982	9.87 734	11	56
5	9.81 767	14	9.94 044	25	0.05 956	9.87 723	11	55
6	9.81 781	15	9.94 069	26	0.05 931	9.87 712	11	54
7	9.81 796	14	9.94 095̄	25	0.05 905	9.87 701	11	53
8	9.81 810	15	9.94 120	26	0.05 880	9.87 690	11	52
9	9.81 825̄	14	9.94 146	25	0.05 854	9.87 679	11	51
10	9.81 839	15	9.94 171	26	0.05 829	9.87 668	11	**50**
11	9.81 854	14	9.94 197	25	0.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	26	0.05 778	9.87 646	11	48
13	9.81 882	15	9.94 248	25	0.05 752	9.87 635̄	11	47
14	9.81 897	14	9.94 273	26	0.05 727	9.87 624	11	46
15	9.81 911	15	9.94 299	25	0.05 701	9.87 613	12	45
16	9.81 926	14	9.94 324	26	0.05 676	9.87 601	11	44
17	9.81 940	15	9.94 350̄	25	0.05 650	9.87 590	11	43
18	9.81 955̄	14	9.94 375	26	0.05 625̄	9.87 579	11	42
19	9.81 969	14	9.94 401	25	0.05 599	9.87 568	11	41
20	9.81 983	15	9.94 426	26	0.05 574	9.87 557	11	**40**
21	9.81 998	14	9.94 452	25	0.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	26	0.05 523	9.87 535̄	11	38
23	9.82 026	15	9.94 503	25	0.05 497	9.87 524	11	37
24	9.82 041	14	9.94 528	26	0.05 472	9.87 513	12	36
25	9.82 055̄	14	9.94 554	25	0.05 446	9.87 501	11	35
26	9.82 069	15	9.94 579	25	0.05 421	9.87 490	11	34
27	9.82 084	14	9.94 604	26	0.05 396	9.87 479	11	33
28	9.82 098	14	9.94 630	25	0.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	26	0.05 345̄	9.87 457	11	31
30	9.82 126		9.94 681		0.05 319	9.87 446		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

48° 30′.

PP	26	25		15	14		12	11
.1	2.6	2.5	.1	1.5	1.4	.1	1.2	1.1
.2	5.2	5.0	.2	3.0	2.8	.2	2.4	2.2
.3	7.8	7.5	.3	4.5	4.2	.3	3.6	3.3
.4	10.4	10.0	.4	6.0	5.6	.4	4.8	4.4
.5	13.0	12.5	.5	7.5	7.0	.5	6.0	5.5
.6	15.6	15.0	.6	9.0	8.4	.6	7.2	6.6
.7	18.2	17.5	.7	10.5	9.8	.7	8.4	7.7
.8	20.8	20.0	.8	12.0	11.2	.8	9.6	8.8
.9	23.4	22.5	.9	13.5	12.6	.9	10.8	9.9

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.82 126	15	9.94 681	25	0.05 319	9.87 446	12	**30**
31	9.82 141	14	9.94 706	26	0.05 294	9.87 434	11	29
32	9.82 155̄	14	9.94 732	25	0.05 268	9.87 423	11	28
33	9.82 169	15	9.94 757	26	0.05 243	9.87 412	11	27
34	9.82 184	14	9.94 783	25	0.05 217	9.87 401	11	26
35	9.82 198	14	9.94 808	26	0.05 192	9.87 390	12	25
36	9.82 212	14	9.94 834	25	0.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23
38	9.82 240	15	9.94 884	26	0.05 116	9.87 356	11	22
39	9.82 255̄	14	9.94 910	25	0.05 090	9.87 345̄	11	21
40	9.82 269	14	9.94 935	26	0.05 065̄	9.87 334	12	**20**
41	9.82 283	14	9.94 961	25	0.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	26	0.05 014	9.87 311	11	18
43	9.82 311	15	9.95 012	25	0.04 988	9.87 300	12	17
44	9.82 326	14	9.95 037	25	0.04 963	9.87 288	11	16
45	9.82 340	14	9.95 062	26	0.04 938	9.87 277	11	15
46	9.82 354	14	9.95 088	25	0.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	26	0.04 887	9.87 255̄	12	13
48	9.82 382	14	9.95 139	25	0.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	26	0.04 836	9.87 232	11	11
50	9.82 410	14	9.95 190	25	0.04 810	9.87 221	12	**10**
51	9.82 424	15	9.95 215	25	0.04 785̄	9.87 209	11	9
52	9.82 439	14	9.95 240	26	0.04 760	9.87 198	11	8
53	9.82 453	14	9.95 266	25	0.04 734	9.87 187	12	7
54	9.82 467	14	9.95 291	26	0.04 709	9.87 175	11	6
55	9.82 481	14	9.95 317	25	0.04 683	9.87 164	11	5
56	9.82 495̄	14	9.95 342	26	0.04 658	9.87 153	12	4
57	9.82 509	14	9.95 368	25	0.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	26	0.04 582	9.87 119	12	1
60	9.82 551		9.95 444		0.04 556	9.87 107		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

48°.

PP	26	25		15	14		12	11
.1	2.6	2.5	1	1.5	1.4	.1	1.2	1.1
.2	5.2	5.0	2	3.0	2.8	.2	2.4	2.2
3	7.8	7.5	.3	4.5	4.2	.3	3.6	3.3
4	10.4	10.0	.4	6.0	5.6	.4	4.8	4.4
.5	13.0	12.5	.5	7.5	7.0	.5	6.0	5.5
.6	15.6	15.0	.6	9.0	8.4	.6	7.2	6.6
.7	18.2	17.5	.7	10.5	9.8	.7	8.4	7.7
.8	20.8	20.0	.8	12.0	11.2	.8	9.6	8.8
.9	23.4	22.5	.9	13.5	12.6	.9	10.8	9.9

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.82 551	14	9.95 444	25	0.04 556	9.87 107	11	**60**
1	9.82 565	14	9.95 469	26	0.04 531	9.87 096	11	59
2	9.82 579	14	9.95 495̄	25	0.04 505	9.87 085̄	12	58
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	26	0.04 455̄	9.87 062	12	56
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050	11	55
6	9.82 635	14	9.95 596	26	0.04 404	9.87 039	11	54
7	9.82 649	14	9.95 622	25	0.04 378	9.87 028	12	53
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	26	0.04 328	9.87 005̄	12	51
10	9.82 691	14	9.95 698	25	0.04 302	9.86 993	11	**50**
11	9.82 705̄	14	9.95 723	25	0.04 277	9.86 982	12	49
12	9.82 719	14	9.95 748	26	0.04 252	9.86 970	11	48
13	9.82 733	14	9.95 774	25	0.04 226	9.86 959	12	47
14	9.82 747	14	9.95 799	26	0.04 201	9.86 947	11	46
15	9.82 761	14	9.95 825̄	25	0.04 175	9.86 936	12	45
16	9.82 775̄	13	9.95 850	25	0.04 150̄	9.86 924	11	44
17	9.82 788	14	9.95 875	26	0.04 125̄	9.86 913	11	43
18	9.82 802	14	9.95 901	25	0.04 099	9.86 902	12	42
19	9.82 816	14	9.95 926	26	0.04 074	9.86 890	11	41
20	9.82 830	14	9.95 952	25	0.04 048	9.86 879	12	**40**
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	12	39
22	9.82 858	14	9.96 002	26	0.03 998	9.86 855	11	38
23	9.82 872	13	9.96 028	25	0.03 972	9.86 844	12	37
24	9.82 885	14	9.96 053	25	0.03 947	9.86 832	11	36
25	9.82 899	14	9.96 078	26	0.03 922	9.86 821	12	35
26	9.82 913	14	9.96 104	25	0.03 896	9.86 809	11	34
27	9.82 927	14	9.96 129	26	0.03 871	9.86 798	12	33
28	9.82 941	14	9.96 155̄	25	0.03 845	9.86 786	11	32
29	9.82 955̄	13	9.96 180	25	0.03 820	9.86 775̄	12	31
30	9.82 968		9.96 205		0.03 795̄	9.86 763		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

47° 30′.

PP	26	25		14	13		12	11
.1	2.6	2.5	1	1.4	1.3	.1	1.2	1.1
.2	5.2	5.0	2	2.8	2.6	.2	2.4	2.2
.3	7.8	7.5	.3	4.2	3.9	.3	3.6	3.3
.4	10.4	10.0	.4	5.6	5.2	.4	4.8	4.4
.5	13.0	12.5	.5	7.0	6.5	.5	6.0	5.5
.6	15.6	15.0	.6	8.4	7.8	.6	7.2	6.6
.7	18.2	17.5	.7	9.8	9.1	.7	8.4	7.7
.8	20.8	20.0	.8	11.2	10.4	.8	9.6	8.8
.9	23.4	22.5	.9	12.6	11.7	.9	10.8	9.9

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.82 968	14	9.96 205	26	0.03 795	9.86 763	11	**30**
31	9.82 982	14	9.96 231	25	0.03 769	9.86 752	12	29
32	9.82 996	14	9.96 256	25	0.03 744	9.86 740	12	28
33	9.83 010	13	9.96 281	26	0.03 719	9.86 728	11	27
34	9.83 023	14	9.96 307	25	0.03 693	9.86 717	12	26
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	11	25
36	9.83 051	14	9.96 357	26	0.03 643	9.86 694	12	24
37	9.83 065	13	9.96 383	25	0.03 617	9.86 682	12	23
38	9.83 078	14	9.96 408	25	0.03 592	9.86 670	11	22
39	9.83 092	14	9.96 433	26	0.03 567	9.86 659	12	21
40	9.83 106	14	9.96 459	25	0.03 541	9.86 647	12	**20**
41	9.83 120	13	9.96 484	26	0.03 516	9.86 635	11	19
42	9.83 133	14	9.96 510	25	0.03 490	9.86 624	12	18
43	9.83 147	14	9.96 535	25	0.03 465	9.86 612	12	17
44	9.83 161	13	9.96 560	26	0.03 440	9.86 600	11	16
45	9.83 174	14	9.96 586	25	0.03 414	9.86 589	12	15
46	9.83 188	14	9.96 611	25	0.03 389	9.86 577	12	14
47	9.83 202	13	9.96 636	26	0.03 364	9.86 565	11	13
48	9.83 215	14	9.96 662	25	0.03 338	9.86 554	12	12
49	9.83 229	13	9.96 687	25	0.03 313	9.86 542	12	11
50	9.83 242	14	9.96 712	26	0.03 288	9.86 530	12	**10**
51	9.83 256	14	9.96 738	25	0.03 262	9.86 518	11	9
52	9.83 270	13	9.96 763	25	0.03 237	9.86 507	12	8
53	9.83 283	14	9.96 788	26	0.03 212	9.86 495	12	7
54	9.83 297	13	9.96 814	25	0.03 186	9.86 483	11	6
55	9.83 310	14	9.96 839	25	0.03 161	9.86 472	12	5
56	9.83 324	14	9.96 864	26	0.03 136	9.86 460	12	4
57	9.83 338	13	9.96 890	25	0.03 110	9.86 448	12	3
58	9.83 351	14	9.96 915	25	0.03 085	9.86 436	11	2
59	9.83 365	13	9.96 940	26	0.03 060	9.86 425	12	1
60	9.83 378		9.96 966		0.03 034	9.86 413		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

47°.

PP	26	25		14	13		12	11
.1	2.6	2.5	.1	1.4	1.3	.1	1.2	1.1
.2	5.2	5.0	.2	2.8	2.6	.2	2.4	2.2
.3	7.8	7.5	.3	4.2	3.9	.3	3.6	3.3
.4	10.4	10.0	.4	5.6	5.2	.4	4.8	4.4
.5	13.0	12.5	.5	7.0	6.5	.5	6.0	5.5
.6	15.6	15.0	.6	8.4	7.8	.6	7.2	6.6
.7	18.2	17.5	.7	9.8	9.1	.7	8.4	7.7
.8	20.8	20.0	.8	11.2	10.4	.8	9.6	8.8
.9	23.4	22.5	.9	12.6	11.7	.9	10.8	9.9

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.83 378	14	9.96 966	25	0.03 034	9.86 413	12	**60**
1	9.83 392	13	9.96 991	25	0.03 009	9.86 401	12	59
2	9.83 405	14	9.97 016	26	0.02 984	9.86 389	12	58
3	9.83 419	13	9.97 042	25	0.02 958	9.86 377	11	57
4	9.83 432	14	9.97 067	25	0.02 933	9.86 366	12	56
5	9.83 446	13	9.97 092	26	0.02 908	9.86 354	12	55
6	9.83 459	14	9.97 118	25	0.02 882	9.86 342	12	54
7	9.83 473	13	9.97 143	25	0.02 857	9.86 330	12	53
8	9.83 486	14	9.97 168	25	0.02 832	9.86 318	12	52
9	9.83 5̄00	13	9.97 193	26	0.02 807	9.86 306	11	51
10	9.83 513	14	9.97 219	25	0.02 781	9.86 295̄	12	**50**
11	9.83 527	13	9.97 244	25	0.02 756	9.86 283	12	49
12	9.83 540	14	9.97 269	26	0.02 731	9.86 271	12	48
13	9.83 554	13	9.97 295̄	25	0.02 705	9.86 259	12	47
14	9.83 567	14	9.97 320	25	0.02 680	9.86 247	12	46
15	9.83 581	13	9.97 345	26	0.02 655̄	9.86 235	12	45
16	9.83 594	14	9.97 371	25	0.02 629	9.86 223	12	44
17	9.83 608	13	9.97 396	25	0.02 604	9.86 211	11	43
18	9.83 621	13	9.97 421	26	0.02 579	9.86 200	12	42
19	9.83 634	14	9.97 447	25	0.02 553	9.86 188	12	41
20	9.83 648	13	9.97 472	25	0.02 528	9.86 176	12	**40**
21	9.83 661	13	9.97 497	26	0.02 503	9.86 164	12	39
22	9.83 674	14	9.97 523	25	0.02 477	9.86 152	12	38
23	9.83 688	13	9.97 548	25	0.02 452	9.86 140	12	37
24	9.83 701	14	9.97 573	25	0.02 427	9.86 128	12	36
25	9.83 715̄	13	9.97 598	26	0.02 402	9.86 116	12	35
26	9.83 728	13	9.97 624	25	0.02 376	9.86 104	12	34
27	9.83 741	14	9.97 649	25	0.02 351	9.86 092	12	33
28	9.83 755̄	13	9.97 674	26	0.02 326	9.86 080	12	32
29	9.83 768	13	9.97 700	25	0.02 300	9.86 068	12	31
30	9.83 781		9.97 725̄		0.02 275	9.86 056		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

46° 30′.

PP	26	25		14	13		12	11
.1	2.6	2.5	.1	1.4	1.3	.1	1.2	1.1
.2	5.2	5.0	.2	2.8	2.6	.2	2.4	2.2
.3	7.8	7.5	.3	4.2	3.9	.3	3.6	3.3
.4	10.4	10.0	.4	5.6	5.2	.4	4.8	4.4
.5	13.0	12.5	.5	7.0	6.5	.5	6.0	5.5
.6	15.6	15.0	.6	8.4	7.8	.6	7.2	6.6
.7	18.2	17.5	.7	9.8	9.1	.7	8.4	7.7
.8	20.8	20.0	.8	11.2	10.4	.8	9.6	8.8
.9	23.4	22.5	.9	12.6	11.7	.9	10.8	9.9

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.83 781	14	9.97 725	25	0.02 275	9.86 056	12	**30**
31	9.83 795	13	9.97 750	26	0.02 250	9.86 044	12	29
32	9.83 808	13	9.97 776	25	0.02 224	9.86 032	12	28
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27
34	9.83 834	14	9.97 826	25	0.02 174	9.86 008	12	26
35	9.83 848	13	9.97 851	26	0.02 149	9.85 996	12	25
36	9.83 861	13	9.97 877	25	0.02 123	9.85 984	12	24
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23
38	9.83 887	14	9.97 927	26	0.02 073	9.85 960	12	22
39	9.83 901	13	9.97 953	25	0.02 047	9.85 948	12	21
40	9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	**20**
41	9.83 927	13	9.98 003	26	0.01 997	9.85 924	12	19
42	9.83 940	14	9.98 029	25	0.01 971	9.85 912	12	18
43	9.83 954	13	9.98 054	25	0.01 946	9.85 900	12	17
44	9.83 967	13	9.98 079	25	0.01 921	9.85 888	12	16
45	9.83 980	13	9.98 104	26	0.01 896	9.85 876	12	15
46	9.83 993	13	9.98 130	25	0.01 870	9.85 864	13	14
47	9.84 006	14	9.98 155	25	0.01 845	9.85 851	12	13
48	9.84 020	13	9.98 180	26	0.01 820	9.85 839	12	12
49	9.84 033	13	9.98 206	25	0.01 794	9.85 827	12	11
50	9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	**10**
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9
52	9.84 072	13	9.98 281	26	0.01 719	9.85 791	12	8
53	9.84 085	13	9.98 307	25	0.01 693	9.85 779	13	7
54	9.84 098	14	9.98 332	25	0.01 668	9.85 766	12	6
55	9.84 112	13	9.98 357	26	0.01 643	9.85 754	12	5
56	9.84 125	13	9.98 383	25	0.01 617	9.85 742	12	4
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2
59	9.84 164	13	9.98 458	26	0.01 542	9.85 706	13	1
60	9.84 177		9.98 484		0.01 516	9.85 693		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

46°.

PP	26	25		14	13		12
.1	2.6	2.5	.1	1.4	1.3	.1	1.2
.2	5.2	5.0	.2	2.8	2.6	.2	2.4
.3	7.8	7.5	.3	4.2	3.9	.3	3.6
.4	10.4	10.0	.4	5.6	5.2	.4	4.8
.5	13.0	12.5	.5	7.0	6.5	.5	6.0
.6	15.6	15.0	.6	8.4	7.8	.6	7.2
.7	18.2	17.5	.7	9.8	9.1	.7	8.4
.8	20.8	20.0	.8	11.2	10.4	.8	9.6
.9	23.4	22.5	.9	12.6	11.7	.9	10.8

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
0	9.84 177	13	9.98 484	25	0.01 516	9.85 693	12	**60**
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59
2	9.84 203	13	9.98 534	26	0.01 466	9.85 669	12	58
3	9.84 216	13	9.98 560	25	0.01 440	9.85 657	12	57
4	9.84 229	13	9.98 585̄	25	0.01 415	9.85 645̄	13	56
5	9.84 242	13	9.98 610	25	0.01 390	9.85 632	12	55
6	9.84 255	14	9.98 635	26	0.01 365̄	9.85 620	12	54
7	9.84 269	13	9.98 661	25	0.01 339	9.85 608	12	53
8	9.84 282	13	9.98 686	25	0.01 314	9.85 596	13	52
9	9.84 295̄	13	9.98 711	26	0.01 289	9.85 583	12	51
10	9.84 308	13	9.98 737	25	0.01 263	9.85 571	12	**50**
11	9.84 321	13	9.98 762	25	0.01 238	9.85 559	12	49
12	9.84 334	13	9.98 787	25	0.01 213	9.85 547	13	48
13	9.84 347	13	9.98 812	26	0.01 188	9.85 534	12	47
14	9.84 360	13	9.98 838	25	0.01 162	9.85 522	12	46
15	9.84 373	12	9.98 863	25	0.01 137	9.85 510	13	45
16	9.84 385	13	9.98 888	25	0.01 112	9.85 497	12	44
17	9.84 398	13	9.98 913	26	0.01 087	9.85 485̄	12	43
18	9.84 411	13	9.98 939	25	0.01 061	9.85 473	13	42
19	9.84 424	13	9.98 964	25	0.01 036	9.85 460	12	41
20	9.84 437	13	9.98 989	26	0.01 011	9.85 448	12	**40**
21	9.84 450	13	9.99 015̄	25	0.00 985	9.85 436	13	39
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	12	38
23	9.84 476	13	9.99 065	25	0.00 935̄	9.85 411	12	37
24	9.84 489	13	9.99 090	26	0.00 910	9.85 399	13	36
25	9.84 502	13	9.99 116	25	0.00 884	9.85 386	12	35
26	9.84 515̄	13	9.99 141	25	0.00 859	9.85 374	13	34
27	9.84 528	12	9.99 166	25	0.00 834	9.85 361	12	33
28	9.84 540	13	9.99 191	26	0.00 809	9.85 349	12	32
29	9.84 553	13	9.99 217	25	0.00 783	9.85 337	13	31
30	9.84 566		9.99 242		0.00 758	9.85 324		**30**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

45° 30′.

PP	26	25		14	13		12
.1	2.6	2.5	.1	1.4	1.3	.1	1.2
.2	5.2	5.0	.2	2.8	2.6	.2	2.4
.3	7.8	7.5	.3	4.2	3.9	.3	3.6
.4	10.4	10.0	.4	5.6	5.2	.4	4.8
.5	13.0	12.5	.5	7.0	6.5	.5	6.0
.6	15.6	15.0	.6	8.4	7.8	.6	7.2
.7	18.2	17.5	.7	9.8	9.1	.7	8.4
.8	20.8	20.0	.8	11.2	10.4	.8	9.6
.9	23.4	22.5	.9	12.6	11.7	.9	10.8

′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	12	**30**
31	9.84 579	13	9.99 267	26	0.00 733	9.85 312	13	29
32	9.84 592	13	9.99 293	25	0.00 707	9.85 299	12	28
33	9.84 605	13	9.99 318	25	0.00 682	9.85 287	13	27
34	9.84 618	12	9.99 343	25	0.00 657	9.85 274	12	26
35	9.84 630	13	9.99 368	26	0.00 632	9.85 262	12	25
36	9.84 643	13	9.99 394	25	0.00 606	9.85 250	13	24
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	12	23
38	9.84 669	13	9.99 444	25	0.00 556	9.85 225	13	22
39	9.84 682	12	9.99 469	26	0.00 531	9.85 212	12	21
40	9.84 694	13	9.99 495	25	0.00 505	9.85 200	13	**20**
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	12	19
42	9.84 720	13	9.99 545	25	0.00 455	9.85 175	13	18
43	9.84 733	12	9.99 570	26	0.00 430	9.85 162	12	17
44	9.84 745	13	9.99 596	25	0.00 404	9.85 150	13	16
45	9.84 758	13	9.99 621	25	0.00 379	9.85 137	12	15
46	9.84 771	13	9.99 646	26	0.00 354	9.85 125	13	14
47	9.84 784	12	9.99 672	25	0.00 328	9.85 112	12	13
48	9.84 796	13	9.99 697	25	0.00 303	9.85 100	13	12
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	13	11
50	9.84 822	13	9.99 747	26	0.00 253	9.85 074	12	**10**
51	9.84 835	12	9.99 773	25	0.00 227	9.85 062	13	9
52	9.84 847	13	9.99 798	25	0.00 202	9.85 049	12	8
53	9.84 860	13	9.99 823	25	0.00 177	9.85 037	13	7
54	9.84 873	12	9.99 848	26	0.00 152	9.85 024	12	6
55	9.84 885	13	9.99 874	25	0.00 126	9.85 012	13	5
56	9.84 898	13	9.99 899	25	0.00 101	9.84 999	13	4
57	9.84 911	12	9.99 924	25	0.00 076	9.84 986	12	3
58	9.84 923	13	9.99 949	26	0.00 051	9.84 974	13	2
59	9.84 936	13	9.99 975	25	0.00 025	9.84 961	12	1
60	9.84 949		0.00 000		0.00 000	9.84 949		**0**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′

45°.

PP	26	25		14	13		12
.1	2.6	2.5	.1	1.4	1.3	.1	1.2
.2	5.2	5.0	.2	2.8	2.6	.2	2.4
.3	7.8	7.5	.3	4.2	3.9	.3	3.6
.4	10.4	10.0	.4	5.6	5.2	.4	4.8
.5	13.0	12.5	.5	7.0	6.5	.5	6.0
.6	15.6	15.0	.6	8.4	7.8	.6	7.2
.7	18.2	17.5	.7	9.8	9.1	.7	8.4
.8	20.8	20.0	.8	11.2	10.4	.8	9.6
.9	23.4	22.5	.9	12.6	11.7	.9	10.8

TABLE III

FIVE-PLACE LOGARITHMS
OF THE
SINE AND TANGENT OF SMALL ANGLES

THE SINE AND TANGENT TO EVERY SECOND FROM 0° TO 8′; TO EVERY TEN SECONDS FROM 0° TO 2°.

THE COSINE AND COTANGENT TO EVERY SECOND FROM 90° TO 89° 52′; TO EVERY TEN SECONDS FROM 90° TO 88°.

0°. LOGARITHMIC SINE AND TANGENT.

′ ″	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″		
0 0	4. —	68557	98660	*16270	*28763	*38454	*46373	*53067	*58866	*63982	*68557	50	
10	5. 68557	72697	76476	79952	83170	86167	88969	91602	94085̄	96433	98660	40	
20	98660	*00779	*02800	*04730	*06579	*08351	*10055̄	*11694	*13273	*14797	*16270	30	
30	6. 16270	17694	19072	20409	21705	22964	24188	25378	26536	27664	28763	20	
40	28763	29836	30882	31904	32903	33879	34833	35767	36682	37577	38454	10	
50	38454	39315̄	40158	40985	41797	42594	43376	44145̄	44900	45643	46373	0	59
1 0	6.4 6373	7090	7797	8492	9175	9849	*0512	*1165̄	*1808	*2442	*3067	50	
10	6.5 3067	3683	4291	4890	5481	6064	6639	7207	7767	8320	8866	40	
20	8866	9406	9939	*0465	*0985	*1499	*2007	*2509	*3006	*3496	*3982	30	
30	6.6 3982	4462	4936	5406	5870	6330	6785̄	7235̄	7680	8121	8557	20	
40	8557	8990	9418	9841	*0261	*0676	*1088	*1496	*1900	*2300	*2697	10	
50	6.7 2697	3090	3479	3865	4248	4627	5003	5376	5746	6112	6476	0	58
2 0	6476	6836	7193	7548	7900	8248	8595̄	8938	9278	9616	9952	50	
10	9952	*0285̄	*0615̄	*0943	*1268	*1591	*1911	*2230	*2545	*2859	*3170	40	
20	6.8 3170	3479	3786	4091	4394	4694	4993	5289	5584	5876	6167	30	
30	6167	6455	6742	7027	7310	7591	7870	8147	8423	8697	8969	20	
40	8969	9240	9509	9776	*0042	*0306	*0568	*0829	*1088	*1346	*1602	10	
50	6.9 1602	1857	2110	2362	2612	2861	3109	3355̄	3599	3843	4085̄	0	57
3 0	4085̄	4325	4565̄	4803	5039	5275̄	5509	5742	5973	6204	6433	50	
10	6433	6661	6888	7113	7338	7561	7783	8004	8224	8443	8660	40	
20	8660	8877	9093	9307	9520	9733	9944	*0155̄	*0364	*0572	*0779	30	
30	7.0 0779	0986	1191	1395	1599	1801	2003	2203	2403	2602	2800	20	
40	2800	2997	3193	3388	3582	3776	3968	4160	4351	4541	4730	10	
50	4730	4919	5106	5293	5479	5664	5849	6032	6215	6397	6579	0	56
4 0	6579	6759	6939	7118	7296	7474	7651	7827	8003	8177	8351	50	
10	8351	8525̄	8698	8870	9041	9211	9381	9551	9719	9887	*0055̄	40	
20	7.1 0055̄	0222	0388	0553	0718	0882	1046	1209	1371	1533	1694	30	
30	1694	1854	2014	2174	2333	2491	2648	2805	2962	3118	3273	20	
40	3273	3428	3582	3736	3889	4042	4194	4346	4497	4647	4797	10	
50	4797	4947	5096	5244	5392	5540	5687	5833	5979	6125̄	6270	0	55
5 0	7.1 6270	6414	6558	6702	6845	6987	7130	7271	7413	7553	7694	50	
10	7694	7834	7973	8112	8250	8389	8526	8663	8800	8937	9072	40	
20	9072	9208	9343	9478	9612	9746	9879	*0012	*0145̄	*0277	*0409	30	
30	7.2 0409	0540	0671	0802	0932	1062	1191	1320	1449	1577	1705	20	
40	1705	1833	1960	2087	2213	2339	2465	2590	2715	2840	2964	10	
50	2964	3088	3212	3335̄	3458	3580	3702	3824	3946	4067	4188	0	54
6 0	4188	4308	4428	4548	4668	4787	4906	5024	5142	5260	5378	50	
10	5378	5495̄	5612	5728	5845̄	5961	6076	6192	6307	6421	6536	40	
20	6536	6650̄	6764	6877	6991	7104	7216	7329	7441	7552	7664	30	
30	7664	7775	7886	7997	8107	8217	8327	8437	8546	8655̄	8763	20	
40	8763	8872	8980	9088	9196	9303	9410	9517	9623	9730	9836	10	
50	9836	9942	*0047	*0152	*0257	*0362	*0467	*0571	*0675	*0779	*0882	0	53
7 0	7.3 0882	0986	1089	1191	1294	1396	1498	1600	1702	1803	1904	50	
10	1904	2005	2106	2206	2306	2406	2506	2606	2705	2804	2903	40	
20	2903	3001	3100	3198	3296	3393	3491	3588	3685	3782	3879	30	
30	3879	3975	4071	4167	4263	4359	4454	4549	4644	4739	4833	20	
40	4833	4928	5022	5116	5209	5303	5396	5489	5582	5675̄	5767	10	
50	5767	5860	5952	6044	6135	6227	6318	6409	6500	6591	6682	0	52
	10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	

 LOGARITHMIC COSINE AND COTANGENT. **89°.**

FUNCTIONS OF SMALL ANGLES.

0°.

′ ″	L. Sin.	L. Tang.	
0 0	—	—	0 **60**
10	5.68 557	5.68 557	50
20	5.98 660	5.98 660	40
30	6.16 270	6.16 270	30
40	6.28 763	6.28 763	20
50	6.38 454	6.38 454	10
1 0	6.46 373	6.46 373	0 **59**
10	6.53 067	6.53 067	50
20	6.58 866	6.58 866	40
30	6.63 982	6.63 982	30
40	6.68 557	6.68 557	20
50	6.72 697	6.72 697	10
2 0	6.76 476	6.76 476	0 **58**
10	6.79 952	6.79 952	50
20	6.83 170	6.83 170	40
30	6.86 167	6.86 167	30
40	6.88 969	6.88 969	20
50	6.91 602	6.91 602	10
3 0	6.94 08$\bar{5}$	6.94 08$\bar{5}$	0 **57**
10	6.96 433	6.96 433	50
20	6.98 660	6.98 660	40
30	7.00 779	7.00 779	30
40	7.02 800	7.02 800	20
50	7.04 730	7.04 730	10
4 0	7.06 579	7.06 579	0 **56**
10	7.08 351	7.08 352	50
20	7.10 05$\bar{5}$	7.10 05$\bar{5}$	40
30	7.11 694	7.11 694	30
40	7.13 273	7.13 273	20
50	7.14 797	7.14 797	10
5 0	7.16 270	7.16 270	0 **55**
10	7.17 694	7.17 694	50
20	7.19 072	7.19 073	40
30	7.20 409	7.20 409	30
40	7.21 705	7.21 705	20
50	7.22 964	7.22 964	10
6 0	7.24 188	7.24 188	0 **54**
10	7.25 378	7.25 378	50
20	7.26 536	7.26 536	40
30	7.27 664	7.27 664	30
40	7.28 763	7.28 764	20
50	7.29 836	7.29 836	10
7 0	7.30 882	7.30 882	0 **53**
10	7.31 904	7.31 904	50
20	7.32 903	7.32 903	40
30	7.33 879	7.33 879	30 **52**
	L. Cos.	L. Cotg.	″ ′

′ ″	L. Sin.	L. Tang.	
7 30	7.33 879	7.33 879	30
40	7.34 833	7.34 833	20
50	7.35 767	7.35 767	10
8 0	7.36 682	7.36 682	0 **52**
10	7.37 577	7.37 577	50
20	7.38 454	7.38 45$\bar{5}$	40
30	7.39 314	7.39 31$\bar{5}$	30
40	7.40 158	7.40 158	20
50	7.40 985	7.40 985	10
9 0	7.41 797	7.41 797	0 **51**
10	7.42 594	7.42 594	50
20	7.43 376	7.43 376	40
30	7.44 14$\bar{5}$	7.44 145	30
40	7.44 900	7.44 900	20
50	7.45 643	7.45 643	10
10 0	7.46 373	7.46 373	0 **50**
10	7.47 090	7.47 091	50
20	7.47 797	7.47 797	40
30	7.48 491	7.48 492	30
40	7.49 175	7.49 176	20
50	7.49 849	7.49 849	10
11 0	7.50 512	7.50 512	0 **49**
10	7.51 16$\bar{5}$	7.51 165	50
20	7.51 808	7.51 809	40
30	7.52 442	7.52 443	30
40	7.53 067	7.53 067	20
50	7.53 683	7.53 683	10
12 0	7.54 291	7.54 291	0 **48**
10	7.54 890	7.54 890	50
20	7.55 481	7.55 481	40
30	7.56 064	7.56 064	30
40	7.56 639	7.56 639	20
50	7.57 206	7.57 207	10
13 0	7.57 767	7.57 767	0 **47**
10	7.58 320	7.58 320	50
20	7.58 866	7.58 867	40
30	7.59 406	7.59 406	30
40	7.59 939	7.59 939	20
50	7.60 465	7.60 466	10
14 0	7.60 985	7.60 986	0 **46**
10	7.61 499	7.61 $\bar{5}$00	50
20	7.62 007	7.62 008	40
30	7.62 509	7.62 510	30
40	7.63 006	7.63 006	20
50	7.63 496	7.63 497	10
15 0	7.63 982	7.63 982	0 **45**
	L. Cos.	L. Cotg.	″ ′

89°.

0°.

′ ″	L. Sin.	L. Tang.	
15 0	7.63 982	7.63 982	0 **45**
10	7.64 461	7.64 462	50
20	7.64 936	7.64 937	40
30	7.65 406	7.65 406	30
40	7.65 870	7.65 871	20
50	7.66 330	7.66 330	10
16 0	7.66 784	7.66 785	0 **44**
10	7.67 235	7.67 235	50
20	7.67 680	7.67 680	40
30	7.68 121	7.68 121	30
40	7.68 557	7.68 558	20
50	7.68 989	7.68 990	10
17 0	7.69 417	7.69 418	0 **43**
10	7.69 841	7.69 842	50
20	7.70 261	7.70 261	40
30	7.70 676	7.70 677	30
40	7.71 088	7.71 088	20
50	7.71 496	7.71 496	10
18 0	7.71 900	7.71 900	0 **42**
10	7.72 300	7.72 301	50
20	7.72 697	7.72 697	40
30	7.73 090	7.73 090	30
40	7.73 479	7.73 480	20
50	7.73 865	7.73 866	10
19 0	7.74 248	7.74 248	0 **41**
10	7.74 627	7.74 628	50
20	7.75 003	7.75 004	40
30	7.75 376	7.75 377	30
40	7.75 745	7.75 746	20
50	7.76 112	7.76 113	10
20 0	7.76 475	7.76 476	0 **40**
10	7.76 836	7.76 837	50
20	7.77 193	7.77 194	40
30	7.77 548	7.77 549	30
40	7.77 899	7.77 900	20
50	7.78 248	7.78 249	10
21 0	7.78 594	7.78 595	0 **39**
10	7.78 938	7.78 938	50
20	7.79 278	7.79 279	40
30	7.79 616	7.79 617	30
40	7.79 952	7.79 952	20
50	7.80 284	7.80 285	10
22 0	7.80 615	7.80 615	0 **38**
10	7.80 942	7.80 943	50
20	7.81 268	7.81 269	40
30	7.81 591	7.81 591	30 **37**
	L. Cos.	L. Cotg.	″ ′

′ ″	L. Sin.	L. Tang.	
22 30	7.81 591	7.81 591	30
40	7.81 911	7.81 912	20
50	7.82 229	7.82 230	10
23 0	7.82 545	7.82 546	0 **37**
10	7.82 859	7.82 860	50
20	7.83 170	7.83 171	40
30	7.83 479	7.83 480	30
40	7.83 786	7.83 787	20
50	7.84 091	7.84 092	10
24 0	7.84 393	7.84 394	0 **36**
10	7.84 694	7.84 695	50
20	7.84 992	7.84 993	40
30	7.85 289	7.85 290	30
40	7.85 583	7.85 584	20
50	7.85 876	7.85 877	10
25 0	7.86 166	7.86 167	0 **35**
10	7.86 455	7.86 456	50
20	7 86 741	7.86 743	40
30	7.87 026	7.87 027	30
40	7.87 309	7.87 310	20
50	7.87 590	7.87 591	10
26 0	7.87 870	7.87 871	0 **34**
10	7.88 147	7.88 148	50
20	7.88 423	7.88 424	40
30	7.88 697	7.88 698	30
40	7.88 969	7.88 970	20
50	7.89 240	7.89 241	10
27 0	7.89 509	7.89 510	0 **33**
10	7.89 776	7.89 777	50
20	7.90 041	7.90 043	40
30	7.90 305	7.90 307	30
40	7.90 568	7.90 569	20
50	7.90 829	7.90 830	10
28 0	7.91 088	7.91 089	0 **32**
10	7.91 346	7.91 347	50
20	7.91 602	7.91 603	40
30	7.91 857	7.91 858	30
40	7.92 110	7.92 111	20
50	7.92 362	7.92 363	10
29 0	7.92 612	7.92 613	0 **31**
10	7.92 861	7.92 862	50
20	7.93 108	7.93 110	40
30	7.93 354	7.93 356	30
40	7.93 599	7.93 601	20
50	7.93 842	7.93 844	10
30 0	7.94 084	7.94 086	0 **30**
	L. Cos.	L. Cotg.	″ ′

89°.

0°.

′ ″	L. Sin.	L. Tang.	
30 0	7.94 084	7.94 086	0 30
10	7.94 325	7.94 326	50
20	7.94 564	7.94 566	40
30	7.94 802	7.94 804	30
40	7.95 039	7.95 040	20
50	7.95 274	7.95 276	10
31 0	7.95 508	7.95 510	0 29
10	7.95 741	7.95 743	50
20	7.95 973	7.95 974	40
30	7.96 203	7.96 205	30
40	7.96 432	7.96 434	20
50	7.96 660	7.96 662	10
32 0	7.96 887	7.96 889	0 28
10	7.97 113	7.97 114	50
20	7.97 337	7.97 339	40
30	7.97 560	7.97 562	30
40	7.97 782	7.97 784	20
50	7.98 003	7.98 005	10
33 0	7.98 223	7.98 225	0 27
10	7.98 442	7.98 444	50
20	7.98 660	7.98 662	40
30	7.98 876	7.98 878	30
40	7.99 092	7.99 094	20
50	7.99 306	7.99 308	10
34 0	7.99 520	7.99 522	0 26
10	7.99 732	7.99 734	50
20	7.99 943	7.99 946	40
30	8.00 154	8.00 156	30
40	8.00 363	8.00 365	20
50	8.00 571	8.00 574	10
35 0	8.00 779	8.00 781	0 25
10	8.00 985	8.00 987	50
20	8.01 190	8.01 193	40
30	8.01 395	8.01 397	30
40	8.01 598	8.01 600	20
50	8.01 801	8.01 803	10
36 0	8.02 002	8.02 004	0 24
10	8.02 203	8.02 205	50
20	8.02 402	8.02 405	40
30	8.02 601	8.02 604	30
40	8.02 799	8.02 801	20
50	8.02 996	8.02 998	10
37 0	8.03 192	8.03 194	0 23
10	8.03 387	8.03 390	50
20	8.03 581	8.03 584	40
30	8.03 775	8.03 777	30 22
	L. Cos.	L. Cotg.	″ ′

′ ″	L. Sin.	L. Tang.	
37 30	8.03 775	8.03 777	30
40	8.03 967	8.03 970	20
50	8.04 159	8.04 162	10
38 0	8.04 350	8.04 353	0 22
10	8.04 540	8.04 543	50
20	8.04 729	8.04 732	40
30	8.04 918	8.04 921	30
40	8.05 105	8.05 108	20
50	8.05 292	8.05 295	10
39 0	8.05 478	8.05 481	0 21
10	8.05 663	8.05 666	50
20	8.05 848	8.05 851	40
30	8.06 031	8.06 034	30
40	8.06 214	8.06 217	20
50	8.06 396	3.06 399	10
40 0	8.06 578	8.06 581	0 20
10	8.06 758	8.06 761	50
20	8.06 938	8.06 941	40
30	8.07 117	8.07 120	30
40	8.07 295	8.07 298	20
50	8.07 473	8.07 476	10
41 0	8.07 650	8.07 653	0 19
10	8.07 826	8.07 829	50
20	8.08 002	8.08 005	40
30	8.08 176	8.08 180	30
40	8.08 350	8.08 354	20
50	8.08 524	8.08 527	10
42 0	8.08 696	8.08 700	0 18
10	8.08 868	8.08 872	50
20	8.09 040	8.09 043	40
30	8.09 210	8.09 214	30
40	8.09 380	8.09 384	20
50	8.09 550	8.09 553	10
43 0	8.09 718	8.09 722	0 17
10	8.09 886	8.09 890	50
20	8.10 054	8.10 057	40
30	8.10 220	8.10 224	30
40	8.10 386	8.10 390	20
50	8.10 552	8.10 555	10
44 0	8.10 717	8.10 720	0 16
10	8.10 881	8.10 884	50
20	8.11 044	8.11 048	40
30	8.11 207	8.11 211	30
40	8.11 370	8.11 373	20
50	8.11 531	8.11 535	10
45 0	8.11 693	8.11 696	0 15
	L. Cos.	L. Cotg.	″ ′

89°.

0°.

′ ″	L. Sin.	L. Tang.	
45 0	8.11 693	8.11 696	0 **15**
10	8.11 853	8.11 857	50
20	8.12 013	8.12 017	40
30	8.12 172	8.12 176	30
40	8.12 331	8.12 335	20
50	8.12 489	8.12 493	10
46 0	8.12 647	8.12 651	0 **14**
10	8.12 804	8.12 808	50
20	8.12 961	8.12 965	40
30	8.13 117	8.13 121	30
40	8.13 272	8.13 276	20
50	8.13 427	8.13 431	10
47 0	8.13 581	8.13 585	0 **13**
10	8.13 735	8.13 739	50
20	8.13 888	8.13 892	40
30	8.14 041	8.14 045	30
40	8.14 193	8.14 197	20
50	8.14 344	8.14 348	10
48 0	8.14 495	8.14 500	0 **12**
10	8.14 646	8.14 650	50
20	8.14 796	8.14 800	40
30	8.14 945	8.14 950	30
40	8.15 094	8.15 099	20
50	8.15 243	8.15 247	10
49 0	8.15 391	8.15 395	0 **11**
10	8.15 538	8.15 543	50
20	8.15 685	8.15 690	40
30	8.15 832	8.15 836	30
40	8.15 978	8.15 982	20
50	8.16 123	8.16 128	10
50 0	8.16 268	8.16 273	0 **10**
10	8.16 413	8.16 417	50
20	8.16 557	8.16 561	40
30	8.16 700	8.16 705	30
40	8.16 843	8.16 848	20
50	8.16 986	8.16 991	10
51 0	8.17 128	8.17 133	0 **9**
10	8.17 270	8.17 275	50
20	8.17 411	8.17 416	40
30	8.17 552	8.17 557	30
40	8.17 692	8.17 697	20
50	8.17 832	8.17 837	10
52 0	8.17 971	8.17 976	0 **8**
10	8.18 110	8.18 115	50
20	8.18 249	8.18 254	40
30	8.18 387	8.18 392	30 **7**
	L. Cos.	L. Cotg.	″ ′

′ ″	L. Sin.	L. Tang.	
52 30	8.18 387	8.18 392	30
40	8.18 524	8.18 530	20
50	8.18 662	8.18 667	10
53 0	8.18 798	8.18 804	0 **7**
10	8.18 935	8.18 940	50
20	8.19 071	8.19 076	40
30	8.19 206	8.19 212	30
40	8.19 341	8.19 347	20
50	8.19 476	8.19 481	10
54 0	8.19 610	8.19 616	0 **6**
10	8.19 744	8.19 749	50
20	8.19 877	8.19 883	40
30	8.20 010	8.20 016	30
40	8.20 143	8.20 149	20
50	8.20 275	8.20 281	10
55 0	8.20 407	8.20 413	0 **5**
10	8.20 538	8.20 544	50
20	8.20 669	8.20 675	40
30	8.20 800	8.20 806	30
40	8.20 930	8.20 936	20
50	8.21 060	8.21 066	10
56 0	8.21 189	8.21 195	0 **4**
10	8.21 319	8.21 324	50
20	8.21 447	8.21 453	40
30	8.21 576	8.21 581	30
40	8.21 703	8.21 709	20
50	8.21 831	8.21 837	10
57 0	8.21 958	8.21 964	0 **3**
10	8.22 085	8.22 091	50
20	8.22 211	8.22 217	40
30	8.22 337	8.22 343	30
40	8.22 463	8.22 469	20
50	8.22 588	8.22 595	10
58 0	8.22 713	8.22 720	0 **2**
10	8.22 838	8.22 844	50
20	8.22 962	8.22 968	40
30	8.23 086	8.23 092	30
40	8.23 210	8.23 216	20
50	8.23 333	8.23 339	10
59 0	8.23 456	8.23 462	0 **1**
10	8.23 578	8.23 585	50
20	8.23 700	8.23 707	40
30	8.23 822	8.23 829	30
40	8.23 944	8.23 950	20
50	8.24 065	8.24 071	10
60 0	8.24 186	8.24 192	0 **0**
	L. Cos.	L. Cotg.	″ ′

89°.

1°.

′ ″	L. Sin.	L. Tang.	
0 0	8.24 186	8.24 192	0 **60**
10	8.24 306	8.24 313	50
20	8.24 426	8.24 433	40
30	8.24 546	8.24 553	30
40	8.24 665	8.24 672	20
50	8.24 785	8.24 791	10
1 0	8.24 903	8.24 910	0 **59**
10	8.25 022	8.25 029	50
20	8.25 140	8.25 147	40
30	8.25 258	8.25 265	30
40	8.25 375	8.25 382	20
50	8.25 493	8.25 500	10
2 0	8.25 609	8.25 616	0 **58**
10	8.25 726	8.25 733	50
20	8.25 842	8.25 849	40
30	8.25 958	8.25 965	30
40	8.26 074	8.26 081	20
50	8.26 189	8.26 196	10
3 0	8.26 304	8.26 312	0 **57**
10	8.26 419	8.26 426	50
20	8.26 533	8.26 541	40
30	8.26 648	8.26 655	30
40	8.26 761	8.26 769	20
50	8.26 875	8.26 882	10
4 0	8.26 988	8.26 996	0 **56**
10	8.27 101	8.27 109	50
20	8.27 214	8.27 221	40
30	8.27 326	8.27 334	30
40	8.27 438	8.27 446	20
50	8.27 550	8.27 558	10
5 0	8.27 661	8.27 669	0 **55**
10	8.27 773	8.27 780	50
20	8.27 883	8.27 891	40
30	8.27 994	8.28 002	30
40	8.28 104	8.28 112	20
50	8.28 215	8.28 223	10
6 0	8.28 324	8.28 332	0 **54**
10	8.28 434	8.28 442	50
20	8.28 543	8.28 551	40
30	8.28 652	8.28 660	30
40	8.28 761	8.28 769	20
50	8.28 869	8.28 877	10
7 0	8.28 977	8.28 986	0 **53**
10	8.29 085	8.29 094	50
20	8.29 193	8.29 201	40
30	8.29 300	8.29 309	30 **52**
	L. Cos.	L. Cotg.	″ ′

′ ″	L. Sin.	L. Tang.	
7 30	8.29 300	8.29 309	30
40	8.29 407	8.29 416	20
50	8.29 514	8.29 523	10
8 0	8.29 621	8.29 629	0 **52**
10	8.29 727	8.29 736	50
20	8.29 833	8.29 842	40
30	8.29 939	8.29 947	30
40	8.30 044	8.30 053	20
50	8.30 150	8.30 158	10
9 0	8.30 255	8.30 263	0 **51**
10	8.30 359	8.30 368	50
20	8.30 464	8.30 473	40
30	8.30 568	8.30 577	30
40	8.30 672	8.30 681	20
50	8.30 776	8.30 785	10
10 0	8.30 879	8.30 888	0 **50**
10	8.30 983	8.30 992	50
20	8.31 086	8.31 095	40
30	8.31 188	8.31 198	30
40	8.31 291	8.31 300	20
50	8.31 393	8.31 403	10
11 0	8.31 495	8.31 505	0 **49**
10	8.31 597	8.31 606	50
20	8.31 699	8.31 708	40
30	8.31 800	8.31 809	30
40	8.31 901	8.31 911	20
50	8.32 002	8.32 012	10
12 0	8.32 103	8.32 112	0 **48**
10	8.32 203	8.32 213	50
20	8.32 303	8.32 313	40
30	8.32 403	8.32 413	30
40	8.32 503	8.32 513	20
50	8.32 602	8.32 612	10
13 0	8.32 702	8.32 711	0 **47**
10	8.32 801	8.32 811	50
20	8.32 899	8.32 909	40
30	8.32 998	8.33 008	30
40	8.33 096	8.33 106	20
50	8.33 195	8.33 205	10
14 0	8.33 292	8.33 302	0 **46**
10	8.33 390	8.33 400	50
20	8.33 488	8.33 498	40
30	8.33 585	8.33 595	30
40	8.33 682	8.33 692	20
50	8.33 779	8.33 789	10
15 0	8.33 875	8.33 886	0 **45**
	L. Cos.	L. Cotg.	″ ′

88°.

1°.

′ ″	L. Sin.	L. Tang.	
15 0	8.33 875	8.33 886	0 **45**
10	8.33 972	8.33 982	50
20	8.34 068	8.34 078	40
30	8.34 164	8.34 174	30
40	8.34 260	8.34 270	20
50	8.34 355	8.34 366	10
16 0	8.34 450	8.34 461	0 **44**
10	8.34 546	8.34 556	50
20	8.34 640	8.34 651	40
30	8.34 735	8.34 746	30
40	8.34 830	8.34 840	20
50	8.34 924	8.34 935	10
17 0	8.35 018	8.35 029	0 **43**
10	8.35 112	8.35 123	50
20	8.35 206	8.35 217	40
30	8.35 299	8.35 310	30
40	8.35 392	8.35 403	20
50	8.35 485	8.35 497	10
18 0	8.35 578	8.35 590	0 **42**
10	8.35 671	8.35 682	50
20	8.35 764	8.35 775	40
30	8.35 856	8.35 867	30
40	8.35 948	8.35 959	20
50	8.36 040	8.36 051	10
19 0	8.36 131	8.36 143	0 **41**
10	8.36 223	8.36 235	50
20	8.36 314	8.36 326	40
30	8.36 405	8.36 417	30
40	8.36 496	8.36 508	20
50	8.36 587	8.36 599	10
20 0	8.36 678	8.36 689	0 **40**
10	8.36 768	8.36 780	50
20	8.36 858	8.36 870	40
30	8.36 948	8.36 960	30
40	8.37 038	8.37 050	20
50	8.37 128	8.37 140	10
21 0	8.37 217	8.37 229	0 **39**
10	8.37 306	8.37 318	50
20	8.37 395	8.37 408	40
30	8.37 484	8.37 497	30
40	8.37 573	8.37 585	20
50	8.37 662	8.37 674	10
22 0	8.37 750	8.37 762	0 **38**
10	8.37 838	8.37 850	50
20	8.37 926	8.37 938	40
30	8.38 014	8.38 026	30 **37**
	L. Cos.	L. Cotg.	″ ′

′ ″	L. Sin.	L. Tang.	
22 30	8.38 014	8.38 026	30
40	8.38 101	8.38 114	20
50	8.38 189	8.38 202	10
23 0	8.38 276	8.38 289	0 **37**
10	8.38 363	8.38 376	50
20	8.38 450	8.38 463	40
30	8.38 537	8.38 550	30
40	8.38 624	8.38 636	20
50	8.38 710	8.38 723	10
24 0	8.38 796	8.38 809	0 **36**
10	8.38 882	8.38 895	50
20	8.38 968	8.38 981	40
30	8.39 054	8.39 067	30
40	8.39 139	8.39 153	20
50	8.39 225	8.39 238	10
25 0	8.39 310	8.39 323	0 **35**
10	8.39 395	8.39 408	50
20	8.39 480	8.39 493	40
30	8.39 565	8.39 578	30
40	8.39 649	8.39 663	20
50	8.39 734	8.39 747	10
26 0	8.39 818	8.39 832	0 **34**
10	8.39 902	8.39 916	50
20	8.39 986	8.40 000	40
30	8.40 070	8.40 083	30
40	8.40 153	8.40 167	20
50	8.40 237	8.40 251	10
27 0	8.40 320	8.40 334	0 **33**
10	8.40 403	8.40 417	50
20	8.40 486	8.40 500	40
30	8.40 569	8.40 583	30
40	8.40 651	8.40 665	20
50	8.40 734	8.40 748	10
28 0	8.40 816	8.40 830	0 **32**
10	8.40 898	8.40 913	50
20	8.40 980	8.40 995	40
30	8.41 062	8.41 077	30
40	8.41 144	8.41 158	20
50	8.41 225	8.41 240	10
29 0	8.41 307	8.41 321	0 **31**
10	8.41 388	8.41 403	50
20	8.41 469	8.41 484	40
30	8.41 550	8.41 565	30
40	8.41 631	8.41 646	20
50	8.41 711	8.41 726	10
30 0	8.41 792	8.41 807	0 **30**
	L. Cos.	L. Cotg.	′ ″

88°.

FUNCTIONS OF SMALL ANGLES.

1°.

′ ″	L. Sin.	L. Tang.	
30 0	8.41 792	8.41 807	0 30
10	8.41 872	8.41 887	50
20	8.41 952	8.41 967	40
30	8.42 032	8.42 048	30
40	8.42 112	8.42 127	20
50	8.42 192	8.42 207	10
31 0	8.42 272	8.42 287	0 29
10	8.42 351	8.42 366	50
20	8.42 430	8.42 446	40
30	8.42 510	8.42 525	30
40	8.42 589	8.42 604	20
50	8.42 667	8.42 683	10
32 0	8.42 746	8.42 762	0 28
10	8.42 825	8.42 840	50
20	8.42 903	8.42 919	40
30	8.42 982	8.42 997	30
40	8.43 060	8.43 075	20
50	8.43 138	8.43 154	10
33 0	8.43 216	8.43 232	0 27
10	8.43 293	8.43 309	50
20	8.43 371	8.43 387	40
30	8.43 448	8.43 464	30
40	8.43 526	8.43 542	20
50	8.43 603	8.43 619	10
34 0	8.43 680	8.43 696	0 26
10	8.43 757	8.43 773	50
20	8.43 834	8.43 850	40
30	8.43 910	8.43 927	30
40	8.43 987	8.44 003	20
50	8.44 063	8.44 080	10
35 0	8.44 139	8.44 156	0 25
10	8.44 216	8.44 232	50
20	8.44 292	8.44 308	40
30	8.44 367	8.44 384	30
40	8.44 443	8.44 460	20
50	8.44 519	8.44 536	10
36 0	8.44 594	8.44 611	0 24
10	8.44 669	8.44 686	50
20	8.44 745	8.44 762	40
30	8.44 820	8.44 837	30
40	8.44 895	8.44 912	20
50	8.44 969	8.44 987	10
37 0	8.45 044	8.45 061	0 23
10	8.45 119	8.45 136	50
20	8.45 193	8.45 210	40
30	8.45 267	8.45 285	30 22
	L. Cos.	L. Cotg.	″ ′

′ ″	L. Sin.	L. Tang.	
37 30	8.45 267	8.45 285	30
40	8.45 341	8.45 359	20
50	8.45 415	8.45 433	10
38 0	8.45 489	8.45 507	0 22
10	8.45 563	8.45 581	50
20	8.45 637	8.45 655	40
30	8.45 710	8.45 728	30
40	8.45 784	8.45 802	20
50	8.45 857	8.45 875	10
39 0	8.45 930	8.45 948	0 21
10	8.46 003	8.46 021	50
20	8.46 076	8.46 094	40
30	8.46 149	8.46 167	30
40	8.46 222	8.46 240	20
50	8.46 294	8.46 312	10
40 0	8.46 366	8.46 385	0 20
10	8.46 439	8.46 457	50
20	8.46 511	8.46 529	40
30	8.46 583	8.46 602	30
40	8.46 655	8.46 674	20
50	8.46 727	8.46 745	10
41 0	8.46 799	8.46 817	0 19
10	8.46 870	8.46 889	50
20	8.46 942	8.46 960	40
30	8.47 013	8.47 032	30
40	8.47 084	8.47 103	20
50	8.47 155	8.47 174	10
42 0	8.47 226	8.47 245	0 18
10	8.47 297	8.47 316	50
20	8.47 368	8.47 387	40
30	8.47 439	8.47 458	30
40	8.47 509	8.47 528	20
50	8.47 580	8.47 599	10
43 0	8.47 650	8.47 669	0 17
10	8.47 720	8.47 740	50
20	8.47 790	8.47 810	40
30	8.47 860	8.47 880	30
40	8.47 930	8.47 950	20
50	8.48 000	8.48 020	10
44 0	8.48 069	8.48 090	0 16
10	8.48 139	8.48 159	50
20	8.48 208	8.48 228	40
30	8.48 278	8.48 298	30
40	8.48 347	8.48 367	20
50	8.48 416	8.48 436	10
45 0	8.48 485	8.48 505	0 15
	L. Cos.	L. Cotg.	″ ′

88°.

1°.

′ ″	L. Sin.	L. Tang.	
45 0	8.48 485	8.48 505	0 15
10	8.48 554	8.48 574	50
20	8.48 622	8.48 643	40
30	8.48 691	8.48 711	30
40	8.48 760	8.48 780	20
50	8.48 828	8.48 849	10
46 0	8.48 896	8.48 917	0 14
10	8.48 965	8.48 985	50
20	8.49 033	8.49 053	40
30	8.49 101	8.49 121	30
40	8.49 169	8.49 189	20
50	8.49 236	8.49 257	10
47 0	8.49 304	8.49 325	0 13
10	8.49 372	8.49 393	50
20	8.49 439	8.49 460	40
30	8.49 506	8.49 528	30
40	8.49 574	8.49 595	20
50	8.49 641	8.49 662	10
48 0	8.49 708	8.49 729	0 12
10	8.49 775	8.49 796	50
20	8.49 842	8.49 863	40
30	8.49 908	8.49 930	30
40	8.49 975	8.49 997	20
50	8.50 042	8.50 063	10
49 0	8.50 108	8.50 130	0 11
10	8.50 174	8.50 196	50
20	8.50 241	8.50 263	40
30	8.50 307	8.50 329	30
40	8.50 373	8.50 395	20
50	8.50 439	8.50 461	10
50 0	8.50 504	8.50 527	0 10
10	8.50 570	8.50 593	50
20	8.50 636	8.50 658	40
30	8.50 701	8.50 724	30
40	8.50 767	8.50 789	20
50	8.50 832	8.50 855	10
51 0	8.50 897	8.50 920	0 9
10	8.50 963	8.50 985	50
20	8.51 028	8.51 050	40
30	8.51 092	8.51 115	30
40	8.51 157	8.51 180	20
50	8.51 222	8.51 245	10
52 0	8.51 287	8.51 310	0 8
10	8.51 351	8.51 374	50
20	8.51 416	8.51 439	40
30	8.51 480	8.51 503	30 7
	L. Cos.	L. Cotg.	″ ′

′ ″	L. Sin.	L. Tang.	
52 30	8.51 480	8.51 503	30
40	8.51 544	8.51 568	20
50	8.51 609	8.51 632	10
53 0	8.51 673	8.51 696	0 7
10	8.51 737	8.51 760	50
20	8.51 801	8.51 824	40
30	8.51 864	8.51 888	30
40	8.51 928	8.51 952	20
50	8.51 992	8.52 015	10
54 0	8.52 055	8.52 079	0 6
10	8.52 119	8.52 143	50
20	8.52 182	8.52 206	40
30	8.52 245	8.52 269	30
40	8.52 308	8.52 332	20
50	8.52 371	8.52 396	10
55 0	8.52 434	8.52 459	0 5
10	8.52 497	8.52 522	50
20	8.52 560	8.52 584	40
30	8.52 623	8.52 647	30
40	8.52 685	8.52 710	20
50	8.52 748	8.52 772	10
56 0	8.52 810	8.52 835	0 4
10	8.52 872	8.52 897	50
20	8.52 935	8.52 960	40
30	8.52 997	8.53 022	30
40	8.53 059	8.53 084	20
50	8.53 121	8.53 146	10
57 0	8.53 183	8.53 208	0 3
10	8.53 245	8.53 270	50
20	8.53 306	8.53 332	40
30	8.53 368	8.53 393	30
40	8.53 429	8.53 455	20
50	8.53 491	8.53 516	10
58 0	8.53 552	8.53 578	0 2
10	8.53 614	8.53 639	50
20	8.53 675	8.53 700	40
30	8.53 736	8.53 762	30
40	8.53 797	8.53 823	20
50	8.53 858	8.53 884	10
59 0	8.53 919	8.53 945	0 1
10	8.53 979	8.54 005	50
20	8.54 040	8.54 066	40
30	8.54 101	8.54 127	30
40	8.54 161	8.54 187	20
50	8.54 222	8.54 248	10
60 0	8.54 282	8.54 308	0 0
	L. Cos.	L. Cotg.	″ ′

88°.

TABLE IV

FOUR-PLACE NAPERIAN LOGARITHMS

NAPERIAN LOGARITHMS.

LOGARITHMS OF POWERS OF 10.

Num.	Log.	Num.	Log.
10	2.3026	.1	$\bar{3}.6974$
100	4.6052	.01	$\bar{5}.3948$
1000	6.9078	.001	$\bar{7}.0922$
10000	9.2103	.0001	$\overline{10}.7897$
100000	11.5129	.00001	$\overline{12}.4871$
1000000	13.8155	.000001	$\overline{14}.1845$
10000000	16.1181	.0000001	$\overline{17}.8819$
100000000	18.4207	.00000001	$\overline{19}.5793$
1000000000	20.7233	.000000001	$\overline{21}.2767$
Num.	**Log.**	**Num.**	**Log.**

LOGARITHMS OF NUMBERS FROM 1 TO 10.

N	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0100	0198	0296	0392	0488	0583	0677	0770	0862
1.1	0.0953	1044	1133	1222	1310	1398	1484	1570	1655	1740
1.2	0.1823	1906	1989	2070	2151	2231	2311	2390	2469	2546
1.3	0.2624	2700	2776	2852	2927	3001	3075	3148	3221	3293
1.4	0.3365	3436	3507	3577	3646	3716	3784	3853	3920	3988
1.5	0.4055	4121	4187	4253	4318	4383	4447	4511	4574	4637
1.6	0.4700	4762	4824	4886	4947	5008	5068	5128	5188	5247
1.7	0.5306	5365	5423	5481	5539	5596	5653	5710	5766	5822
1.8	0.5878	5933	5988	6043	6098	6152	6206	6259	6313	6366
1.9	0.6419	6471	6523	6575	6627	6678	6729	6780	6831	6881
2.0	0.6931	6981	7031	7080	7129	7178	7227	7275	7324	7372
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

NAPERIAN LOGARITHMS.

N	0	1	2	3	4	5	6	7	8	9
2.0	0.6931	6981	7031	7080	7129	7178	7227	7275	7324	7372
2.1	0.7419	7467	7514	7561	7608	7655	7701	7747	7793	7839
2.2	0.7885	7930	7975	8020	8065	8109	8154	8198	8242	8286
2.3	0.8329	8372	8416	8459	8502	8544	8587	8629	8671	8713
2.4	0.8755	8796	8838	8879	8920	8961	9002	9042	9083	9123
2.5	0.9163	9203	9243	9282	9322	9361	9400	9439	9478	9517
2.6	0.9555	9594	9632	9670	9708	9746	9783	9821	9858	9895
2.7	0.9933	9969	0̄006	0̄043	0̄080	0̄116	0̄152	0188	0̄225	0̄260
2.8	1.0296	0332	0367	0403	0438	0473	0508	0543	0578	0613
2.9	1.0647	0682	0716	0750	0784	0818	0852	0886	0919	0953
3.0	1.0986	1019	1053	1086	1119	1151	1184	1217	1249	1282
3.1	1.1314	1346	1378	1410	1442	1474	1506	1537	1569	1600
3.2	1.1632	1663	1694	1725	1756	1787	1817	1848	1878	1909
3.3	1.1939	1969	2000	2030	2060	2090	2119	2149	2179	2208
3.4	1.2238	2267	2296	2326	2355	2384	2413	2442	2470	2499
3.5	1.2528	2556	2585	2613	2641	2669	2698	2726	2754	2782
3.6	1.2809	2837	2865	2892	2920	2947	2975	3002	3029	3056
3.7	1.3083	3110	3137	3164	3191	3218	3244	3271	3297	3324
3.8	1.3350	3376	3403	3429	3455	3481	3507	3533	3558	3584
3.9	1.3610	3635	3661	3686	3712	3737	3762	3788	3813	3838
4.0	1.3863	3888	3913	3938	3962	3987	4012	4036	4061	4085
4.1	1.4110	4134	4159	4183	4207	4231	4255	4279	4303	4327
4.2	1.4351	4375	4398	4422	4446	4469	4493	4516	4540	4563
4.3	1.4586	4609	4633	4656	4679	4702	4725	4748	4770	4793
4.4	1.4816	4839	4861	4884	4907	4929	4951	4974	4996	5019
4.5	1.5041	5063	5085	5107	5129	5151	5173	5195	5217	5239
4.6	1.5261	5282	5304	5326	5347	5369	5390	5412	5433	5454
4.7	1.5476	5497	5518	5539	5560	5581	5602	5623	5644	5665
4.8	1.5686	5707	5728	5748	5769	5790	5810	5831	5851	5872
4.9	1.5892	5913	5933	5953	5974	5994	6014	6034	6054	6074
5.0	1.6094	6114	6134	6154	6174	6194	6214	6233	6253	6273
5.1	1.6292	6312	6332	6351	6371	6390	6409	6429	6448	6467
5.2	1.6487	6506	6525	6544	6563	6582	6601	6620	6639	6658
5.3	1.6677	6696	6715	6734	6752	6771	6790	6808	6827	6845
5.4	1.6864	6882	6901	6919	6938	6956	6974	6993	7011	7029
5.5	1.7047	7066	7084	7102	7120	7138	7156	7174	7192	7210
5.6	1.7228	7246	7263	7281	7299	7317	7334	7352	7370	7387
5.7	1.7405	7422	7440	7457	7475	7492	7509	7527	7544	7561
5.8	1.7579	7596	7613	7630	7647	7664	7681	7699	7716	7733
5.9	1.7750	7766	7783	7800	7817	7834	7851	7867	7884	7901
6.0	1.7918	7934	7951	7967	7984	8001	8017	8034	8050	8066
	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
6.0	1.7918	7934	7951	7967	7984	8001	8017	8034	8050	8066
6.1	1.8083	8099	8116	8132	8148	8165	8181	8197	8213	8229
6.2	1.8245	8262	8278	8294	8310	8326	8342	8358	8374	8390
6.3	1.8405	8421	8437	8453	8469	8485	8500	8516	8532	8547
6.4	1.8563	8579	8594	8610	8625	8641	8656	8672	8687	8703
6.5	1.8718	8733	8749	8764	8779	8795	8810	8825	8840	8856
6.6	1.8871	8886	8901	8916	8931	8946	8961	8976	8991	9006
6.7	1.9021	9036	9051	9066	9081	9095	9110	9125	9140	9155
6.8	1.9169	9184	9199	9213	9228	9242	9257	9272	9286	9301
6.9	1.9315	9330	9344	9359	9373	9387	9402	9416	9430	9445
7.0	1.9459	9473	9488	9502	9516	9530	9544	9559	9573	9587
7.1	1.9601	9615	9629	9643	9657	9671	9685	9699	9713	9727
7.2	1.9741	9755	9769	9782	9796	9810	9824	9838	9851	9865
7.3	1.9879	9892	9906	9920	9933	9947	9961	9974	9988	$\bar{0}001$
7.4	2.0015	0028	0042	0055	0069	0082	0096	0109	0122	0136
7.5	2.0149	0162	0176	0189	0202	0215	0229	0242	0255	0268
7.6	2.0281	0295	0308	0321	0334	0347	0360	0373	0386	0399
7.7	2.0412	0425	0438	0451	0464	0477	0490	0503	0516	0528
7.8	2.0541	0554	0567	0580	0592	0605	0618	0631	0643	0656
7.9	2.0669	0681	0694	0707	0719	0732	0744	0757	0769	0782
8.0	2.0794	0807	0819	0832	0844	0857	0869	0882	0894	0906
8.1	2.0919	0931	0943	0956	0968	0980	0992	1005	1017	1029
8.2	2.1041	1054	1066	1078	1090	1102	1114	1126	1138	1150
8.3	2.1163	1175	1187	1199	1211	1223	1235	1247	1258	1270
8.4	2.1282	1294	1306	1318	1330	1342	1353	1365	1377	1389
8.5	2.1401	1412	1424	1436	1448	1459	1471	1483	1494	1506
8.6	2.1518	1529	1541	1552	1564	1576	1587	1599	1610	1622
8.7	2.1633	1645	1656	1668	1679	1691	1702	1713	1725	1736
8.8	2.1748	1759	1770	1782	1793	1804	1815	1827	1838	1849
8.9	2.1861	1872	1883	1894	1905	1917	1928	1939	1950	1961
9.0	2.1972	1983	1994	2006	2017	2028	2039	2050	2061	2072
9.1	2.2083	2094	2105	2116	2127	2138	2148	2159	2170	2181
9.2	2.2192	2203	2214	2225	2235	2246	2257	2268	2279	2289
9.3	2.2300	2311	2322	2332	2343	2354	2364	2375	2386	2396
9.4	2.2407	2418	2428	2439	2450	2460	2471	2481	2492	2502
9.5	2.2513	2523	2534	2544	2555	2565	2576	2586	2597	2607
9.6	2.2618	2628	2638	2649	2659	2670	2680	2690	2701	2711
9.7	2.2721	2732	2742	2752	2762	2773	2783	2793	2803	2814
9.8	2.2824	2834	2844	2854	2865	2875	2885	2895	2905	2915
9.9	2.2925	2935	2946	2956	2966	2976	2986	2996	3006	3016
10.0	2.3026	3126	3224	3322	3418	3514	3609	3703	3796	3888
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

TABLE V

FOUR-PLACE LOGARITHMS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9
10	0000	043	086	128	170	212	253	294	334	374
11	414	453	492	531	569	607	645	682	719	755
12	792	828	864	899	934	969	1004	1038	1072	1106
13	1139	173	206	239	271	303	335	367	399	430
14	461	492	523	553	584	614	644	673	703	732
15	1761	790	818	847	875	903	931	959	987	2014
16	2041	068	095	122	148	175	201	227	253	279
17	304	330	355	380	405	430	455	480	504	529
18	553	577	601	625	648	672	695	718	742	765
19	788	810	833	856	878	900	923	945	967	989
20	3010	032	054	075	096	118	139	160	181	201
21	222	243	263	284	304	324	345	365	385	404
22	424	444	464	483	502	522	541	560	579	598
23	617	636	655	674	692	711	729	747	766	784
24	802	820	838	856	874	892	909	927	945	962
25	3979	997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	166	183	200	216	232	249	265	281	298
27	314	330	346	362	378	393	409	425	440	456
28	472	487	502	518	533	548	564	579	594	609
29	624	639	654	669	683	698	713	728	742	757
30	4771	786	800	814	829	843	857	871	886	900
31	914	928	942	955	969	983	997	5011	5024	5038
32	5051	065	079	092	105	119	132	145	159	172
33	185	198	211	224	237	250	263	276	289	302
34	315	328	340	353	366	378	391	403	416	428
35	5441	453	465	478	490	502	514	527	539	551
36	563	575	587	599	611	623	635	647	658	670
37	682	694	705	717	729	740	752	763	775	786
38	798	809	821	832	843	855	866	877	888	899
39	911	922	933	944	955	966	977	988	999	6010
40	6021	031	042	053	064	075	085	096	107	117
N	0	1	2	3	4	5	6	7	8	9

PP	38	32	28	25		22	21	19		18	17	16
.1	3.8	3.2	2.8	2.5	.1	2.2	2.1	1.9	.1	1.8	1.7	1.6
.2	7.6	6.4	5.6	5.0	.2	4.4	4.2	3.8	.2	3.6	3.4	3.2
.3	11.4	9.6	8.4	7.5	.3	6.6	6.3	5.7	.3	5.4	5.1	4.8
.4	15.2	12.8	11.2	10.0	.4	8.8	8.4	7.6	.4	7.2	6.8	6.4
.5	19.0	16.0	14.0	12.5	.5	11.0	10.5	9.5	.5	9.0	8.5	8.0
.6	22.8	19.2	16.8	15.0	.6	13.2	12.6	11.4	.6	10.8	10.2	9.6
.7	26.6	22.4	19.6	17.5	.7	15.4	14.7	13.3	.7	12.6	11.9	11.2
.8	30.4	25.6	22.4	20.0	.8	17.6	16.8	15.2	.8	14.4	13.6	12.8
.9	34.2	28.8	25.2	22.5	.9	19.8	18.9	17.1	.9	16.2	15.3	14.4

N	0	1	2	3	4	5	6	7	8	9
40	6021	031	042	053	064	075	085	096	107	117
41	128	138	149	160	170	180	191	201	212	222
42	232	243	253	263	274	284	294	304	314	325
43	335	345	355	365	375	385	395	405	415	425
44	435	444	454	464	474	484	493	503	513	522
45	6532	542	551	561	571	580	590	599	609	618
46	628	637	646	656	665	675	684	693	702	712
47	721	730	739	749	758	767	776	785	794	803
48	812	821	830	839	848	857	866	875	884	893
49	902	911	920	928	937	946	955	964	972	981
50	6990	998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	084	093	101	110	118	126	135	143	152
52	160	168	177	185	193	202	210	218	226	235
53	243	251	259	267	275	284	292	300	308	316
54	324	332	340	348	356	364	372	380	388	396
55	7404	412	419	427	435	443	451	459	466	474
56	482	490	497	505	513	520	528	536	543	551
57	559	566	574	582	589	597	604	612	619	627
58	634	642	649	657	664	672	679	686	694	701
59	709	716	723	731	738	745	752	760	767	774
60	7782	789	796	803	810	818	825	832	839	846
61	853	860	868	875	882	889	896	903	910	917
62	924	931	938	945	952	959	966	973	980	987
63	993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	069	075	082	089	096	102	109	116	122
65	8129	136	142	149	156	162	169	176	182	189
66	195	202	209	215	222	228	235	241	248	254
67	261	267	274	280	287	293	299	306	312	319
68	325	331	338	344	351	357	363	370	376	382
69	388	395	401	407	414	420	426	432	439	445
70	451	457	463	470	476	482	488	494	500	506
N	0	1	2	3	4	5	6	7	8	9

PP	15	14	13	12		11	10	9		8	7	6
.1	1.5	1.4	1.3	1.2	.1	1.1	1.0	0.9	.1	0.8	0.7	0.6
.2	3.0	2.8	2.6	2.4	.2	2.2	2.0	1.8	.2	1.6	1.4	1.2
.3	4.5	4.2	3.9	3.6	.3	3.3	3.0	2.7	.3	2.4	2.1	1.8
.4	6.0	5.6	5.2	4.8	.4	4.4	4.0	3.6	.4	3.2	2.8	2.4
.5	7.5	7.0	6.5	6.0	.5	5.5	5.0	4.5	.5	4.0	3.5	3.0
.6	9.0	8.4	7.8	7.2	.6	6.6	6.0	5.4	.6	4.8	4.2	3.6
.7	10.5	9.8	9.1	8.4	.7	7.7	7.0	6.3	.7	5.6	4.9	4.2
.8	12.0	11.2	10.4	9.6	.8	8.8	8.0	7.2	.8	6.4	5.6	4.8
.9	13.5	12.6	11.7	10.8	.9	9.9	9.0	8.1	.9	7.2	6.3	5.4

N	0	1	2	3	4	5	6	7	8	9
70	8451	457	463	470	476	482	488	494	500	506
71	513	519	525	531	537	543	549	555	561	567
72	573	579	585	591	597	603	609	615	621	627
73	633	639	645	651	657	663	669	675	681	686
74	692	698	704	710	716	722	727	733	739	745
75	8751	756	762	768	774	779	785	791	797	802
76	808	814	820	825	831	837	842	848	854	859
77	865	871	876	882	887	893	899	904	910	915
78	921	927	932	938	943	949	954	960	965	971
79	976	982	987	993	998	9004	9009	9015	9020	9025
80	9031	036	042	047	053	058	063	069	074	079
81	085	090	096	101	106	112	117	122	128	133
82	138	143	149	154	159	165	170	175	180	186
83	191	196	201	206	212	217	222	227	232	238
84	243	248	253	258	263	269	274	279	284	289
85	9294	299	304	309	315	320	325	330	335	340
86	345	350	355	360	365	370	375	380	385	390
87	395	400	405	410	415	420	425	430	435	440
88	445	450	455	460	465	469	474	479	484	489
89	494	499	504	509	513	518	523	528	533	538
90	9542	547	552	557	562	566	571	576	581	586
91	590	595	600	605	609	614	619	624	628	633
92	638	643	647	652	657	661	666	671	675	680
93	685	689	694	699	703	708	713	717	722	727
94	731	736	741	745	750	754	759	763	768	773
95	9777	782	786	791	795	800	805	809	814	818
96	823	827	832	836	841	845	850	854	859	863
97	868	872	877	881	886	890	894	899	903	908
98	912	917	921	926	930	934	939	943	948	952
99	956	961	965	969	974	978	983	987	991	996
100	0000	004	009	013	017	022	026	030	035	040
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

PP	7	6		5	4
.1	0.7	0.6	.1	0.5	0.4
.2	1.4	1.2	.2	1.0	0.8
.3	2.1	1.8	.3	1.5	1.2
.4	2.8	2.4	.4	2.0	1.6
.5	3.5	3.0	.3	2.5	2.0
.6	4.2	3.6	.6	3.0	2.4
.7	4.9	4.2	.7	3.5	2.8
.8	5.6	4.8	.8	4.0	3.2
.9	6.3	5.4	.9	4.5	3.6

TABLE VI

FOUR-PLACE LOGARITHMS
OF THE
TRIGONOMETRIC FUNCTIONS

TO EVERY TEN MINUTES

°	′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	′	°
0	0	——		——		——	0.0000	0	0	90
	10	7.4637	3011	7.4637	3011	2.5363	0.0000	0	50	
	20	7.7648	1760	7.7648	1761	2.2352	0.0000	0	40	
	30	7.9408	1250	7.9409	1249	2.0591	0.0000	0	30	
	40	8.0658	969	8.0658	969	1.9342	0.0000	0	20	
	50	8.1627	792	8.1627	792	1.8373	0.0000	1	10	
1	0	8.2419	669	8.2419	670	1.7581	9.9999	0	0	89
	10	8.3088	580	8.3089	580	1.6911	0.9999	0	50	
	20	8.3668	511	8.3669	512	1.6331	9.9999	0	40	
	30	8.4179	458	8.4181	457	1.5819	9.9999	1	30	
	40	8.4637	413	8.4638	415	1.5362	9.9998	0	20	
	50	8.5050	378	8.5053	378	1.4947	9.9998	1	10	
2	0	8.5428	348	8.5431	348	1.4569	9.9997	0	0	88
	10	8.5776	321	8.5779	322	1.4221	9.9997	1	50	
	20	8.6097	300	8.6101	300	1.3899	9.9996	0	40	
	30	8.6397	280	8.6401	281	1.3599	9.9996	1	30	
	40	8.6677	263	8.6682	263	1.3318	9.9995	0	20	
	50	8.6940	248	8.6945	249	1.3055	9.9995	1	10	
3	0	8.7188	235	8.7194	235	1.2806	9.9994	1	0	87
	10	8.7423	222	8.7429	223	1.2571	9.9993	0	50	
	20	8.7645	212	8.7652	213	1.2348	9.9993	1	40	
	30	8.7857	202	8.7865	202	1.2135	9.9992	1	30	
	40	8.8059	192	8.8067	194	1.1933	9.9991	1	20	
	50	8.8251	185	8.8261	185	1.1739	9.9990	1	10	
4	0	8.8436	177	8.8446	178	1.1554	9.9989	0	0	86
	10	8.8613	170	8.8624	171	1.1376	9.9989	1	50	
	20	8.8783	163	8.8795	165	1.1205	9.9988	1	40	
	30	8.8946	158	8.8960	158	1.1040	9.9987	1	30	
	40	8.9104	152	8.9118	154	1.0882	9.9986	1	20	
	50	8.9256	147	8.9272	148	1.0728	9.9985	2	10	
5	0	8.9403		8.9420		1.0580	9.9983		0	85
		L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′	°

PP	348	300	263		235	213	185		171	158	147
.1	34.8	30	26.3	.1	23.5	21.3	18.5	.1	17.1	15.8	14.7
.2	69.6	60	52.6	.2	47.0	42.6	37.0	.2	34.2	31.6	29.4
.3	104.4	90	78.9	.3	70.5	63.9	55.5	.3	51.3	47.4	44.1
.4	139.2	120	105.2	.4	94.0	85.2	74.0	.4	68.4	63.2	58.8
.5	174.0	150	131.5	.5	117.5	106.5	92.5	.5	85.5	79.0	73.5
.6	208.8	180	157.8	.6	141.0	127.8	111.0	.6	102.6	94.8	88.2
.7	243.6	210	184.1	.7	164.5	149.1	129.5	.7	119.7	110.6	102.9
.8	278.4	240	210.4	.8	188.0	170.4	148.0	.8	136.8	126.4	117.6
.9	313.2	270	236.7	.9	211.5	191.7	166.5	.9	153.9	142.2	132.3

° ′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
5 0	8.9403	142	8.9420	143	1.0580	9.9983	1	0 85
10	8.9545	137	8.9563	138	1.0437	9.9982	1	50
20	8.9682	134	8.9701	135	1.0299	9.9981	1	40
30	8.9816	129	8.9836	130	1.0164	9.9980	1	30
40	8.9945	125	8.9966	127	1.0034	9.9979	2	20
50	9.0070	122	9.0093	123	0.9907	9.9977	1	10
6 0	9.0192	119	9.0216	120	0.9784	9.9976	1	0 84
10	9.0311	115	9.0336	117	0.9664	9.9975	2	50
20	9.0426	113	9.0453	114	0.9547	9.9973	1	40
30	9.0539	109	9.0567	111	0.9433	9.9972	1	30
40	9.0648	107	9.0678	108	0.9322	9.9971	2	20
50	9.0755	104	9.0786	105	0.9214	9.9969	1	10
7 0	9.0859	102	9.0891	104	0.9109	9.9968	2	0 83
10	9.0961	99	9.0995	101	0.9005	9.9966	2	50
20	9.1060	97	9.1096	98	0.8904	9.9964	1	40
30	9.1157	95	9.1194	97	0.8806	9.9963	2	30
40	9.1252	93	9.1291	94	0.8709	9.9961	2	20
50	9.1345	91	9.1385	93	0.8615	9.9959	1	10
8 0	9.1436	89	9.1478	91	0.8522	9.9958	2	0 82
10	9.1525	87	9.1569	89	0.8431	9.9956	2	50
20	9.1612	85	9.1658	87	0.8342	9.9954	2	40
30	9.1697	84	9.1745	86	0.8255	9.9952	2	30
40	9.1781	82	9.1831	84	0.8169	9.9950	2	20
50	9.1863	80	9.1915	82	0.8085	9.9948	2	10
9 0	9.1943	79	9.1997	81	0.8003	9.9946	2	0 81
10	9.2022	78	9.2078	80	0.7922	9.9944	2	50
20	9.2100	76	9.2158	78	0.7842	9.9942	2	40
30	9.2176	75	9.2236	77	0.7764	9.9940	2	30
40	9.2251	73	9.2313	76	0.7687	9.9938	2	20
50	9.2324	73	9.2389	74	0.7611	9.9936	2	10
10 0	9.2397		9.2463		0.7537	9.9934		0 80
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′ °

PP	138	125	117		104	97	89		84	78	73
.1	13.8	12.5	11.7	.1	10.4	9.7	8.9	.1	8.4	7.8	7.3
.2	27.6	25.0	23.4	.2	20.8	19.4	17.8	.2	16.8	15.6	14.6
.3	41.4	37.5	35.1	.3	31.2	29.1	26.7	.3	25.2	23.4	21.9
.4	55.2	50.0	46.8	.4	41.6	38.8	35.6	.4	33.6	31.2	29.2
.5	69.0	62.5	58.5	.5	52.0	48.5	44.5	.5	42.0	39.0	36.5
.6	82.8	75.0	70.2	.6	62.4	58.2	53.4	.6	50.4	46.8	43.8
.7	96.6	87.5	81.9	.7	72.8	67.9	62.3	.7	58.8	54.6	51.1
.8	110.4	100.0	93.6	.8	83.2	77.6	71.2	.8	67.2	62.4	58.4
.9	124.2	112.5	105.3	.9	93.6	87.3	80.1	.9	75.6	70.2	65.7

°	′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	′	°
10	0	9.2397	71	9.2463	73	0.7537	9.9934	3	0	**80**
	10	9.2468	70	9.2536	73	0.7464	9.9931	2	50	
	20	9.2538	68	9.2609	71	0.7391	9.9929	2	40	
	30	9.2606	68	9.2680	70	0.7320	9.9927	3	30	
	40	9.2674	66	9.2750	69	0.7250	9.9924	2	20	
	50	9.2740	66	9.2819	68	0.7181	9.9922	3	10	
11	0	9.2806	64	9.2887	66	0.7113	9.9919	2	0	**79**
	10	9.2870	64	9.2953	67	0.7047	9.9917	3	50	
	20	9.2934	63	9.3020	65	0.6980	9.9914	2	40	
	30	9.2997	61	9.3085	64	0.6915	9.9912	3	30	
	40	9.3058	61	9.3149	63	0.6851	9.9909	2	20	
	50	9.3119	60	9.3212	63	0.6788	9.9907	3	10	
12	0	9.3179	59	9.3275	61	0.6725	9.9904	3	0	**78**
	10	9.3238	58	9.3336	61	0.6664	9.9901	2	50	
	20	9.3296	57	9.3397	61	0.6603	9.9899	3	40	
	30	9.3353	57	9.3458	59	0.6542	9.9896	3	30	
	40	9.3410	56	9.3517	59	0.6483	9.9893	3	20	
	50	9.3466	55	9.3576	58	0.6424	9.9890	3	10	
13	0	9.3521	54	9.3634	57	0.6366	9.9887	3	0	**77**
	10	9.3575	54	9.3691	57	0.6309	9.9884	3	50	
	20	9.3629	53	9.3748	56	0.6252	9.9881	3	40	
	30	9.3682	52	9.3804	55	0.6196	9.9878	3	30	
	40	9.3734	52	9.3859	55	0.6141	9.9875	3	20	
	50	9.3786	51	9.3914	54	0.6086	9.9872	3	10	
14	0	9.3837	50	9.3968	53	0.6032	9.9869	3	0	**76**
	10	9.3887	50	9.4021	53	0.5979	9.9866	3	50	
	20	9.3937	49	9.4074	53	0.5926	9.9863	4	40	
	30	9.3986	49	9.4127	51	0.5873	9.9859	3	30	
	40	9.4035	48	9.4178	52	0.5822	9.9856	3	20	
	50	9.4083	47	9.4230	51	0.5770	9.9853	4	10	
15	0	9.4130		9.4281		0.5719	9.9849		0	**75**
		L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′	°

PP	71	68	66		64	61	58		55	53	51
.1	7.1	6.8	6.6	.1	6.4	6.1	5.8	.1	5.5	5.3	5.1
.2	14.2	13.6	13.2	.2	12.8	12.2	11.6	.2	11.0	10.6	10.2
.3	21.3	20.4	19.8	.3	19.2	18.3	17.4	.3	16.5	15.9	15.3
.4	28.4	27.2	26.4	.4	25.6	24.4	23.2	.4	22.0	21.2	20.4
.5	35.5	34.0	33.0	.5	32.0	30.5	29.0	.5	27.5	26.5	25.5
.6	42.6	40.8	39.6	.6	38.4	36.6	34.8	.6	33.0	31.8	30.6
.7	49.7	47.6	46.2	.7	44.8	42.7	40.6	.7	38.5	37.1	35.7
.8	56.8	54.4	52.8	.8	51.2	48.8	46.4	.8	44.0	42.4	40.8
.9	63.9	61.2	59.4	.9	57.6	54.9	52.2	.9	49.5	47.7	45.9

° ′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
15 0	9.4130	47	9.4281	50	0.5719	9.9849	3	0 **75**
10	9.4177	46	9.4331	50	0.5669	9.9846	3	50
20	9.4223	46	9.4381	49	0.5619	9.9843	4	40
30	9.4269	45	9.4430	49	0.5570	9.9839	3	30
40	9.4314	45	9.4479	48	0.5521	9.9836	4	20
50	9.4359	44	9.4527	48	0.5473	9.9832	4	10
16 0	9.4403	44	9.4575	47	0.5425	9.9828	3	0 **74**
10	9.4447	44	9.4622	47	0.5378	9.9825	4	50
20	9.4491	42	9.4669	47	0.5331	9.9821	4	40
30	9.4533	43	9.4716	46	0.5284	9.9817	3	30
40	9.4576	42	9.4762	46	0.5238	9.9814	4	20
50	9.4618	41	9.4808	45	0.5192	9.9810	4	10
17 0	9.4659	41	9.4853	45	0.5147	9.9806	4	0 **73**
10	9.4700	41	9.4898	45	0.5102	9.9802	4	50
20	9.4741	40	9.4943	44	0.5057	9.9798	4	40
30	9.4781	40	9.4987	44	0.5013	9.9794	4	30
40	9.4821	40	9.5031	44	0.4969	9.9790	4	20
50	9.4861	39	9.5075	43	0.4925	9.9786	4	10
18 0	9.4900	39	9.5118	43	0.4882	9.9782	4	0 **72**
10	9.4939	38	9.5161	42	0.4839	9.9778	4	50
20	9.4977	38	9.5203	42	0.4797	9.9774	4	40
30	9.5015	37	9.5245	42	0.4755	9.9770	5	30
40	9.5052	38	9.5287	42	0.4713	9.9765	4	20
50	9.5090	36	9.5329	41	0.4671	9.9761	4	10
19 0	9.5126	37	9.5370	41	0.4630	9.9757	5	0 **71**
10	9.5163	36	9.5411	40	0.4589	9.9752	4	50
20	9.5199	36	9.5451	40	0.4549	9.9748	5	40
30	9.5235	35	9.5491	40	0.4509	9.9743	4	30
40	9.5270	36	9.5531	40	0.4469	9.9739	5	20
50	9.5306	35	9.5571	40	0.4429	9.9734	4	10
20 0	9.5341		9.5611		0.4389	9.9730		0 **70**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′ °

PP	49	47	45		44	43	41		40	38	36
.1	4.9	4.7	4.5	.1	4.4	4.3	4.1	.1	4.0	3.8	3.6
.2	9.8	9.4	9.0	.2	8.8	8.6	8.2	.2	8.0	7.6	7.2
.3	14.7	14.1	13.5	.3	13.2	12.9	12.3	.3	12.0	11.4	10.8
.4	19.6	18.8	18.0	.4	17.6	17.2	16.4	.4	16.0	15.2	14.4
.5	24.5	23.5	22.5	.5	22.0	21.5	20.5	.5	20.0	19.0	18.0
.6	29.4	28.2	27.0	.6	26.4	25.8	24.6	.6	24.0	22.8	21.6
.7	34.3	32.9	31.5	.7	30.8	30.1	28.7	.7	28.0	26.6	25.2
.8	39.2	37.6	36.0	.8	35.2	34.4	32.8	.8	32.0	30.4	28.8
.9	44.1	42.3	40.5	.9	39.6	38.7	36.9	.9	36.0	34.2	32.4

° ′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
20 0	9.5341	34	9.5611	39	0.4389	9.9730	5	0 **70**
10	9.5375	34	9.5650	39	0.4350	9.9725	4	50
20	9.5409	34	9.5689	38	0.4311	9.9721	5	40
30	9.5443	34	9.5727	39	0.4273	9.9716	5	30
40	9.5477	33	9.5766	38	0.4234	9.9711	5	20
50	9.5510	33	9.5804	38	0.4196	9.9706	4	10
21 0	9.5543	33	9.5842	37	0.4158	9.9702	5	0 **69**
10	9.5576	33	9.5879	38	0.4121	9.9697	5	50
20	9.5609	32	9.5917	37	0.4083	9.9692	5	40
30	9.5641	32	9.5954	37	0.4046	9.9687	5	30
40	9.5673	31	9.5991	37	0.4009	9.9682	5	20
50	9.5704	32	9.6028	36	0.3972	9.9677	5	10
22 0	9.5736	31	9.6064	36	0.3936	9.9672	5	0 **68**
10	9.5767	31	9.6100	36	0.3900	9.9667	6	50
20	9.5798	30	9.6136	36	0.3864	9.9661	5	40
30	9.5828	31	9.6172	36	0.3828	9.9656	5	30
40	9.5859	30	9.6208	35	0.3792	9.9651	5	20
50	9.5889	30	9.6243	36	0.3757	9.9646	6	10
23 0	9.5919	29	9.6279	35	0.3721	9.9640	5	0 **67**
10	9.5948	30	9.6314	34	0.3686	9.9635	6	50
20	9.5978	29	9.6348	35	0.3652	9.9629	5	40
30	9.6007	29	9.6383	34	0.3617	9.9624	6	30
40	9.6036	29	9.6417	35	0.3583	9.9618	5	20
50	9.6065	28	9.6452	34	0.3548	9.9613	6	10
24 0	9.6093	28	9.6486	34	0.3514	9.9607	5	0 **66**
10	9.6121	28	9.6520	33	0.3480	9.9602	6	50
20	9.6149	28	9.6553	34	0.3447	9.9596	6	40
30	9.6177	28	9.6587	33	0.3413	9.9590	6	30
40	9.6205	27	9.6620	34	0.3380	9.9584	5	20
50	9.6232	27	9.6654	33	0.3346	9.9579	6	10
25 0	9.6259		9.6687		0.3313	9.9573		0 **65**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′ °

PP	39	37	35		34	33	32		31	30	29
.1	3.9	3.7	3.5	.1	3.4	3.3	3.2	.1	3.1	3.0	2.9
.2	7.8	7.4	7.0	.2	6.8	6.6	6.4	.2	6.2	6.0	5.8
.3	11.7	11.1	10.5	.3	10.2	9.9	9.6	.3	9.3	9.0	8.7
.4	15.6	14.8	14.0	.4	13.6	13.2	12.8	.4	12.4	12.0	11.6
.5	19.5	18.5	17.5	.5	17.0	16.5	16.0	.5	15.5	15.0	14.5
.6	23.4	22.2	21.0	.6	20.4	19.8	19.2	.6	18.6	18.0	17.4
.7	27.3	25.9	24.5	.7	23.8	23.1	22.4	.7	21.7	21.0	20.3
.8	31.2	29.6	28.0	.8	27.2	26.4	25.6	.8	24.8	24.0	23.2
.9	35.1	33.3	31.5	.9	30.6	29.7	28.8	.9	27.9	27.0	26.1

° ′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
25 0	9.6259	27	9.6687	33	0.3313	9.9573	6	0 **65**
10	9.6286	27	9.6720	32	0.3280	9.9567	6	50
20	9.6313	27	9.6752	33	0.3248	9.9561	6	40
30	9.6340	26	9.6785	32	0.3215	9.9555	6	30
40	9.6366	26	9.6817	33	0.3183	9.9549	6	20
50	9.6392	26	9.6850	32	0.3150	9.9543	6	10
26 0	9.6418	26	9.6882	32	0.3118	9.9537	7	0 **64**
10	9.6444	26	9.6914	32	0.3086	9.9530	6	50
20	9.6470	25	9.6946	31	0.3054	9.9524	6	40
30	9.6495	26	9.6977	32	0.3023	9.9518	6	30
40	9.6521	25	9.7009	31	0.2991	9.9512	7	20
50	9.6546	24	9.7040	32	0.2960	9.9505	6	10
27 0	9.6570	25	9.7072	31	0.2928	9.9499	7	0 **63**
10	9.6595	25	9.7103	31	0.2897	9.9492	6	50
20	9.6620	24	9.7134	31	0.2866	9.9486	7	40
30	9.6644	24	9.7165	31	0.2835	9.9479	6	30
40	9.6668	24	9.7196	30	0.2804	9.9473	7	20
50	9.6692	24	9.7226	31	0.2774	9.9466	7	10
28 0	9.6716	24	9.7257	30	0.2743	9.9459	6	0 **62**
10	9.6740	23	9.7287	30	0.2713	9.9453	7	50
20	9.6763	24	9.7317	31	0.2683	9.9446	7	40
30	9.6787	23	9.7348	30	0.2652	9.9439	7	30
40	9.6810	23	9.7378	30	0.2622	9.9432	7	20
50	9.6833	23	9.7408	30	0.2592	9.9425	7	10
29 0	9.6856	22	9.7438	29	0.2562	9.9418	7	0 **61**
10	9.6878	23	9.7467	30	0.2533	9.9411	7	50
20	9.6901	22	9.7497	29	0.2503	9.9404	7	40
30	9.6923	23	9.7526	30	0.2474	9.9397	7	30
40	9.6946	22	9.7556	29	0.2444	9.9390	7	20
50	9.6968	22	9.7585	29	0.2415	9.9383	8	10
30 0	9.6990		9.7614		0.2386	9.9375		0 **60**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′ °

PP	28	27	26	25	24	23	22	7	6
.1	2.8	2.7	2.6	2.5	2.4	2.3	2.2	0.7	0.6
.2	5.6	5.4	5.2	5.0	4.8	4.6	4.4	1.4	1.2
.3	8.4	8.1	7.8	7.5	7.2	6.9	6.6	2.1	1.8
.4	11.2	10.8	10.4	10.0	9.6	9.2	8.8	2.8	2.4
.5	14.0	13.5	13.0	12.5	12.0	11.5	11.0	3.5	3.0
.6	16.8	16.2	15.6	15.0	14.4	13.8	13.2	4.2	3.6
.7	19.6	18.9	18.2	17.5	16.8	16.1	15.4	4.9	4.2
.8	22.4	21.6	20.8	20.0	19.2	18.4	17.6	5.6	4.8
.9	25.2	24.3	23.4	22.5	21.6	20.7	19.8	6.3	5.4

°	′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	′	°
30	0	9.6990	22	9.7614	30	0.2386	9.9375	7	0	**60**
	10	9.7012	21	9.7644	29	0.2356	9.9368	7	50	
	20	9.7033	22	9.7673	28	0.2327	9.9361	8	40	
	30	9.7055	21	9.7701	29	0.2299	9.9353	7	30	
	40	9.7076	21	9.7730	29	0.2270	9.9346	8	20	
	50	9.7097	21	9.7759	29	0.2241	9.9338	7	10	
31	0	9.7118	21	9.7788	28	0.2212	9.9331	8	0	**59**
	10	9.7139	21	9.7816	29	0.2184	9.9323	8	50	
	20	9.7160	21	9.7845	28	0.2155	9.9315	7	40	
	30	9.7181	20	9.7873	29	0.2127	9.9308	8	30	
	40	9.7201	21	9.7902	28	0.2098	9.9300	8	20	
	50	9.7222	20	9.7930	28	0.2070	9.9292	8	10	
32	0	9.7242	20	9.7958	28	0.2042	9.9284	8	0	**58**
	10	9.7262	20	9.7986	28	0.2014	9.9276	8	50	
	20	9.7282	20	9.8014	28	0.1986	9.9268	8	40	
	30	9.7302	20	9.8042	28	0.1958	9.9260	8	30	
	40	9.7322	20	9.8070	27	0.1930	9.9252	8	20	
	50	9.7342	19	9.8097	28	0.1903	9.9244	8	10	
33	0	9.7361	19	9.8125	28	0.1875	9.9236	8	0	**57**
	10	9.7380	20	9.8153	27	0.1847	9.9228	9	50	
	20	9.7400	19	9.8180	28	0.1820	9.9219	8	40	
	30	9.7419	19	9.8208	27	0.1792	9.9211	8	30	
	40	9.7438	19	9.8235	28	0.1765	9.9203	9	20	
	50	9.7457	19	9.8263	27	0.1737	9.9194	8	10	
34	0	9.7476	18	9.8290	27	0.1710	9.9186	9	0	**56**
	10	9.7494	19	9.8317	27	0.1683	9.9177	8	50	
	20	9.7513	18	9.8344	27	0.1656	9.9169	9	40	
	30	9.7531	19	9.8371	27	0.1629	9.9160	9	30	
	40	9.7550	18	9.8398	27	0.1602	9.9151	9	20	
	50	9.7568	18	9.8425	27	0.1575	9.9142	8	10	
35	0	9.7586		9.8452		0.1548	9.9134		0	**55**
		L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′	°

PP	29	28	27		22	21	20		19	8	7
.1	2.9	2.8	2.7	.1	2.2	2.1	2.0	.1	1.9	0.8	0.7
.2	5.8	5.6	5.4	.2	4.4	4.2	4.0	.2	3.8	1.6	1.4
.3	8.7	8.4	8.1	.3	6.6	6.3	6.0	.3	5.7	2.4	2.1
.4	11.6	11.2	10.8	.4	8.8	8.4	8.0	.4	7.6	3.2	2.8
.5	14.5	14.0	13.5	.5	11.0	10.5	10.0	.5	9.5	4.0	3.5
.6	17.4	16.8	16.2	.6	13.2	12.6	12.0	.6	11.4	4.8	4.2
.7	20.3	19.6	18.9	.7	15.4	14.7	14.0	.7	13.3	5.6	4.9
.8	23.2	22.4	21.6	.8	17.6	16.8	16.0	.8	15.2	6.4	5.6
.9	26.1	25.2	24.3	.9	19.8	18.9	18.0	.9	17.1	7.2	6.3

° ′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
35 0	9.7586	18	9.8452	27	0.1548	9.9134	9	0 55
10	9.7604	18	9.8479	27	0.1521	9.9125	9	50
20	9.7622	18	9.8506	27	0.1494	9.9116	9	40
30	9.7640	17	9.8533	26	0.1467	9.9107	9	30
40	9.7657	18	9.8559	27	0.1441	9.9098	9	20
50	9.7675	17	9.8586	27	0.1414	9.9089	9	10
36 0	9.7692	18	9.8613	26	0.1387	9.9080	10	0 54
10	9.7710	17	9.8639	27	0.1361	9.9070	9	50
20	9.7727	17	9.8666	26	0.1334	9.9061	9	40
30	9.7744	17	9.8692	26	0.1308	9.9052	10	30
40	9.7761	17	9.8718	27	0.1282	9.9042	9	20
50	9.7778	17	9.8745	26	0.1255	9.9033	10	10
37 0	9.7795	16	9.8771	26	0.1229	9.9023	9	0 53
10	9.7811	17	9.8797	27	0.1203	9.9014	10	50
20	9.7828	16	9.8824	26	0.1176	9.9004	9	40
30	9.7844	17	9.8850	26	0.1150	9.8995	10	30
40	9.7861	16	9.8876	26	0.1124	9.8985	10	20
50	9.7877	16	9.8902	26	0.1098	9.8975	10	10
38 0	9.7893	17	9.8928	26	0.1072	9.8965	10	0 52
10	9.7910	16	9.8954	26	0.1046	9.8955	10	50
20	9.7926	15	9.8980	26	0.1020	9.8945	10	40
30	9.7941	16	9.9006	26	0.0994	9.8935	10	30
40	9.7957	16	9.9032	26	0.0968	9.8925	10	20
50	9.7973	16	9.9058	26	0.0942	9.8915	10	10
39 0	9.7989	15	9.9084	26	0.0916	9.8905	10	0 51
10	9.8004	16	9.9110	25	0.0890	9.8895	11	50
20	9.8020	15	9.9135	26	0.0865	9.8884	10	40
30	9.8035	15	9.9161	26	0.0839	9.8874	10	30
40	9.8050	16	9.9187	25	0.0813	9.8864	11	20
50	9.8066	15	9.9212	26	0.0788	9.8853	10	10
40 0	9.8081		9.9238		0.0762	9.8843		0 50
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′ °

PP	26	25	18	17	16	15	11	10	9
.1	2.6	2.5	1.8	1.7	1.6	1.5	1.1	1.0	0.9
.2	5.2	5.0	3.6	3.4	3.2	3.0	2.2	2.0	1.8
.3	7.8	7.5	5.4	5.1	4.8	4.5	3.3	3.0	2.7
.4	10.4	10.0	7.2	6.8	6.4	6.0	4.4	4.0	3.6
.5	13.0	12.5	9.0	8.5	8.0	7.5	5.5	5.0	4.5
.6	15.6	15.0	10.8	10.2	9.6	9.0	6.6	6.0	5.4
.7	18.2	17.5	12.6	11.9	11.2	10.5	7.7	7.0	6.3
.8	20.8	20.0	14.4	13.6	12.8	12.0	8.8	8.0	7.2
.9	23.4	22.5	16.2	15.3	14.4	13.5	9.9	9.0	8.1

° ′	L. Sin.	d.	L. Tang.	d.	L. Cotg.	L. Cos.	d.	
40 0	9.8081	15	9.9238	26	0.0762	9.8843	11	0 **50**
10	9.8096	15	9.9264	25	0.0736	9.8832	11	50
20	9.8111	14	9.9289	26	0.0711	9.8821	11	40
30	9.8125	15	9.9315	26	0.0685	9.8810	10	30
40	9.8140	15	9.9341	25	0.0659	9.8800	11	20
50	9.8155	14	9.9366	26	0.0634	9.8789	11	10
41 0	9.8169	15	9.9392	25	0.0608	9.8778	11	0 **49**
10	9.8184	14	9.9417	26	0.0583	9.8767	11	50
20	9.8198	15	9.9443	25	0.0557	9.8756	11	40
30	9.8213	14	9.9468	26	0.0532	9.8745	12	30
40	9.8227	14	9.9494	25	0.0506	9.8733	11	20
50	9.8241	14	9.9519	25	0.0481	9.8722	11	10
42 0	9.8255	14	9.9544	26	0.0456	9.8711	12	0 **48**
10	9.8269	14	9.9570	25	0.0430	9.8699	11	50
20	9.8283	14	9.9595	26	0.0405	9.8688	12	40
30	9.8297	14	9.9621	25	0.0379	9.8676	11	30
40	9.8311	13	9.9646	25	0.0354	9.8665	12	20
50	9.8324	14	9.9671	26	0.0329	9.8653	12	10
43 0	9.8338	13	9.9697	25	0.0303	9.8641	12	0 **47**
10	9.8351	14	9.9722	25	0.0278	9.8629	11	50
20	9.8365	13	9.9747	25	0.0253	9.8618	12	40
30	9.8378	13	9.9772	25	0.0228	9.8606	12	30
40	9.8391	14	9.9798	25	0.0202	9.8594	12	20
50	9.8405	13	9.9823	25	0.0177	9.8582	13	10
44 0	9.8418	13	9.9848	26	0.0152	9.8569	12	0 **46**
10	9.8431	13	9.9874	25	0.0126	9.8557	12	50
20	9.8444	13	9.9899	25	0.0101	9.8545	13	40
30	9.8457	12	9.9924	25	0.0076	9.8532	12	30
40	9.8469	13	9.9949	26	0.0051	9.8520	13	20
50	9.8482	13	9.9975	25	0.0025	9.8507	12	10
45 0	9.8495		0.0000		0.0000	9.8495		0 **45**
	L. Cos.	d.	L. Cotg.	d.	L. Tang.	L. Sin.	d.	′ °

PP	26	25	15		14	13	12		11	10
.1	2.6	2.5	1.5	.1	1.4	1.3	1.2	.1	1.1	1.0
.2	5.2	5.0	3.0	.2	2.8	2.6	2.4	.2	2.2	2.0
.3	7.8	7.5	4.5	.3	4.2	3.9	3.6	.3	3.3	3.0
.4	10.4	10.0	6.0	.4	5.6	5.2	4.8	.4	4.4	4.0
.5	13.0	12.5	7.5	.5	7.0	6.5	6.0	.5	5.5	5.0
.6	15.6	15.0	9.0	.6	8.4	7.8	7.2	.6	6.6	6.0
.7	18.2	17.5	10.5	.7	9.8	9.1	8.4	.7	7.7	7.0
.8	20.8	20.0	12.0	.8	11.2	10.4	9.6	.8	8.8	8.0
.9	23.4	22.5	13.5	.9	12.6	11.7	10.8	.9	9.9	9.0

TABLE VII

FOUR-PLACE NATURAL TRIGONOMETRIC FUNCTIONS

TO EVERY TEN MINUTES

° ′	Sin.	d.	Tang.	d.	Cotg.	d.	Cos.	d.	
0 0	0.0000	29	0.0000	29	infinit.	—	1.0000	0	0 **90**
10	0.0029	29	0.0029	29	343.7737	—	1.0000	0	50
20	0.0058	29	0.0058	29	171.8854	—	1.0000	0	40
30	0.0087	29	0.0087	29	114.5887	—	1.0000	1	30
40	0.0116	29	0.0116	29	85.9398	—	0.9999	0	20
50	0.0145	30	0.0145	30	68.7501	114601	0.9999	1	10
1 0	0.0175	29	0.0175	29	57.2900	81861	0.9998	0	0 **89**
10	0.0204	29	0.0204	29	49.1039	61398	0.9998	1	50
20	0.0233	29	0.0233	29	42.9641	47756	0.9997	0	40
30	0.0262	29	0.0262	29	38.1885	38207	0.9997	1	30
40	0.0291	29	0.0291	29	34.3678	31262	0.9996	1	20
50	0.0320	29	0.0320	29	31.2416	26053	0.9995	1	10
2 0	0.0349	29	0.0349	29	28.6363	22047	0.9994	1	0 **88**
10	0.0378	29	0.0378	29	26.4316	18898	0.9993	1	50
20	0.0407	29	0.0407	30	24.5418	16380	0.9992	2	40
30	0.0436	29	0.0437	29	22.9038	14334	0.9990	1	30
40	0.0465	29	0.0466	29	21.4704	12648	0.9989	1	20
50	0.0494	29	0.0495	29	20.2056	11245	0.9988	2	10
3 0	0.0523	29	0.0524	29	19.0811	10061	0.9986	1	0 **87**
10	0.0552	29	0.0553	29	18.0750	9057	0.9985	2	50
20	0.0581	29	0.0582	30	17.1693	8194	0.9983	2	40
30	0.0610	30	0.0612	29	16.3499	7451	0.9981	1	30
40	0.0640	29	0.0641	29	15.6048	6804	0.9980	2	20
50	0.0669	29	0.0670	29	14.9244	6237	0.9978	2	10
4 0	0.0698	29	0.0699	30	14.3007	5740	0.9976	2	0 **86**
10	0.0727	29	0.0729	29	13.7267	5298	0.9974	3	50
20	0.0756	29	0.0758	29	13.1969	4907	0.9971	2	40
30	0.0785	29	0.0787	29	12.7062	4557	0.9969	2	30
40	0.0814	29	0.0816	30	12.2505	4243	0.9967	3	20
50	0.0843	29	0.0846	29	11.8262	3961	0.9964	2	10
5 0	0.0872		0.0875		11.4301		0.9962		0 **85**
	Cos.	d.	Cotg.	d.	Tang.	d.	Sin.	d.	′ °

PP	26053	16380	11245		8194	6237	4907		3961	30	29
.1	2605	1638	1125	.1	819.4	623.7	490.7	.1	396.1	3.0	2.9
.2	5211	3276	2249	.2	1638.8	1247.4	981.4	.2	792.2	6.0	5.8
.3	7816	4914	3374	.3	2458.2	1871.1	1472.1	.3	1188.3	9.0	8.7
.4	10421	6552	4498	.4	3277.6	2494.8	1962.8	.4	1584.4	12.0	11.6
.5	13027	8190	5623	.5	4097.0	3118.5	2453.5	.5	1980.5	15.0	14.5
.6	15632	9828	6747	.6	4916.4	3742.2	2944.2	.6	2376.6	18.0	17.4
.7	18237	11466	7872	.7	5735.8	4365.9	3434.9	.7	2772.7	21.0	20.3
.8	20842	13104	8996	.8	6555.2	4989.6	3925.6	.8	3168.8	24.0	23.2
.9	23448	14742	10121	.9	7374.6	5613.3	4416.3	.9	3564.9	27.0	26.1

° ′	Sin.	d.	Tang.	d.	Cotg.	d.	Cos.	d.	
5 0	0.0872	29	0.0875	29	11.4301	3707	0.9962	3	0 **85**
10	0.0901	28	0.0904	30	11.0594	3475	0.9959	2	50
20	0.0929	29	0.0934	29	10.7119	3265	0.9957	3	40
30	0.0958	29	0.0963	29	10.3854	3074	0.9954	3	30
40	0.0987	29	0.0992	30	10.0780	2898	0.9951	3	20
50	0.1016	29	0.1022	29	9.7882	2738	0.9948	3	10
6 0	0.1045	29	0.1051	29	9.5144	2591	0.9945	3	0 **84**
10	0.1074	29	0.1080	30	9.2553	2455	0.9942	3	50
20	0.1103	29	0.1110	29	9.0098	2329	0.9939	3	40
30	0.1132	29	0.1139	30	8.7769	2214	0.9936	4	30
40	0.1161	29	0.1169	29	8.5555	2105	0.9932	3	20
50	0.1190	29	0.1198	30	8.3450	2007	0.9929	4	10
7 0	0.1219	29	0.1228	29	8.1443	1913	0.9925	3	0 **83**
10	0.1248	28	0.1257	30	7.9530	1826	0.9922	4	50
20	0.1276	29	0.1287	30	7.7704	1746	0.9918	4	40
30	0.1305	29	0.1317	29	7.5958	1671	0.9914	3	30
40	0.1334	29	0.1346	30	7.4287	1600	0.9911	4	20
50	0.1363	29	0.1376	29	7.2687	1533	0.9907	4	10
8 0	0.1392	29	0.1405	30	7.1154	1472	0.9903	4	0 **82**
10	0.1421	28	0.1435	30	6.9682	1413	0.9899	5	50
20	0.1449	29	0.1465	30	6.8269	1357	0.9894	4	40
30	0.1478	29	0.1495	29	6.6912	1306	0.9890	4	30
40	0.1507	29	0.1524	30	6.5606	1258	0.9886	5	20
50	0.1536	28	0.1554	30	6.4348	1210	0.9881	4	10
9 0	0.1564	29	0.1584	30	6.3138	1168	0.9877	5	0 **81**
10	0.1593	29	0.1614	30	6.1970	1126	0.9872	4	50
20	0.1622	28	0.1644	29	6.0844	1086	0.9868	5	40
30	0.1650	29	0.1673	30	5.9758	1050	0.9863	5	30
40	0.1679	29	0.1703	30	5.8708	1014	0.9858	5	20
50	0.1708	28	0.1733	30	5.7694	981	0.9853	5	10
10 0	0.1736		0.1763		5.6713		0.9848		0 **80**
	Cos.	d.	Cotg.	d.	Tang.	d.	Sin.	d.	′ °

PP	2738	1533	981		30	29	28		5	4	3
.1	273.8	153.3	98.1	.1	3.0	2.9	2.8	.1	0.5	0.4	0.3
.2	547.6	306.6	196.2	.2	6.0	5.8	5.6	.2	1.0	0.8	0.6
.3	821.4	459.9	294.3	.3	9.0	8.7	8.4	.3	1.5	1.2	0.9
.4	1095.2	613.2	392.4	.4	12.0	11.6	11.2	.4	2.0	1.6	1.2
.5	1369.0	766.5	490.5	.5	15.0	14.5	14.0	.5	2.5	2.0	1.5
.6	1642.8	919.8	588.6	.6	18.0	17.4	16.8	.6	3.0	2.4	1.8
.7	1916.6	1073.1	686.7	.7	21.0	20.3	19.6	.7	3.5	2.8	2.1
.8	2190.4	1226.4	784.8	.8	24.0	23.2	22.4	.8	4.0	3.2	2.4
.9	2464.2	1379.7	882.9	.9	27.0	26.1	25.2	.9	4.5	3.6	2.7

° ′	Sin.	d.	Tang.	d.	Cotg.	d.	Cos.	d.	
10 0	0.1736	29	0.1763	30	5.6713	949	0.9848	5	0 **80**
10	0.1765	29	0.1793	30	5.5764	919	0.9843	5	50
20	0.1794	28	0.1823	30	5.4845	890	0.9838	5	40
30	0.1822	29	0.1853	30	5.3955	862	0.9833	6	30
40	0.1851	29	0.1883	31	5.3093	836	0.9827	5	20
50	0.1880	28	0.1914	30	5.2257	811	0.9822	6	10
11 0	0.1908	29	0.1944	30	5.1446	788	0.9816	5	0 **79**
10	0.1937	28	0.1974	30	5.0658	764	0.9811	6	50
20	0.1965	29	0.2004	31	4.9894	742	0.9805	6	40
30	0.1994	28	0.2035	30	4.9152	722	0.9799	6	30
40	0.2022	29	0.2065	30	4.8430	701	0.9793	6	20
50	0.2051	28	0.2095	31	4.7729	683	0.9787	6	10
12 0	0.2079	29	0.2126	30	4.7046	664	0.9781	6	0 **78**
10	0.2108	28	0.2156	30	4.6382	646	0.9775	6	50
20	0.2136	28	0.2186	31	4.5736	629	0.9769	6	40
30	0.2164	29	0.2217	30	4.5107	613	0.9763	6	30
40	0.2193	28	0.2247	31	4.4494	597	0.9757	7	20
50	0.2221	29	0.2278	31	4.3897	582	0.9750	6	10
13 0	0.2250	28	0.2309	30	4.3315	568	0.9744	7	0 **77**
10	0.2278	28	0.2339	31	4.2747	554	0.9737	7	50
20	0.2306	28	0.2370	31	4.2193	540	0.9730	6	40
30	0.2334	29	0.2401	31	4.1653	527	0.9724	7	30
40	0.2363	28	0.2432	30	4.1126	515	0.9717	7	20
50	0.2391	28	0.2462	31	4.0611	503	0.9710	7	10
14 0	0.2419	28	0.2493	31	4.0108	491	0.9703	7	0 **76**
10	0.2447	29	0.2524	31	3.9617	481	0.9696	7	50
20	0.2476	28	0.2555	31	3.9136	469	0.9689	8	40
30	0.2504	28	0.2586	31	3.8667	459	0.9681	7	30
40	0.2532	28	0.2617	31	3.8208	448	0.9674	7	20
50	0.2560	28	0.2648	31	3.7760	439	0.9667	8	10
15 0	0.2588		0.2679		3.7321		0.9659		0 **75**
	Cos.	d.	Cotg.	d.	Tang.	d.	Sin.	d.	′ °

PP	742	448	31		30	29	28		7	6	5
.1	74.2	44.8	3.1	.1	3.0	2.9	2.8	.1	0.7	0.6	0.5
.2	148.4	89.6	6.2	.2	6.0	5.8	5.6	.2	1.4	1.2	1.0
.3	222.6	134.4	9.3	.3	9.0	8.7	8.4	.3	2.1	1.8	1.5
.4	296.8	179.2	12.4	.4	12.0	11.6	11.2	.4	2.8	2.4	2.0
.5	371.0	224.0	15.5	.5	15.0	14.5	14.0	.5	3.5	3.0	2.5
.6	445.2	268.8	18.6	.6	18.0	17.4	16.8	.6	4.2	3.6	3.0
.7	519.4	313.6	21.7	.7	21.0	20.3	19.6	.7	4.9	4.2	3.5
.8	593.6	358.4	24.8	.8	24.0	23.2	22.4	.8	5.6	4.8	4.0
.9	667.8	403.2	27.9	.9	27.0	26.1	25.2	.9	6.3	5.4	4.5

° ′	Sin.	d.	Tang.	d.	Cotg.	d.	Cos.	d.	
15 0	0.2588	28	0.2679	32	3.7321	430	0.9659	7	0 **75**
10	0.2616	28	0.2711	31	3.6891	421	0.9652	8	50
20	0.2644	28	0.2742	31	3.6470	411	0.9644	8	40
30	0.2672	28	0.2773	32	3.6059	403	0.9636	8	30
40	0.2700	28	0.2805	31	3.5656	395	0.9628	7	20
50	0.2728	28	0.2836	31	3.5261	387	0.9621	8	10
16 0	0.2756	28	0.2867	32	3.4874	379	0.9613	8	0 **74**
10	0.2784	28	0.2899	32	3.4495	371	0.9605	9	50
20	0.2812	28	0.2931	31	3.4124	365	0.9596	8	40
30	0.2840	28	0.2962	32	3.3759	357	0.9588	8	30
40	0.2868	28	0.2994	32	3.3402	350	0.9580	8	20
50	0.2896	28	0.3026	31	3.3052	343	0.9572	9	10
17 0	0.2924	28	0.3057	32	3.2709	338	0.9563	8	0 **73**
10	0.2952	27	0.3089	32	3.2371	330	0.9555	9	50
20	0.2979	28	0.3121	32	3.2041	325	0.9546	9	40
30	0.3007	28	0.3153	32	3.1716	319	0.9537	9	30
40	0.3035	27	0.3185	32	3.1397	313	0.9528	8	20
50	0.3062	28	0.3217	32	3.1084	307	0.9520	9	10
18 0	0.3090	28	0.3249	32	3.0777	302	0.9511	9	0 **72**
10	0.3118	27	0.3281	33	3.0475	297	0.9502	10	50
20	0.3145	28	0.3314	32	3.0178	291	0.9492	9	40
30	0.3173	28	0.3346	32	2.9887	287	0.9483	9	30
40	0.3201	27	0.3378	33	2.9600	281	0.9474	9	20
50	0.3228	28	0.3411	32	2.9319	277	0.9465	10	10
19 0	0.3256	27	0.3443	33	2.9042	272	0.9455	9	0 **71**
10	0.3283	28	0.3476	32	2.8770	268	0.9446	10	50
20	0.3311	27	0.3508	33	2.8502	263	0.9436	10	40
30	0.3338	27	0.3541	33	2.8239	259	0.9426	9	30
40	0.3365	28	0.3574	33	2.7980	255	0.9417	10	20
50	0.3393	27	0.3607	33	2.7725	250	0.9407	10	10
20 0	0.3420		0.3640		2.7475		0.9397		0 **70**
	Cos.	d.	Cotg.	d.	Tang.	d.	Sin.	d.	′ °

PP	255	33	32		31	28	27		10	9	8
.1	25.5	3.3	3.2	.1	3.1	2.8	2.7	.1	1.0	0.9	0.8
.2	51.0	6.6	6.4	.2	6.2	5.6	5.4	.2	2.0	1.8	1.6
.3	76.5	9.9	9.6	.3	9.3	8.4	8.1	.3	3.0	2.7	2.4
.4	102.0	13.2	12.8	.4	12.4	11.2	10.8	.4	4.0	3.6	3.2
.5	127.5	16.5	16.0	.5	15.5	14.0	13.5	.5	5.0	4.5	4.0
.6	153.0	19.8	19.2	.6	18.6	16.8	16.2	.6	6.0	5.4	4.8
.7	178.5	23.1	22.4	.7	21.7	19.6	18.9	.7	7.0	6.3	5.6
.8	204.0	26.4	25.6	.8	24.8	22.4	21.6	.8	8.0	7.2	6.4
.9	229.5	29.7	28.8	.9	27.9	25.2	24.3	.9	9.0	8.1	7.2

° ′	Sin.	d.	Tang.	d.	Cotg.	d.	Cos.	d.	
20 0	0.3420	28	0.3640	33	2.7475	247	0.9397	10	0 **70**
10	0.3448	27	0.3673	33	2.7228	243	0.9387	10	50
20	0.3475	27	0.3706	33	2.6985	239	0.9377	10	40
30	0.3502	27	0.3739	33	2.6746	235	0.9367	11	30
40	0.3529	28	0.3772	33	2.6511	232	0.9356	10	20
50	0.3557	27	0.3805	34	2.6279	228	0.9346	10	10
21 0	0.3584	27	0.3839	33	2.6051	225	0.9336	11	0 **69**
10	0.3611	27	0.3872	34	2.5826	221	0.9325	10	50
20	0.3638	27	0.3906	33	2.5605	219	0.9315	11	40
30	0.3665	27	0.3939	34	2.5386	214	0.9304	11	30
40	0.3692	27	0.3973	33	2.5172	212	0.9293	10	20
50	0.3719	27	0.4006	34	2.4960	209	0.9283	11	10
22 0	0.3746	27	0.4040	34	2.4751	206	0.9272	11	0 **68**
10	0.3773	27	0.4074	34	2.4545	203	0.9261	11	50
20	0.3800	27	0.4108	34	2.4342	200	0.9250	11	40
30	0.3827	27	0.4142	34	2.4142	197	0.9239	11	30
40	0.3854	27	0.4176	34	2.3945	195	0.9228	12	20
50	0.3881	26	0.4210	35	2.3750	191	0.9216	11	10
23 0	0.3907	27	0.4245	34	2.3559	190	0.9205	11	0 **67**
10	0.3934	27	0.4279	35	2.3369	186	0.9194	12	50
20	0.3961	26	0.4314	34	2.3183	185	0.9182	11	40
30	0.3987	27	0.4348	35	2.2998	181	0.9171	12	30
40	0.4014	27	0.4383	34	2.2817	180	0.9159	12	20
50	0.4041	26	0.4417	35	2.2637	177	0.9147	12	10
24 0	0.4067	27	0.4452	35	2.2460	174	0.9135	11	0 **66**
10	0.4094	26	0.4487	35	2.2286	173	0.9124	12	50
20	0.4120	27	0.4522	35	2.2113	170	0.9112	12	40
30	0.4147	26	0.4557	35	2.1943	168	0.9100	12	30
40	0.4173	27	0.4592	36	2.1775	166	0.9088	13	20
50	0.4200	26	0.4628	35	2.1609	164	0.9075	12	10
25 0	0.4226		0.4663		2.1445		0.9063		0 **65**
	Cos.	d.	Cotg.	d.	Tang.	d.	Sin.	d.	′ °

PP	177	35	34		33	27	26		12	11	10
.1	17.7	3.5	3.4	.1	3.3	2.7	2.6	.1	1.2	1.1	1.0
.2	35.4	7.0	6.8	.2	6.6	5.4	5.2	.2	2.4	2.2	2.0
.3	53.1	10.5	10.2	.3	9.9	8.1	7.8	.3	3.6	3.3	3.0
.4	70.8	14.0	13.6	.4	13.2	10.8	10.4	.4	4.8	4.4	4.0
.5	88.5	17.5	17.0	.5	16.5	13.5	13.0	.5	6.0	5.5	5.0
.6	106.2	21.0	20.4	.6	19.8	16.2	15.6	.6	7.2	6.6	6.0
.7	123.9	24.5	23.8	.7	23.1	18.9	18.2	.7	8.4	7.7	7.0
.8	141.6	28.0	27.2	.8	26.4	21.6	20.8	.8	9.6	8.8	8.0
.9	159.3	31.5	30.6	.9	29.7	24.3	23.4	.9	10.8	9.9	9.0

° ′	Sin.	d.	Tang.	d.	Cotg.	d.	Cos.	d.	
25 0	0.4226	27	0.4663	36	2.1445	162	0.9063	12	0 **65**
10	0.4253	26	0.4699	35	2.1283	160	0.9051	13	50
20	0.4279	26	0.4734	36	2.1123	158	0.9038	12	40
30	0.4305	26	0.4770	36	2.0965	156	0.9026	13	30
40	0.4331	27	0.4806	35	2.0809	154	0.9013	12	20
50	0.4358	26	0.4841	36	2.0655	152	0.9001	13	10
26 0	0.4384	26	0.4877	36	2.0503	150	0.8988	13	0 **64**
10	0.4410	26	0.4913	37	2.0353	149	0.8975	13	50
20	0.4436	26	0.4950	36	2.0204	147	0.8962	13	40
30	0.4462	26	0.4986	36	2.0057	145	0.8949	13	30
40	0.4488	26	0.5022	37	1.9912	144	0.8936	13	20
50	0.4514	26	0.5059	36	1.9768	142	0.8923	13	10
27 0	0.4540	26	0.5095	37	1.9626	140	0.8910	13	0 **63**
10	0.4566	26	0.5132	37	1.9486	139	0.8897	13	50
20	0.4592	25	0.5169	37	1.9347	137	0.8884	14	40
30	0.4617	26	0.5206	37	1.9210	136	0.8870	13	30
40	0.4643	26	0.5243	37	1.9074	134	0.8857	14	20
50	0.4669	26	0.5280	37	1.8940	133	0.8843	14	10
28 0	0.4695	25	0.5317	37	1.8807	131	0.8829	13	0 **62**
10	0.4720	26	0.5354	38	1.8676	130	0.8816	14	50
20	0.4746	26	0.5392	38	1.8546	128	0.8802	14	40
30	0.4772	25	0.5430	37	1.8418	127	0.8788	14	30
40	0.4797	26	0.5467	38	1.8291	126	0.8774	14	20
50	0.4823	25	0.5505	38	1.8165	125	0.8760	14	10
29 0	0.4848	26	0.5543	38	1.8040	123	0.8746	14	0 **61**
10	0.4874	25	0.5581	38	1.7917	121	0.8732	14	50
20	0.4899	25	0.5619	39	1.7796	121	0.8718	14	40
30	0.4924	26	0.5658	38	1.7675	119	0.8704	15	30
40	0.4950	25	0.5696	39	1.7556	119	0.8689	14	20
50	0.4975	25	0.5735	39	1.7437	116	0.8675	15	10
30 0	0.5000		0.5774		1.7321		0.8660		0 **60**
	Cos.	d.	Cotg.	d.	Tang.	d.	Sin.	d.	′ °

PP	149	131	39		38	37	36		25	14	13
.1	14.9	13.1	3.9	.1	3.8	3.7	3.6	.1	2.5	1.4	1.3
.2	29.8	26.2	7.8	.2	7.6	7.4	7.2	.2	5.0	2.8	2.6
.3	44.7	39.3	11.7	.3	11.4	11.1	10.8	.3	7.5	4.2	3.9
.4	59.6	52.4	15.6	.4	15.2	14.8	14.4	.4	10.0	5.6	5.2
.5	74.5	65.5	19.5	.5	19.0	18.5	18.0	.5	12.5	7.0	6.5
.6	89.4	78.6	23.4	.6	22.8	22.2	21.6	.6	15.0	8.4	7.8
.7	104.3	91.7	27.3	.7	26.6	25.9	25.2	.7	17.5	9.8	9.1
.8	119.2	104.8	31.2	.8	30.4	29.6	28.8	.8	20.0	11.2	10.4
.9	134.1	117.9	35.1	.9	34.2	33.3	32.4	.9	22.5	12.6	11.7

° ′	Sin.	d.	Tang.	d.	Cotg.	d.	Cos.	d.	
30 0	0.5000	25	0.5774	38	1.7321	116	0.8660	14	0 **60**
10	0.5025	25	0.5812	39	1.7205	115	0.8646	15	50
20	0.5050	25	0.5851	39	1.7090	113	0.8631	15	40
30	0.5075	25	0.5890	40	1.6977	113	0.8616	15	30
40	0.5100	25	0.5930	39	1.6864	111	0.8601	14	20
50	0.5125	25	0.5969	40	1.6753	110	0.8587	15	10
31 0	0.5150	25	0.6009	39	1.6643	109	0.8572	15	0 **59**
10	0.5175	25	0.6048	40	1.6534	108	0.8557	15	50
20	0.5200	25	0.6088	40	1.6426	107	0.8542	16	40
30	0.5225	25	0.6128	40	1.6319	107	0.8526	15	30
40	0.5250	25	0.6168	40	1.6212	105	0.8511	15	20
50	0.5275	24	0.6208	41	1.6107	104	0.8496	16	10
32 0	0.5299	25	0.6249	40	1.6003	103	0.8480	15	0 **58**
10	0.5324	24	0.6289	41	1.5900	102	0.8465	15	50
20	0.5348	25	0.6330	41	1.5798	101	0.8450	16	40
30	0.5373	25	0.6371	41	1.5697	100	0.8434	16	30
40	0.5398	24	0.6412	41	1.5597	100	0.8418	15	20
50	0.5422	24	0.6453	41	1.5497	98	0.8403	16	10
33 0	0.5446	25	0.6494	42	1.5399	98	0.8387	16	0 **57**
10	0.5471	24	0.6536	41	1.5301	97	0.8371	16	50
20	0.5495	24	0.6577	42	1.5204	96	0.8355	16	40
30	0.5519	25	0.6619	42	1.5108	95	0.8339	16	30
40	0.5544	24	0.6661	42	1.5013	94	0.8323	16	20
50	0.5568	24	0.6703	42	1.4919	93	0.8307	17	10
34 0	0.5592	24	0.6745	42	1.4826	93	0.8290	16	0 **56**
10	0.5616	24	0.6787	43	1.4733	92	0.8274	16	50
20	0.5640	24	0.6830	43	1.4641	91	0.8258	17	40
30	0.5664	24	0.6873	43	1.4550	90	0.8241	16	30
40	0.5688	24	0.6916	43	1.4460	90	0.8225	17	20
50	0.5712	24	0.6959	43	1.4370	89	0.8208	16	10
35 0	0.5736		0.7002		1.4281		0.8192		0 **55**
	Cos.	d.	Cotg.	d.	Tang.	d.	Sin.	d.	′ °

PP	43	42	41		40	25	24		17	16	15
.1	4.3	4.2	4.1	.1	4.0	2.5	2.4	.1	1.7	1.6	1.5
.2	8.6	8.4	8.2	.2	8.0	5.0	4.8	.2	3.4	3.2	3.0
.3	12.9	12.6	12.3	.3	12.0	7.5	7.2	.3	5.1	4.8	4.5
.4	17.2	16.8	16.4	.4	16.0	10.0	9.6	.4	6.8	6.4	6.0
.5	21.5	21.0	20.5	.5	20.0	12.5	12.0	.5	8.5	8.0	7.5
.6	25.8	25.2	24.6	.6	24.0	15.0	14.4	.6	10.2	9.6	9.0
.7	30.1	29.4	28.7	.7	28.0	17.5	16.8	.7	11.9	11.2	10.5
.8	34.4	33.6	32.8	.8	32.0	20.0	19.2	.8	13.6	12.8	12.0
.9	38.7	37.8	36.9	.9	36.0	22.5	21.6	.9	15.3	14.4	13.5

°	′	Sin.	d.	Tang.	d.	Cotg.	d.	Cos.	d.	′	°
35	0	0.5736	24	0.7002	44	1.4281	88	0.8192	17	0	55
	10	0.5760	23	0.7046	43	1.4193	87	0.8175	17	50	
	20	0.5783	24	0.7089	44	1.4106	87	0.8158	17	40	
	30	0.5807	24	0.7133	44	1.4019	85	0.8141	17	30	
	40	0.5831	23	0.7177	44	1.3934	86	0.8124	17	20	
	50	0.5854	24	0.7221	44	1.3848	84	0.8107	17	10	
36	0	0.5878	23	0.7265	45	1.3764	84	0.8090	17	0	54
	10	0.5901	24	0.7310	45	1.3680	83	0.8073	17	50	
	20	0.5925	23	0.7355	45	1.3597	83	0.8056	17	40	
	30	0.5948	24	0.7400	45	1.3514	82	0.8039	18	30	
	40	0.5972	23	0.7445	45	1.3432	81	0.8021	17	20	
	50	0.5995	23	0.7490	46	1.3351	81	0.8004	18	10	
37	0	0.6018	23	0.7536	45	1.3270	80	0.7986	17	0	53
	10	0.6041	24	0.7581	46	1.3190	79	0.7969	18	50	
	20	0.6065	23	0.7627	46	1.3111	79	0.7951	17	40	
	30	0.6088	23	0.7673	47	1.3032	78	0.7934	18	30	
	40	0.6111	23	0.7720	46	1.2954	78	0.7916	18	20	
	50	0.6134	23	0.7766	47	1.2876	77	0.7898	18	10	
38	0	0.6157	23	0.7813	47	1.2799	76	0.7880	18	0	52
	10	0.6180	22	0.7860	47	1.2723	76	0.7862	18	50	
	20	0.6202	23	0.7907	47	1.2647	75	0.7844	18	40	
	30	0.6225	23	0.7954	48	1.2572	75	0.7826	18	30	
	40	0.6248	23	0.8002	48	1.2497	74	0.7808	18	20	
	50	0.6271	22	0.8050	48	1.2423	74	0.7790	19	10	
39	0	0.6293	23	0.8098	48	1.2349	73	0.7771	18	0	51
	10	0.6316	22	0.8146	49	1.2276	73	0.7753	18	50	
	20	0.6338	23	0.8195	48	1.2203	72	0.7735	19	40	
	30	0.6361	22	0.8243	49	1.2131	72	0.7716	18	30	
	40	0.6383	23	0.8292	50	1.2059	71	0.7698	19	20	
	50	0.6406	22	0.8342	49	1.1988	70	0.7679	19	10	
40	0	0.6428		0.8391		1.1918		0.7660		0	50
		Cos.	d.	Cotg.	d.	Tang.	d.	Sin.	d.	′	°

PP	48	47	46	45	44	23	22	19	18
.1	4.8	4.7	4.6	4.5	4.4	2.3	2.2	1.9	1.8
.2	9.6	9.4	9.2	9.0	8.8	4.6	4.4	3.8	3.6
.3	14.4	14.1	13.8	13.5	13.2	6.9	6.6	5.7	5.4
.4	19.2	18.8	18.4	18.0	17.6	9.2	8.8	7.6	7.2
.5	24.0	23.5	23.0	22.5	22.0	11.5	11.0	9.5	9.0
.6	28.8	28.2	27.6	27.0	26.4	13.8	13.2	11.4	10.8
.7	33.6	32.9	32.2	31.5	30.8	16.1	15.4	13.3	12.6
.8	38.4	37.6	36.8	36.0	35.2	18.4	17.6	15.2	14.4
.9	43.2	42.3	41.4	40.5	39.6	20.7	19.8	17.1	16.2

° ′	Sin.	d.	Tang.	d.	Cotg.	d.	Cos.	d.	
40 0	0.6428	22	0.8391	50	1.1918	71	0.7660	18	0 **50**
10	0.6450	22	0.8441	50	1.1847	69	0.7642	19	50
20	0.6472	22	0.8491	50	1.1778	70	0.7623	19	40
30	0.6494	23	0.8541	50	1.1708	68	0.7604	19	30
40	0.6517	22	0.8591	51	1.1640	69	0.7585	19	20
50	0.6539	22	0.8642	51	1.1571	67	0.7566	19	10
41 0	0.6561	22	0.8693	51	1.1504	68	0.7547	19	0 **49**
10	0.6583	21	0.8744	52	1.1436	67	0.7528	19	50
20	0.6604	22	0.8796	51	1.1369	66	0.7509	19	40
30	0.6626	22	0.8847	52	1.1303	66	0.7490	20	30
40	0.6648	22	0.8899	53	1.1237	66	0.7470	19	20
50	0.6670	21	0.8952	52	1.1171	65	0.7451	20	10
42 0	0.6691	22	0.9004	53	1.1106	65	0.7431	19	0 **48**
10	0.6713	21	0.9057	53	1.1041	64	0.7412	20	50
20	0.6734	22	0.9110	53	1.0977	64	0.7392	19	40
30	0.6756	21	0.9163	54	1.0913	63	0.7373	20	30
40	0.6777	22	0.9217	54	1.0850	64	0.7353	20	20
50	0.6799	21	0.9271	54	1.0786	62	0.7333	19	10
43 0	0.6820	21	0.9325	55	1.0724	63	0.7314	20	0 **47**
10	0.6841	21	0.9380	55	1.0661	62	0.7294	20	50
20	0.6862	22	0.9435	55	1.0599	61	0.7274	20	40
30	0.6884	21	0.9490	55	1.0538	61	0.7254	20	30
40	0.6905	21	0.9545	56	1.0477	61	0.7234	20	20
50	0.6926	21	0.9601	56	1.0416	61	0.7214	21	10
44 0	0.6947	20	0.9657	56	1.0355	60	0.7193	20	0 **46**
10	0.6967	21	0.9713	57	1.0295	60	0.7173	20	50
20	0.6988	21	0.9770	57	1.0235	59	0.7153	20	40
30	0.7009	21	0.9827	57	1.0176	59	0.7133	21	30
40	0.7030	20	0.9884	58	1.0117	59	0.7112	20	20
50	0.7050	21	0.9942	58	1.0058	58	0.7092	21	10
45 0	0.7071		1.0000		1.0000		0.7071		0 **45**
	Cos.	d.	Cotg.	d.	Tang.	d.	Sin.	d.	′ °

PP	57	55	54		53	51	22		21	20	19
.1	5.7	5.5	5.4	.1	5.3	5.1	2.2	.1	2.1	2.0	1.9
.2	11.4	11.0	10.8	.2	10.6	10.2	4.4	.2	4.2	4.0	3.8
.3	17.1	16.5	16.2	.3	15.9	15.3	6.6	.3	6.3	6.0	5.7
.4	22.8	22.0	21.6	.4	21.2	20.4	8.8	.4	8.4	8.0	7.6
.5	28.5	27.5	27.0	.5	26.5	25.5	11.0	.5	10.5	10.0	9.5
.6	34.2	33.0	32.4	.6	31.8	30.6	13.2	.6	12.6	12.0	11.4
.7	39.9	38.5	37.8	.7	37.1	35.7	15.4	.7	14.7	14.0	13.3
.8	45.6	44.0	43.2	.8	42.4	40.8	17.6	.8	16.8	16.0	15.2
.9	51.3	49.5	48.6	.9	47.7	45.9	19.8	.9	18.9	18.0	17.1

TABLE VIII.

SQUARES AND SQUARE ROOTS OF NUMBERS.

SQUARES OF INTEGERS FROM 10 TO 100.

N	0	1	2	3	4	5	6	7	8	9
10	100	121	144	169	196	225	256	289	324	361
20	400	441	484	529	576	625	676	729	784	841
30	900	961	1024	1089	1156	1225	1296	1369	1444	1521
40	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
50	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
60	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
70	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
80	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
90	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801

SQUARE ROOTS OF NUMBERS FROM 0 TO 10, AT INTERVALS OF .1.

N	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0	.316	.447	.548	.632	.707	.775	.837	.894	.949
1	1.000	1.049	1.095	1.140	1.183	1.225	1.265	1.304	1.342	1.378
2	1.414	1.449	1.483	1.517	1.549	1.581	1.612	1.643	1.673	1.703
3	1.732	1.761	1.789	1.817	1.844	1.871	1.897	1.924	1.949	1.975
4	2.000	2.025	2.049	2.074	2.098	2.121	2.145	2.168	2.191	2.214
5	2.236	2.258	2.280	2.302	2.324	2.345	2.366	2.387	2.408	2.429
6	2.449	2.470	2.490	2.510	2.530	2.550	2.569	2.588	2.608	2.627
7	2.646	2.665	2.683	2.702	2.720	2.739	2.757	2.775	2.793	2.811
8	2.828	2.846	2.864	2.881	2.898	2.915	2.933	2.950	2.966	2.983
9	3.000	3.017	3.033	3.050	3.066	3.082	3.098	3.114	3.130	3.146

SQUARE ROOTS OF INTEGERS FROM 10 TO 100.

N	0	1	2	3	4	5	6	7	8	9
10	3.162	3.317	3.464	3.606	3.742	3.873	4.000	4.123	4.243	4.359
20	4.472	4.583	4.690	4.796	4.899	5.000	5.099	5.196	5.292	5.385
30	5.477	5.568	5.657	5.745	5.831	5.916	6.000	6.083	6.164	6.245
40	6.325	6.403	6.481	6.557	6.633	6.708	6.782	6.856	6.928	7.000
50	7.071	7.141	7.211	7.280	7.348	7.416	7.483	7.550	7.616	7.681
60	7.746	7.810	7.874	7.937	8.000	8.062	8.124	8.185	8.246	8.307
70	8.367	8.426	8.485	8.544	8.602	8.660	8.718	8.775	8.832	8.888
80	8.944	9.000	9.055	9.110	9.165	9.220	9.274	9.327	9.381	9.434
90	9.487	9.539	9.592	9.644	9.695	9.747	9.798	9.849	9.899	9.950

TABLE IX.

THE HYPERBOLIC AND EXPONENTIAL FUNCTIONS OF NUMBERS FROM 0 TO 2.5, AT INTERVALS OF .1.

x	cosh x	sinh x	tanh x	e^x	e^{-x}
0	1.000	0	0	1.000	1.000
.1	1.005	.100	.100	1.105	.905
.2	1.020	.201	.197	1.221	.819
.3	1.045	.305	.291	1.350	.741
.4	1.081	.411	.380	1.492	.670
.5	1.128	.521	.462	1.649	.607
.6	1.185	.637	.537	1.822	.549
.7	1.255	.759	.604	2.014	.497
.8	1.337	.888	.664	2.226	.449
.9	1.433	1.027	.716	2.460	.407
1.0	1.543	1.175	.762	2.718	.368
1.1	1.669	1.336	.801	3.004	.333
1.2	1.811	1.509	.834	3.320	.301
1.3	1.971	1.698	.862	3.669	.273
1.4	2.151	1.904	.885	4.055	.247
1.5	2.352	2.129	.905	4.482	.223
1.6	2.577	2.376	.922	4.953	.202
1.7	2.828	2.646	.935	5.474	.183
1.8	3.107	2.942	.947	6.050	.165
1.9	3.418	3.268	.956	6.686	.150
2.0	3.762	3.627	.964	7.389	.135
2.1	4.144	4.022	.970	8.166	.122
2.2	4.568	4.457	.976	9.025	.111
2.3	5.037	4.937	.980	9.974	.100
2.4	5.557	5.466	.984	11.023	.091
2.5	6.132	6.050	.987	12.182	.082

TABLE X

CONSTANTS

MEASURES AND WEIGHTS AND OTHER CONSTANTS

MEASURES AND WEIGHTS

English Measures

LENGTH

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
16½ feet = 1 rod (rd.)
5280 feet = 1 mile (mi.)
6080.2 feet = 1 nautical mile
5½ yards = 1 rod
4 rods = 1 chain (ch.)

1 inch = 2.54 centimeters
1 yard = 0.9144 meter
1 mile = 1.6093 kilometers

AREA

144 sq. inches = 1 sq. foot
9 sq. feet = 1 sq. yard
30¼ sq. yards = 1 sq. rod
160 sq. rods = 1 acre
43560 sq. feet = 1 acre
640 acres = 1 sq. mile

1 sq. inch = 6.4516 sq. centimeters
1 sq. foot = 0.0929 sq. meter
1 sq. yard = 0.8361 sq. meter
1 acre = 0.4047 hectare

VOLUME

1728 cu. inches = 1 cu. foot
27 cu. feet = 1 cu. yard
128 cu. feet = 1 cord (cd.)

1 cu. inch = 16.387 cu. centimeters
1 cu. foot = 0.0283 cu. meter
1 cu. yard = 0.7646 cu. meter
1 cord = 3.625 steres

CAPACITY

1 liq. gal. = 3.785 liters = 231 cu. in.
1 dry gal. = 4.405 liters = 268.8 cu. in.
1 bushel = 0.3524 hkl. = 2150.42 cu. in.

AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.)
100 lbs. = 1 hundredweight (cwt.)
20 hundredweight = 1 ton (T.)

1 pound = 0.4536 kilo. = 7000 grains
1 ton = 0.9072 tonneau (t.)

TROY WEIGHT

1 pound = 5760 grains = 12 ounces

Metric Measures

LENGTH

10 millimeters (mm.) = 1 centimeter (cm.)
10 centimeters = 1 decimeter (dm.)
10 decimeters = 1 meter (m.)
10 meters = 1 dekameter (dkm.)
10 dekameters = 1 hektometer (hkm.)
10 hektometers = 1 kilometer (km.)

1 meter = 39.37 inches = 3.2808 feet
1 kilometer = 0.6214 mile

AREA

100 sq. millimeters = 1 sq. centimeter
100 sq. centimeters = 1 sq. decimeter
100 sq. decimeters = 1 sq. meter = 1 centare (ca.)
100 sq. meters = 1 are (a.)
100 ares = 1 hektare (hka.)

1 sq. centimeter = 0.1550 sq. inch
1 sq. meter = 1.196 sq. yards = 10.764 sq. feet
1 are = 1076.46 sq. feet
1 hektare = 2.471 acres

VOLUME

1000 cu. millimeters = 1 cu. centimeter
1000 cu. centimeters = 1 cu. decimeter
1000 cu. decimeters = 1 cu. meter = 1 stere (st.)

1 cu. centimeter = 0.061 cu. inch
1 cu. meter = 35.314 cu. feet = 1.308 cu. yards
1 stere = 0.2759 cord

CAPACITY

100 centiliters (cl.) = 1 liter (l.)
100 liters = 1 hektoliter (hkl.)

1 liter = 1.0567 liq. qts. = 1 cu. dcm.

METRIC WEIGHT

1000 grams (gm.) = 1 kilogram (kilo.)
1000 kilograms = 1 tonneau (t.)

1 gram = 15.432 grains
1 kilogram = 2.2046 pounds
1 tonneau = 1.1023 tons

ARCS

[illegible]

CONSTANTS

[illegible]

p. 15, 3 Given sec $A = m$, find the other functions
p. 16, 15 " $\csc 60 = 2/3\sqrt{3}$ "
p. 16, 16 " $\tan 15 = 2 - \sqrt{3}$ "
p. 16, 20 " $\tan 90 = \infty$
p. 16, 22 Express the values of all other functions in terms of cos A
p. 16, 23 Express the values of all other functions in terms of tan A.

p. 90, Ex 28, take 125.32 in place of 115.3
memorize 37 formulae.

p 91 Ex. 32
p. 91 Ex De Moivres.

89 Ex. 15 Page 89, 1) Ex
-84 odd on top, p 85, 12 - 24
Page 84, even, Solution of Δ, complex., cos 6x, sin 6x
top
Curves - p. 63-64, 66, Page 89 Ex 14, 13.
Formula 32 - 41; Page 40, last six. Page 87 E
Formulae 1 - 32 - Page 89 Ex, 17. Page 40, 1-1

11-14 - Right Δ, Imag. + De Moivre's theorem
Given $\cos x = m$, and x in ~~I~~ II quadrant, find all
area of reg. poly. of 77 sides is 3/4 sq. foot
find the perimeter, the radius and the apothem

$\cos 7x$, $\sin 7x$
De Moivre's Theorem
Page 90 Ex. 28

+ memorize p. 18